# DBT-STAR COLLEGE SCHEME MULTIDISCIPLINARY NATIONAL SEMINAR ON BIOMATHEMATICS 



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SRI SARADA COLLEGE FOR WOMEN (AUTONOMOUS)
Reaccredited with "B++" Grade by NAAC (Affiliated to Periyar University)

Salem-636 016, India

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## PREFACE

Biomathematics is the use of mathematical models to help understand phenomena in biology. The techniques of Biomathematics are widely prevalent in areas such as cellular neurobiology, epidemic modeling, and population genetics.

While Biomathematics may sound like a narrow discipline, in fact it encompasses all of biology and virtually all of the mathematical sciences, including statistics, operations research, and scientific computing.

The book entitled, "DBT-STAR College Scheme Sponsored Multidisciplinary National Seminar on "Biomathematics", is a compilation of various articles prevailing in all areas of arts, science and engineering primarily focusing on the importance of Mathematics in solving certain biological problems to a greater extent. Also various results governing graph theory, fluid flows and the efficacy of Vedas in modern science practices are also incorporated.

We express our deep sense of gratitude to the DBT-STAR College Scheme, our benevolent management, our Dynamic Madam Principal Dr. R. Uma Rani M.C.A., M.Phil., Ph.D., and Controller of Examinations Dr. S. Geetha, M.Sc., M.Phil., Ph.D., and DBT-STAR College Scheme Coordinator Dr. P. Umamaheswari, Head \& Associate Professor of Botany for their support and valuable suggestions in bringing out the seminar proceedings successfully.

## Editors

Dr. M. K. Uma
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# PREDICTION OF AIR POLLUTION IN DELHI BY USING DIFFERENCE EQUATIONS 

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#### Abstract

The project entitled "PREDICTION OF AIR POLLUTION IN DELHI BY USING DIFFERENCE EQUATIONS" gives us a detailed study on air quality in Delhi region. In this project, we had analysed the data from 2015-2020 [5] and the prediction of future air quality has been carried out by analysing the pollutants using difference equations.


Keywords Air pollution, Air pollutants, Particulate matter, Predictive analysis

## Introduction

Pure air is a mixture of various gases such as nitrogen, oxygen, argon, carbon dioxide, and small amount of other gases in a fixed proportion. If the composition of air alters by any means; it is known as air pollution, which can lead to effects on human health, environment, and other living creatures. According to The Air (Prevention and Control of Pollution) Act, 1981, "air pollution is the presence of any solid, liquid, or gaseous substance in the atmosphere in such concentration as may be or tend to be injurious to human beings or other living creatures or plants or property or environment. The enormity of air pollution has always been a matter due to rapid development and urbanisation over a long period. The increasing level of pollutants in ambient air in the recent years has deteriorated the air quality of Delhi at an alarming rate. This brought us to focus our study in air quality in Delhi region. In this project, the prediction of future air quality has been carried out by analyzing the pollutants using difference equations.

## Review of Literature

According to world population review, Delhi the National Capital Territory (NCT) of India is the densely populated metropolitan city with a large influx of population from other states of India. As per the last census carried out in 2011, population of Delhi was 16.7 million [3]. In recent years rapid industrialisation and urbanisation posed detrimental effect on environment. Problem of air pollution is increasingly getting more serious. Increasing levels of pollutants in air is causing extreme health disorder. [2] discussed about how the air pollution directly affects a population of millions who are suffering from shortness of breath, eye irritation , chronic respiratory disorders, pneumonia, acute asthma etc. In [1] the authors had analysed the data from 2011-2015 and detailed analysis of air pollutants from 2009-2017 had been proposed along with the
critical observation of 2016-2017 air pollutants trend in Delhi. Motivating by the above, we had analysed the data from 2015-2020[5] and using difference equations to predict the future trends of air pollutants like nitrogen dioxide $\left(\mathrm{NO}_{2}\right)$, sulphur dioxide $\left(\mathrm{SO}_{2}\right)$, suspended particulate matter (PM).

## Sources of Air Pollution [4]

Natural sources: Natural sources of air pollution include volcanic activity, dust, sea-salt, fires, lightening, soil outgassing etc. Anthropogenic sources: These sources include stationary point sources (e.g. emission from industries), mobile sources (e.g. vehicular emission, marine vessels, airplanes etc.), waste disposal landfills, open burning etc.

The substances which are responsible for causing air pollution are called as air pollutants.Those pollutants which are emitted directly from any emission source in the atmosphere are termed as primary air pollutants, e.g. sulphur dioxide $\left(\mathrm{SO}_{2}\right)$, carbon monoxide $(\mathrm{CO})$, lead $(\mathrm{Pb})$, ammonia $\left(\mathrm{NH}_{3}\right)$, etc. Secondary pollutants are formed by the reactions between primary air pollutants and normal atmospheric constituents. In some of the cases, these pollutants are formed by utilizing the solar energy, e.g. ozone, peroxyacetylnitrate (PAN), smog, etc.Pollutants which are in the form of gas are termed as gaseous air pollutants.e.g. Sulphur Dioxide $\left(\mathrm{SO}_{2}\right)$, Nitrogen Dioxide $\left(\mathrm{NO}_{2}\right)$, Ozone $\left(\mathrm{O}_{3}\right)$, Carbon Monoxide (CO), etc.

Particulate air pollutants or particulate matter (PM) can be defined as the microscopic solid or liquid matter suspended in the earth's atmosphere. Total suspended particulate matter (TSPM) is the concentration of particulate matter which is obtained when a high volume bulk sampling is done on a filter substrate. It includes particles of all sizes. $\mathrm{PM}_{10}$ are the particles less than $10 \mu \mathrm{~m}$ in diameter. $\mathrm{PM}_{2.5}$ are the particles less than $2.5 \mu \mathrm{~m}$ in diameter. $\mathrm{PM}_{1.0}$ are the particles less than $1 \mu \mathrm{~m}$ in diameter.Particles which lie between $10 \mu \mathrm{~m}$ to $2.5 \mu \mathrm{~m}$ are termed as 'coarse particles' whereas particles with diameter less than $2.5 \mu \mathrm{~m}$ are called as 'fine particles'. Fine particles also include ultrafine particles of size less than $0.1 \mu \mathrm{~m}\left(\mathrm{PM}_{0.1}\right)$.

## Study Area

Delhi has been considered for the study whose geographical regions are shown. Delhi is one of the most polluted cities in the world according to World Health Organization (WHO). The level of the airborne particulate matter $\mathrm{PM}_{2.5}$ is very high in Delhi. Also, $\mathrm{PM}_{10}$ level is the highest among the 11 mega cities of the world having more than 14 million habitants.PM is considered to
be most harmful pollutants to health. Of late, the air pollution status in Delhi has undergone many changes in terms of the levels of pollutants and the control measures taken to reduce them.


## Analysis

The main objective of this project is to understand the link between air pollution levels and chemical composition and to investigate the effects of air pollution on the distribution of related health impacts and socio-economics. According to the report that Delhi has exceeded the maximum $\mathrm{PM}_{2.5}, \mathrm{NO}_{2}$ and $\mathrm{SO}_{2}$ in the upcoming years (2021-2030). Vehicular emissions and industrial activities were found to be associated with indoor as well as outdoor air pollution in Delhi.

Many studies suggest the quality of air has been significantly improving in the last years in the majority of the world regions. However, air quality still creates a significant problem especially in some densely populated urban areas and during certain weather conditions. Several reports observe the serious of the air pollution on the people's health and many analysis and models have been tested to reduce the problem. In this regard, difference equation is particularly used in this project to predict the future level of pollutants in Delhi.
Data: [5]

| Concentration <br> level/years | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{S O}_{2}$ | 5 | 7 | 7 | 15 | 10 | 12 |
| $\mathbf{N O}_{2}$ | 65 | 66 | 68 | 44 | 59 | 52 |
| $\mathbf{P M}_{2.5}$ | 392 | 289 | 349 | 420 | 458 | 470 |

## Results

The Air pollution in Delhi was determined by the average percent change. The initial concentration of $\mathrm{NO}_{2}, \mathrm{SO}_{2}, \mathrm{PM}_{2.5}$ in the year 2020 was 52,12 and $470\left(\mu / \mathrm{m}^{3}\right)$. The estimated growth rate in the concentration of $\mathrm{NO}_{2}, \mathrm{SO}_{2}$ and $\mathrm{PM}_{2.5}$ in 2015 - 2020 was $15.4074 \%, 26.38 \%$ and $11.236 \%$ respectively.
i) If this trend continues then estimate the concentration of $\mathrm{NO}_{2}$ in the year 2021, 2025 and 2030
ii) Predict the concentration of $\mathrm{SO}_{2}$ in the year 2021, 2025 and 2030
iii) If this growth rate continues, then estimate the concentration of $\mathrm{PM}_{2.5}$ in 2021, 2025 and 2030

## Solution:

i) The initial concentration of $\mathrm{NO}_{2}$ is $\mathrm{P}_{0}=52 \mu / \mathrm{m}^{3}$

The growth rate is $\mathrm{r}=15.4074 \%$ (or) 0.154074
$\mathrm{P}_{\mathrm{n}}=$ Concentration level at the end of the time period
The concentration dynamics can be modelled by the difference equation

$$
\begin{align*}
& \mathrm{P}_{\mathrm{n}+1}=\mathrm{P}_{\mathrm{n}}+\mathrm{r} \mathrm{P}_{\mathrm{n}} \\
& \mathrm{P}_{\mathrm{n}+1}=(1+0.154074) \mathrm{P}_{\mathrm{n}} \\
& \mathrm{P}_{\mathrm{n}+1}=(1.154074) \mathrm{P}_{\mathrm{n}} \tag{1}
\end{align*}
$$

This the first order linear homogenous difference equation with constant coefficient. The analytical solution of the difference equation is

$$
\begin{align*}
& P_{n}=(1.154074)^{n} P_{0} \\
& P_{n}=(1.154074)^{n} 52 \tag{2}
\end{align*}
$$

We can estimate the concentration of $\mathrm{NO}_{2}$ by using equation (2)
In the year 2021, $\mathrm{n}=1$

$$
\begin{aligned}
& \mathrm{P}_{1}=(1.154074)^{1} 52=60.0118 \\
& \mathrm{P}_{1} \approx 60 \mu / \mathrm{m}^{3}
\end{aligned}
$$

In the year $2025, \mathrm{n}=5$

$$
\begin{aligned}
& \mathrm{P}_{5}=(1.154074)^{5} 52=106.456 \\
& \mathrm{P}_{5} \approx 106 \mu / \mathrm{m}^{3}
\end{aligned}
$$

In the year 2030, $\mathrm{n}=10$

$$
\begin{aligned}
\mathrm{P}_{10} & =(1.154074)^{10} 52=217.94148 \\
\mathrm{P}_{10} & \approx 218 \mu / \mathrm{m}^{3}
\end{aligned}
$$



The initial concentration of $\mathrm{SO}_{2}$ is $\mathrm{P}_{0}=12 \mu / \mathrm{m}^{3}$
The growth rate is $\mathrm{r}=26.38 \%$ (or) 0.2638
$\mathrm{P}_{\mathrm{n}}=$ Concentration level at the end of the time period
The concentration dynamics can be modelled by the difference equation

$$
\begin{align*}
& \mathrm{P}_{\mathrm{n}+1}=\mathrm{P}_{\mathrm{n}}+\mathrm{r} \mathrm{P}_{\mathrm{n}}=1+(0.2638) \mathrm{P}_{\mathrm{n}} \\
& \mathrm{P}_{\mathrm{n}+1}=(1.2638) \mathrm{P}_{\mathrm{n}} \tag{3}
\end{align*}
$$

This the first order linear homogenous difference equation with constant coefficient. The analytical solution of the difference equation is

$$
\begin{align*}
& \mathrm{P}_{\mathrm{n}}=(1.2638)^{\mathrm{n}} \mathrm{P}_{0} \\
& \mathrm{P}_{\mathrm{n}}=(1.2638)^{\mathrm{n}} 12 \tag{4}
\end{align*}
$$

We can estimate the concentration of $\mathrm{SO}_{2}$ by using equation (4)
In the year 2021, $\mathrm{n}=1$

$$
\begin{aligned}
& \mathrm{P}_{1}=(1.2638)^{1} 12=15.1656 \\
& \mathrm{P}_{1} \approx 15 \mu / \mathrm{m}^{3}
\end{aligned}
$$

In the year 2025, $\mathrm{n}=5$

$$
\begin{aligned}
& P_{5}=(1.2638)^{5} 12=38.6877 \\
& P_{5} \approx 39 \mu / \mathrm{m}^{3}
\end{aligned}
$$

In the year 2030, $\mathrm{n}=10$

$$
\begin{aligned}
& \mathrm{P}_{10}=(1.2638)^{10} 12=124.728 \\
& \mathrm{P}_{10} \approx 125 \mu / \mathrm{m}^{3}
\end{aligned}
$$

## Chart of a concentration level of $\mathrm{SO}_{2}$ after 10 years

## 

The initial concentration of $\mathrm{PM}_{2.5}$ is $\mathrm{P}_{0}=470 \mu / \mathrm{m}^{3}$
The growth rate is $\mathrm{r}=11.236 \%$ (or) 0.11236
$\mathrm{P}_{\mathrm{n}}=$ Concentration level at the end of the time period
The concentration dynamics can be modelled by the difference equation

$$
\begin{align*}
& P_{n+1}=P_{n}+r P_{n} \\
& P_{n+1}=1+(0.11236) P_{n} \\
& P_{n+1}=(1.11236) P_{n} \tag{5}
\end{align*}
$$

This the first order linear homogenous difference equation with constant coefficient. The analytical solution of the difference equation is

$$
\begin{align*}
& P_{n}=(1.11236)^{n} P_{0} \\
& P_{n}=(1.11236)^{n} 470 \tag{6}
\end{align*}
$$

We can estimate the concentration of $\mathrm{PM}_{2.5}$ by using equation (6)
In the year 2021, $\mathrm{n}=1$

$$
\begin{aligned}
& \mathrm{P}_{1}=(1.11236)^{1} 470=522.809 \\
& \mathrm{P}_{1} \approx 523 \mu / \mathrm{m}^{3}
\end{aligned}
$$

In the year 2025, $\mathrm{n}=5$

$$
\begin{aligned}
& P_{5}=(1.11236)^{5} 470=800.432 \\
& P_{5} \approx 800 \mu / \mathrm{m}^{3}
\end{aligned}
$$

In the year 2030, $\mathrm{n}=10$

$$
\begin{aligned}
& \mathrm{P}_{10}=(1.11236)^{10} 470=1363.1746 \\
& \mathrm{P}_{10} \approx 1363 \mu / \mathrm{m}^{3}
\end{aligned}
$$



From the difference equation, we have predicted the concentration of $\mathrm{No}_{2}, \mathrm{So}_{2}$ and $\mathrm{PM}_{2.5}$ in the upcoming years (i.e) for 2021, 2025 and 2030. Using the above method, the estimated concentration of $\mathrm{No}_{2}, \mathrm{So}_{2}$ and $\mathrm{PM}_{2.5}$

| YEARS | $\mathrm{No}_{2}\left(\mu / \mathrm{m}^{3}\right)$ | $\mathrm{So}_{2}\left(\mu / \mathrm{m}^{3}\right)$ | $\mathrm{PM}_{2.5}\left(\mu / \mathrm{m}^{3}\right)$ |
| :--- | :--- | :--- | :--- |
| 2021 | 60 | 15 | 523 |
| 2025 | 106 | 39 | 800 |
| 2030 | 218 | 125 | 1363 |

## Conclusion

The Central and state governments has taken several steps to reduce the level of air pollution in Delhi. Comparing to the given data the level of $\mathrm{SO}_{2}, \mathrm{NO}_{2}, \mathrm{PM}_{2.5}$ constantly increase in the upcoming years. It is difficult to hold any one particulate form responsible for maximum risk to health. However the air pollution status in Delhi has undergone many changes in terms of the levels of pollutants and the control measures taken to reduces them. The already existing measures need to be strengthened and magnified to a larger scale. The governmental efforts alone are not enough. Participation of the community is crucial in order to make Delhi NCR an air pollution free place.

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# MAXIMISING THE GROWTH RATE OF MEDICINAL PLANTS BY USING SIMPLEX METHOD 

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#### Abstract

In this paper a real world problem on maximising the growth of medicinal plants is formulated. The quantity of seeds is analyzed by following the steps of simplex method and the LPP is solved using TORA Operations Research Software. Few suggestions have also be given for maximizing the growth of Medicinal plants.


## INTRODUCTION

Medicinal plants are considered as a rich resource of ingredients which can be used in drug development pharmacopeial, non- pharmacopeial or synthetic drugs. Apart from that, these plants play a critical role in the development of human cultures around the entire world. Moreover, some plants are considered as important source of nutrition and because of that they are recommended for their therapeutic values. Some plants and their derivatives are considered as important source for active ingredients which are used in aspirin and toothpaste etc.

Medicinal plants such as Aloevera, Tulsi, Neem, Turmeric and Ginger cure several common ailments. These are considered as home remedies in many parts of the country. It is a known fact that lots of consumers are using Basil (Tulsi) for making medicines, black tea and for other activities in their day-to-day life.

The medicinal plants find application in pharmaceutical, cosmetics, agricultural and food industry. The use of the medicinal plants for curing diseases has been documented in history of all civilizations. Man in the pre-historic era was not aware about the health hazards associated with irrational therapy. With the onset of research in medicine, it was concluded that plants contain active principles which are responsible for curative action of the herbs.

In recent years, there have been significant amounts of work involving applications of various mathematical modeling and computational techniques to predict medicinal properties of plants, and thus to provide information based on the selection of plant materials for further studies aiming at potential drug discovery and development. Maximizing medicinal plants is dependent on a paradigm shift globally involving a reduction on medicinal plants and the potential value they hold for global markets as well
as the implementation of sustainable practice. By addressing the key steps required for this paradigm shift, the value of medicinal plants will be fully realized.

Research works on the distribution of Medicinal plants have been carried out by many Mathematicians and Statisticians like Kelly Kindscher, Steve Corbett and Katrina McClure[1], Iris Mateescu, Laura Paun, Stefan Popescu, George Roata, Manuela Sidoroff[2], Shaukat Iqbal Malik, Anum Munir, Ghulam Mujtaba Shah, Azhar Mehmood[3].

In order to develop a country, the production of medicinal plants plays a vital role in health and economy of a country. But there is not that much awareness among the common people about the importance of medicinal plants. The aim of this paper is to identify the quantity of seeds to be planted per hectare to maximize the yield of medicinal plants using Simplex Method.

## FORMULATION OF A REAL-WORLD PROBLEM

The yields of three major medicinal plants Aloe Vera, Tulsi, Turmeric are 15 tons/hectare, 10 ton/hectare, 8 tons/hectare. The farm yard manure (FYM) needed for the three medicinal plants are 10 tons/hectare, 15 tons/hectare, and 10 tons/hectare. [The general need of farm yard manure of medicinal plants is 15 tons/hectare.] The pH measures needed for the growth of three medicinal plants are 8.5, 4.3, 7 [The general pH level for the growth of medicinal plants is 6]. The amount of nitrogen rate in the manure for the growth of three medicinal plants is $40 \mathrm{~kg} /$ hectare, $120 \mathrm{~kg} /$ hectare, 70 kg /hectare [The general amount of nitrogen in medicinal plants manure is 240 $\mathrm{kg} /$ hectare.]. The amount of phosphorus rate in the manure for the growth of the three medicinal plants is $75 \mathrm{~kg} /$ hectare, $65 \mathrm{~kg} /$ hectare, $40 \mathrm{~kg} /$ hectare [The general rate of phosphorus in medicinal plants manure is $110 \mathrm{~kg} /$ hectare.] The amount of potassium rate in the manure for the growth of three medicinal plants is $70 \mathrm{~kg} /$ hectare, 60 $\mathrm{kg} /$ hectare, $37 \mathrm{~kg} /$ hectare [The general rate of potassium in medicinal plants manures $110 \mathrm{~kg} /$ hectare.] These are the main parameters considered for the production of medicinal plants. Formulate a LPP to maximize the yield of the medicinal plants mentioned [Aloe Vera, Tulsi, Turmeric]

SOLUTION

| Medicinal <br> Plants | FYM <br> tons/hectare | Ph | $\mathbf{N}$ <br> $\mathbf{k g} /$ hectare | $\mathbf{P}$ <br> $\mathbf{k g} /$ hectare | $\mathbf{K}$ <br> $\mathbf{k g} /$ hectare |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Aloe Vera | 10 | 8.5 | 40 | 75 | 70 |
| Tulsi | 15 | 4.3 | 120 | 65 | 60 |


| Turmeric | 10 | 7 | 70 | 40 | 37 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| General <br> need | 15 | 6 | 240 | 110 | 100 |

## Step : 1

Our key decision is to determine the quantity of seeds in kg to be planted per hectare.

## Step : 2

The decision variables are denoted by $x_{1}, x_{2}$, and $x_{3}$ which denotes the quantity of seeds in kg to be planted per hectare.

## Step : 3

Since the quantity of seeds cannot be negative, we have $x_{1} \geq 0, x_{2} \geq 0$ and $x_{3} \geq 0$

## Step: 4

The medicinal plant Aloe Vera needs 10 tons/hectare of farm yard manure for its growth. So, the need of farm yard manure for $x_{1}$ number of plants is $10 x_{1}$. In the same way, we have the following constraints.

$$
\begin{gathered}
10 x_{1}+15 x_{2}+10 x_{3} \leq 15 \\
8.5 x_{1}+4.3 x_{2}+7 x_{3} \leq 6 \\
40 x_{1}+120 x_{2}+70 x_{3} \leq 240 \\
75 x_{1}+65 x_{2}+40 x_{3} \leq 110 \\
70 x_{1}+60 x_{2}+37 x_{3} \leq 10
\end{gathered}
$$

## Step : 5

The main objective is to maximize the yield of the three medicinal plants.

## FORMULATION OF LPP

$$
\text { Maximize } z=15 x_{1}+10 x_{2}+8 x_{3}
$$

Subject to the constraints:

$$
\begin{gathered}
10 x_{1}+15 x_{2}+10 x_{3} \leq 15 \\
8.5 x_{1}+4.3 x_{2}+7 x_{3} \leq 6 \\
40 x_{1}+120 x_{2}+70 x_{3} \leq 240 \\
75 x_{1}+65 x_{2}+40 x_{3} \leq 110 \\
70 x_{1}+60 x_{2}+37 x_{3} \leq 10
\end{gathered}
$$

For all $x_{1} \geq 0, x_{2} \geq 0, x_{3} \geq 0$.
The solution is obtained with the help of TORA software which produces the solution for the above maximization problem in a short span of time.

## LINEAR PROGRAM - ORIGINAL DATA

Title: LPP

|  | x1 | x2 | x3 |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Maximize | 15.00 | 10.00 | 8.00 |  |  |
| Subject to |  |  |  | $<$ | 15.00 |
| (1) | 10.00 | 15.00 | 10.00 | 6.00 |  |
| (2) | 8.50 | 4.50 | 7.00 | $<=$ | 240.00 |
| (3) | 40.00 | 120.00 | 70.00 | $<=$ | 110.00 |
| (4) | 75.00 | 65.00 | 40.00 | $<=$ | 100.00 |
| (5) | 70.00 | 60.00 | 37.00 |  |  |
|  |  |  |  |  |  |
| Lower Bound | 0.00 | 0.00 | 0.00 |  |  |
| Upper Bound infinity | infinity | infinity |  |  |  |
| Unrestr'd $(\mathbf{y} / \mathbf{n}) ?$ | n | n | n |  |  |

## SIMPLEX TABLEAUS - (Starting All-Slack Method)

## Title: LPP



| Iteration 3 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Basic | $\times 1$ | $\times 2$ | x3 | $5 \times 4$ | $5 \times 5$ | $5 \times 6$ |
| $z$ (max) | 0.00 | 0.00 | 4.73 | 0.21 | 1.52 | 0.00 |
| $\times 2$ | 0.00 | 1.00 | 0.18 | 0.10 | -0.12 | 0.00 |
| $\times 1$ | 1.00 | 0.00 | 0.73 | -0.05 | 0.18 | 0.00 |
| $5 \times 6$ | 0.00 | 0.00 | 19.09 | -10.18 | 7.27 | 1.00 |
| sx7 | 0.00 | 0.00 | -28.36 | -2.61 | -5.78 | 0.00 |
| sx8 | 0.00 | 0.00 | -24.82 | -2.36 | -5.45 | 0.00 |
| Lower Bound | 0.00 | 0.00 | 0.00 |  |  |  |
| Upper Bound | infinity | infinity | infinity |  |  |  |
| Unrestr'd ( y in)? | $n$ | n | n |  |  |  |
| Basic | $5 \times 7$ | 5xB | Solution |  |  |  |
| $z$ (max) | 0.00 | 0.00 | 12.27 |  |  |  |
| $\times 2$ | 0.00 | 0.00 | 0.82 |  |  |  |
| $\times 1$ | 0.00 | 0.00 | 0.27 |  |  |  |
| $5 \times 6$ | 0.00 | 0.00 | 130.81 |  |  |  |
| 5x7 | 1.00 | 0.00 | 38.36 |  |  |  |
| $5 \times 8$ | 0.00 | 1.00 | 31.82 |  |  |  |

## ANSWER:

$\mathrm{Z}=12.27$ tons $/$ hectare
$x_{1}=0.27$ tons $/$ hectare $=0.27 \times 1000 \mathrm{~kg} /$ hectare $=270 \mathrm{~kg} /$ hectare
$x_{2}=0.82$ tons $/$ hectare $=0.82 \times 1000 \mathrm{~kg} /$ hectare $=820 \mathrm{~kg} /$ hectare
$x_{3}=0$.

## INTERPRETATION:

Thus if we plant 270 kg of Aloe Vera seeds per hectare of land and 820 kg of Tulsi seeds per hectare of land, we get a maximized yield.

## SUGGESTIONS FOR IMPROVING MEDICINAL PLANTS SECTOR

The present worldwide interest in plant-based medicines of Indian origin needs to be harnessed by reframing a clear policy for the promotion of commercial cultivation, research and development, and for the increase in exports of medicinal plants. For the development of the medicinal plant sector, there is a need to develop the coordinated efforts at each stage (e.g. research, cultivation, collection, storage, processing, manufacturing and marketing), which would be supported by an appropriate policy framework.

## FARMING STRATEGIES

Selection of medicinal plant species for cultivation is an initial important step for the development of the medicinal plants sector. Economic feasibility is the major rational for a decision to bring medicinal plant species into cultivation. Apart from the priority species selected by the Planning Commission and the NMPB, the rare species banned for collection from the wild should also be taken on a priority basis for cultivation because a majority of such species are very expensive, have high demand and low supply.

Cultivation may not be economical if a medicinal plant species is abundant in the wild and available easily. A large variation in climatic and soil conditions in India sustain a variety of medicinal plant species, which may be cultivated according to their niche and agro-climatic conditions.

Apart from meeting the present demand, farming may conserve the wild genetic diversity, permit better species identification, improve quality control, and permit production of uniform material, from which standardized products can be consistently obtained. The planting material therefore should be of good quality, rich in active ingredients, pest and disease- resistant and environmental tolerant. Studies conducted on
the agro- forestry of medicinal plants elsewhere suggest that since many medicinal plant species prefer to grow under forest cover, agro forestry offers a convenient strategy for their cultivation as well as conservation through:

* Integrating shade tolerant medicinal plants as lower strata species in multi strara system
* Cultivating short cycle medicinal plants as intercrops in existing stands of tree crops,
* Growing medicinal tree as shade providers and boundary markers, inter- planting medicinal plants with food crops

The medicinal plants sector can be improved if the agricultural support agencies would come forward to help strengthen the medicinal plants growers andif research institutions would help the plant growers by improving their basic knowledge about cultivation practices. Awareness and interest of farmers, supportive government policies, assured markets, profitable price levels, access to simple and appropriate agro-techniques, and availability of trained manpower are some of the key factors for successful medicinal plants cultivation.

## COMMUNITY-BASED: MEDICINAL PLANTS CULTIVATION

The Government of India has also enacted export restrictions on 29 plant species to prevent their further loss in wild. However, the sustainability of these species cannot be achieved merely by promulgating a ban on export or by fencing areas rich in medicinal plants.

The philosophy of joint forest management if applied on medicinal plants it may be useful in mitigating the various conflicts. Giving ownership of natural resources to local people and encouraging profitable uses of those resources may provide a powerful incentive for both medicinal plant conservation and sustainable economic development of the local people. Sustainable harvesting combined with cultivation can improve yields, and it may also reduce pressure in specified areas marked for biodiversity conservation. It has been shown that under cultivation and in a consolidated area the plants give a better yield and assure the supply of quality planting material to the industries.

The Medicinal Plants Conservation Area (MPCAs) may ensure the autonomous development of a rural community by enhancing the people's income. This way they can decide themselves how to use their assets and resources, for which market they wish to produce, and by which services they need to achieve their goals. It also enables them to restore resources depleted by overuse, assume the long term supply of resources, regulate
national and international trade by assuring a continuous supply of quality material, and ensure the conservation of not only the species concerned but also its associated species and ecosystems. The number of linkages in medicinal plants supply, process, drug formulation, trade, transport, and the retail industry can be used to generate employment. The promotion of community- based conservation through the establishment of MPCA and dissemination of awareness among the various stakeholders (e.g., herb cultivators, herb gatherers, herbal practitioners, and traders) about sustainable harvesting techniques and processing through training can be a step forward in the conservation of medicinal plants.

Identification of proper areas for establishing MPCA is one of the important tasks. Areas traditionally known for their medicinal plants richness, occurrence of endemic species, representative of the forest types, high density of prioritized medicinal plant species and minimum level of legal protection may be some of the deciding factors for selection of MPCAs. The number of MPCAs may be increased as per the diversity of habitats and microhabitats of the respective agro-climatic zone as such areas require focused and more attention due to rich botanical diversity.

The major objectives of establishing MPCA are

* to conserve the diversity and richness of medicinal plants in natural habitats across various agro-climatic zones
* to disseminate and strengthen the values of medicinal plants, its significance and conservation education,
* to develop long-term institutionalized mechanism for conservation of medicinal plants, population status along with identifying threatened medicinal plant species and undertaking measures for their recovery,
* to develop sharing of experiences among stakeholders of medicinal plants sector who are actively involved in their conservation, cultivation and sustainable utilization
* to facilitate linkages between the medicinal plant conservation organizations and medicinal plant user groups.

One of the major tasks of MPCA is to involve local communities and assure them for sharing of benefits accrued from the medicinal plants sector and MPCAs. This community-oriented policy is required realizing that rural and tribal communities are among the key custodians of medicinal plants. Apart from conservation aspects, many more activities may be taken up in MPCAs which includes, developing a complete set of
databases on each MPCA. On the basis of such database the strategies may be developed for future course of action in developing such MPCAs.

## CONCLUSION

As our lifestyle is now getting techno-savvy, we are moving away from nature. But we cannot escape from nature because we are part of nature. As herbs are natural products they are free from side effects, they are comparatively safe, eco-friendly and locally available. Traditionally there are lot of herbs used for the ailments related to different seasons. There is a need to promote them to save the human lives. These herbal products are the symbol of safety in contrast to the synthetic drugs that are regarded as unsafe to human being and environment. Although herbs had been priced for their medicinal, flavouring and aromatic qualities for centuries, the synthetic products of the modern age surpassed their importance, for a while. However, the blind dependence on synthetics is over and people are returning to the naturals with hope of safety and security. It's time to promote them globally.

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# MITIGATING THE AMOUNT OF DEBRIS IN THE VILLAGE OF MELTHOMBAI, SALEM DISTRICT WITH THE AID OF DIJKSTRA ALGORITHM 

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#### Abstract

This paper gives a brief survey on "Mitigating the amount of debris in the village of Melthombai, Salem district with the aid of Dijkstra Algorithm" at Melthomabai Village situated in Salem District, Tamilnadu. The suitable methods for collecting household waste are discussed in this paper. Also, an attempt is made to find the shortest path to dispose the waste of Village Melthombai by using Dijkstra's Algorithm.

\subsection*{1.1 INTRODUCTION}

Waste Management (or waste disposal) includes the activities and actions required to manage waste from its inception to its final disposal. This includes the collection, transport, treatment and disposal of waste, together with monitoring and regulation of the waste management process. Waste can be solid, liquid, or gas and each type has different methods of disposal and management. Waste management deals with all types of waste, including industrial, biological and household. In some cases, waste can pose a threat to human health. Waste is produced by human activities. For example, the waste management is intended to reduce adverse effects of waste on human health, the environment or aesthetics. Waste management practices are not uniform among countries (developed and developing nations); regions (urban and rural areas), and residential and industrial sectors can all take different. Waste management is a dynamically evolving sector within the national economy. Within the context of waste management, and from the point of view of the sustainable development of environment friendly policies[3], the issues of collection and sorting are important contributory factory[4]. The efficient of the distribution of waste bins and the cycle of collection represents other aspects of high-quality waste management[5].


### 1.2 PRELIMINARIES:

In this section the basic definitions for this project are studied.

## Definition 1.2.2[1]

A network consists of a set of nodes linked by arcs(or branches). The notation for describing a network is (N.A) where N is a set of nodes and A is the set of arcs.

## Definition 1.2.3[1]

A connected network is such that every 2 distinct nodes are linked by atleastone path.

## Definition 1.2.4[2]

A graph $G$ consists of a pair $(V(G), X(G))$ where $V(G)$ is a non-empty finite setwhose elements are called points (or) vertices and $X(G)$ is a set of unordered pairs of distinct elements of $V(G)$.

## Definition 1.2.5[1]

The elements of $\mathrm{X}(\mathrm{G})$ are called lines or edges of the graph.

## Definition 1.2.6[2]

A walk is called a trial if all its lines are distinct and is called a path if all itspoints are distinct.
Definition 1.2.7[2]
A graph is said to be connected if every pair of its points are connected.

## 2. FINDING THE SHORTEST PATH USING SHORTEST ROUTE NETWORK

In this section, the suitable methods for collecting household waste are discussed in first section then in the second section, an attempt is made to find the shortest path to dispose the waste of Village Melthombai by using Dijkstra's Algorithm.

### 2.1 STUDY AREA

The area taken for study is Melthombai, keeripatty town panchayat, Attur(T.K), Salem(D.T). Melthombai is the small village, with population of about 500 people. It is the small village in Salem District. Total geographical area ofMelthombai village is about 2 km .

### 2.2 Analysis of Dijkstra's algorithm in graph network model:

This section deals with finding the shortest route between the chosen places Melthombai to Attur in order to dispose waste at an easiest way, by using Dijkstra's Algorithm in Graph theory.

## Problem:

Let the places from Melthombai to Atturbe given the node numbers as follows:

| Activity | Distance(km) |
| :---: | :---: |
| $1-2$ | 3 |
| $1-3$ | 4 |
| $2-4$ | 2 |
| $2-5$ | 4 |
| $3-4$ | 5 |
| $3-7$ | 5 |
| $4-6$ | 4 |
| $5-8$ | 8 |
| $6-8$ | 4 |
| $6-9$ | 5 |
| $7-9$ | 6 |
| $8-9$ | 7 |

Node 1-Melthombai; Node 2-Holy mother school; Node 3-Perumal temple; Node 4-Government higher secondary School; Node 5-Keeripatty road; Node 6-Eichampatty; Node 7-Mottur; Node 8-

Maliyagarai and Node 9-Attur
By using Dijkstra's Algorithm, it is easy to find the shortest distance from a node toall other nodes.

## Solution:



## Step:1

The distance matrix summarizing the distance from the start node1 to all other node is given in Figure. In this Figure one can see the actual distance from the start node1 to all its neighbours. The distance for all other nodes from node1 is assumed as


Figure 1

| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 4 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |

## Step:2

In Figure 1 the smallest distance $(x)$ is 3 and the corresponding node $(L)$ is 2 . Since the node 2 is the neighbour of the start node, thicken they are 1-2 asshown is Figure 1.

## Step:3

Since node2 is not the required destination, let us proceed step 4.

## Iteration :2 Step:4

For node $2(\mathrm{~L})$, nodes 4 and 5 are the unselected neighbours. Then $\mathrm{X}+d_{24}=3+2=5$. where $d_{24}$ is the actual distance from node2 to node4. Since this distance is lessthan $\infty$. Update the distance to node 4 is 5 then $\mathrm{X}+d_{25}=3+4=7$. Again since this distance is less than $\infty$. Update the distance to node 5as 7. Transferthe data of node 9 to the position of node this are summarizes in Figure 2


Figure 2

| 9 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\infty$ | 4 | 5 | 7 | $\infty$ | $\infty$ | $\infty$ |

Step:5
Set $\mathrm{k}=2$, let us proceed step 2. In Figure 2 the smallest distance $(\mathrm{x})$ is 4,and the corresponding node $(\mathrm{L})$ is 3 . Since $L$ is neighbour of the start node thickness the arc 1-3. Since the node 3 is not the required destination let us proceed step:4.

## Iteration: 3 Step: 4

For node $3(\mathrm{~L})$, nodes 4 and 7 are the unselected neighbours. Then $\mathrm{X}+d_{34}=4+5=9$ because it is greater, where $d_{34}$ is the actual distance from node 3 to node 4 . Since this distance is less than $\infty$. Update the distance to node 4 is 5 .Then $\mathrm{X}+d_{37}=4+5=9$, where $d_{37}$ is the actual distance from node 3 to node 7 . Since this distance is less than $\infty$. Update the distance to node 7 is 9 .


The results of this step are summarized is Figure 3.

| 9 | 8 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\infty$ | $\infty$ | 5 | 7 | $\infty$ | 9 |

## Step:5

Set $\mathrm{K}=3$, let us proceed step:2. In Figure 3, the smallest distance $(\mathrm{X})$ is 5 and the corresponding node $(\mathrm{L})$ is 4 . Since L is not a neighbour of the start node, perform step 2.2, the iteration among the proceeding iterations in which as the distance to L from starting node is different from X is the iteration 1. The node selected in that iteration is 2 . So, thicken the arc $2-4$. Since the node 4 is not
the required destination let us proceed step: 4

## Iteration: 4Step:4

For node 4(L), nodes 2, 3 and 6. Where 2 and 3 are selected neighbors. Node 6 is the unselected neighbor. Then $X+d_{46}=5+4=9$


The results of this step are summaries as Figure 4.


## Step:5

Set $\mathrm{k}=4$ andlet us proceed step:2. In Figure 4 the smallest distance $(\mathrm{X})$ is 7 and the corresponding node $(\mathrm{L})$ is 5 . Since node 5 is not the required distination, let us proceed step:4

## Iteration :5Step:4

For node $5(\mathrm{~L})$ is 8 are the unselected neighbours. Then $X+d_{58}=7+8=15$.


The results of this step are summarizing in Figure 5.

| 9 | 8 | 7 | 6 |
| :--- | :--- | :--- | :--- |
| $\infty$ | 15 | 9 | 9 |

## Step:5

Set $\mathrm{K}=6$ and let us proceeds. In Figure 5 the smallest distance ( X ) is 9 and the corresponding node $(\mathrm{L})$ is 6 .

## Step:3

Since node 6 is not the required destination.

## Iteration:6 Step:4

For node $6(\mathrm{~L})$ is 9 , and 8 are the unselected neighbours. Then $+d_{69}=9+5=14$ and $X+$ $d_{68}=9+4=13$


The results of this step are summarized in Figure 6.

| 9 | 8 | 7 |
| :--- | :--- | :--- |
| 14 | 13 | 9 |

## Step:5

Set $\mathrm{K}=5$ and let us proceed step:2. In Figure 6 the smallest distance $(\mathrm{X})$ is 9 and the corresponding node ( L ) is 7 . Since, node 6 is not the required destination. let us proceed step:4.

## Iteration:7 Step:4

For node $7(\mathrm{~L})$ is 9 are the unselected neighbours. ThenX $+d_{79}=9+6=15>14$
Because it is greater than 14 .


The results of this step are summarized in Figure 6.

| 9 | 8 |
| :--- | :--- |
| 14 | 13 |

## Step:5

Set $\mathrm{K}=7$ and let us proceed step:2. In Figure 7 the smallest distance ( X ) is 13, and the corresponding node ( L ) is 8 . Since, node 8 is not the required destination let us proceed step:4.

## Iteration:8Step:4

For node $8(\mathrm{~L})$ is 9 are the unselected neighbours. Then $\mathrm{X}+d_{89}=13+7=20>14$. Because it is greater than 14.


The results of this step are summarized in Figure 8.


## Step:5

Set $\mathrm{k}=8$ and let us proceed step:2. In Figure 8 the smallest distance $(\mathrm{X})$ is 14 and the corresponding node $(\mathrm{L})$ is 9.Since L is not a neighbours of the start node, perform the step2.2. 2.2 the iteration among the proceeding iterations in which as the distance to L from starting node is different from X is the iteration 4.The node selected in that iteration is 6 . So, thicken the arc $6-9$.

## Step:3

Since, node 9 is not the required destination. Let us proceed step 6 .

## Step:6

Based on the guidelines of this step, the shortest path is 1-2-4-6-9 and thecorresponding node is node is 14 .


Hence the shortest route is Melthombai-Holy mother school-Government higher secondary school-Eichampatty-Attur.

## Suggestions

People in doing travel activity have an impact on high economic growth. This can cause the level of congestion to increase due to the wrong route selection resulting in the built up of vehicles on certain roads resulting in an in effective travel.

- Many benefits can be obtained when knowing the shortest or fastest route in a travel route.i.e, it can save travel time, power, less fuel consumption and the density of vehicles on certain segments can decompose.
- Shortest path helps to avoid traffic and help to reach the destination on time


## Conclusion

In this paper it is identified that the number of municipal waste can be reduced by splitting the waste which contains papers and plastics. Then in attempt is made to identify the shortest path to dispose the waste from the Village Melthombai to Attur in Salem District and this was achieved by using Dijkstra's Algorithm of Graph Theory.

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# A BIRD'S EYE VIEW ON DNA IN VEDAS 

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The development of Indus valley civilization was absolutely scientific. It has caused a quest to know their source of systematic development. It has been observed that the excavated seals and the tablets are the hoofs of their traces. The Vedic metaphors have been engraved on the seals. The symbols have vast ocean of cytological facts in its background. They discover the reactions at molecular level. Its literary evidences can be traced in the Vedic hymns, where a picture of welldeveloped society has been explored Rig-Veda 1-164. With this awareness the cities like Harappa and Mohenjo-Daro could have been developed. The symbols re-count the origin and evolution of the creation from the fundamental energy Rig-Veda 10-90.

The Indus valley civilisation's representational symbols on the seals are cytological models for teaching and they travel around the cellular genetic evolution on the earth with genetic recombination. Its literary evidences are in the vedic hymns." The hymns, when decoded, relate to the complex processes like the synthesis of chromosome, DNA replication, protein translation, and nucleotide pairs for nuclear reaction.

Demonstrating one of the decoded hymns of the Rig Veda it is explained that, "The Tvasta (as the DNA is referred to in the Rig Veda) that gives shape to all creatures is termed as an omni form (visvarupa) and it proves that DNA is present in all living beings. The same hymn goes further and identifies the 'Brhaspati' which is the nucleus embedded in the DNA. Similarly, there are codes for photosynthesis, the development of eukaryotic cell, bio-geo chemical cycle, oxidation, and reduction, and even global warming, which is elucidated in Atharva Veda, which is what comes at the end of consumption of the world's resources."

Man is viewed as a psycho- physical unity. Thought and extension, mind and body are viewed simply as different dimensions or attributes of the manifest world. This unity is maintained by the doctrine of guņa-s, which, function both on the psycho mental level as well as on the gross physical level. It is astonishing to believe that the fundamental principle of Vedas provides clue to understanding the structure and functions of DNA (De oxy ribo nucleic acid ). The major curiosity of Biologists is to understand and unveil the relationship between consciousness and matter. The understanding the structure and function of DNA is the key to the scientific understanding of the relationship between consciousness and matter.

## STRUCTURE OF DNA IN VEDAS

DNA is a complex molecule, which is the fundamental blueprint of all life forms on earth. It is present in every cell. Each DNA consists of two fibers like strands around themselves; each strand is made of molecules called nucleotides. There are four types of nucleotides called as, A, T, G, C., (Adenine, Thymine, Guanine, Cytosine) from these numerous combinations can be had, which in turn forms the sequence in the DNA strands. The unit of the genetic language is the codon- a sequence of three nucleotides. There are 64 different codons that can be made from the four bases comprising a total of 64 x $3=192$ nucleotides.

We know some functions of some parts of DNA and its relationship to cellular and biochemical processes has been found within the structure of the sounds of Veda - the most ancient tradition of knowledge. The building block of the Creation is Atom and the building block of the living-beings is DNA ( De oxy ribose Nucleic Acid ) Rig-Veda 4-58. The Creation has evolved from the single unified forcefundamental energy under the laws of thermodynamics. Rig-Veda 10-90. The first life has generated in water Rig-Veda 1-163-1.

The living-beings have progressed from the DNA with genetic recombination and the Mendel's laws of inheritance. The life on the earth is supported by the ozone layer O3. The conversion of the hydrogen into Helium H 3 on the surface of the sun, and the water molecules H2O. The triple bond of Hydrogen $\mathrm{H} \Xi \mathrm{H}$ has the key for the genetic recombination of the DNA Rig-Veda 1-163-2, 3, 4.

The horned hood expresses the synthesis of DNA from the atoms and molecules. They have synthesized the cell and the genomic chromosome. The surrounding animals have been evolved from the single cell with genetic recombination and crossing over. The $U$ shape structure is the symbol for the genomic chromosome and the fish like structure is a symbol for the cell. It has been expressed that the life has originated on the earth with synthesis of a cell, it has been expressed by a fish like structure the dot in the centre represent nucleus in the cell. The leaves on the head is a symbol for the photosynthesis, the genetic recombination with crossing over have given the ways for evolution. The horned hood expresses the four horned buffalo emitted Rig-Veda 4-58-1,2,3 The leaves on the head expresses the source of food on the earth is photosynthesis Rig-Veda 1-164-7".

Rig Veda, the button seal indicates origin of the biotic and a-biotic components of the nature from the atmospheric ocean in two different directions. The hollow structure in the centre is symbolic atmospheric ocean and the bifurcation of the hollow structure into two different directions indicates the evolution of the biotic and a-biotic components in two different directions. Atharva Veda 13-1-52.

The hollow space- it is the qualm and quiet pre- cosmic condition of the fundamental energy, when there was nothing in existence only deep darkness was there. There was only one force, which began to disintegrate from infinite to finite with its back ground in the infinite cosmos Rig-Veda10-129 1, 2, 3. Rig-Veda 10-129- 2 "The kine extract milk from his head; clothing ל
themselves in a wrap, they drunk water with the foot Rig-Veda 1-164-7. The seven figures at the base expresses the phenotypes and genotypes. It has been expressed that the creation came into the existence with formation of the atmospheric layers of the earth. Atharvaveda 14-1-5

## THE RELATA AND THE RELATUM:

Veda helps here to further our present level of understanding. The DNA is composed of two long molecular strands. Though the two strands makes copies of them selves, only one of the two strands is involved in the production of proteins. One strand is silent and the other active. Similarly the fundamental structure of the Veda is silence and activity, known in Vedic terminology as Puruşa \& Prakŗti. These two values, infinte, eternal silence, and infinite, eternal dynamism are expressed in the name of the Veda- Ŗk.

R̨k has two values " ŖR̨R̨R̨R-----" (G) reverberating wholeness-dynamism- and 'K' (Mçü),whose pronunciation stops the flow of speech , stands for the absolute stop, point value - Silence; it is obvious that $\underset{\sim}{\mathrm{R} k}$ is the unity of dynamism and silence- $\mathrm{RQ}^{\mathrm{k}}$ is the field of all possibilities. "ŗ" means "to tend towards" (prakrti) and so it is clear that maximum value of dynamism is displayed and " k " means "the spirit" (puruşa) and maximum value of silence is displayed, so from this it is understood that $\underset{\text { ŗk }}{ }$ is the unity of dynamism and silence. The union of prakŗti and puruşa is the beginning of creation.

## DNA IN PRAKRTI \& PURUŞA

To know about the terminology viz., Prakŗ̧ti \& Puruşa, the dualism of classical Sāņkhya centers or focuses around the distinction of the conscious and the unconscious, and is as follows:

The guṇa-s are "reals", they are the "intelligent-stuff", "energy-stuff" and "mass-stuff". In the state of equilibrium, the "reals" are known as prakrti. When this equilibrium is disturbed by the proximity of puruşa, the process of creation takes place. Purussa, by its presence "intelligizes" these "reals" or "subtle stuffs" and renders creation possible. Puruşa is the pure consciousness which "intelligizes" the unconscious Prakṛti.

Puruşa, the self or soul but more precisely the principle, of consciousness, is simply content less witness (Säkşitva), its only function being that of passive presence. When described as "overseeing" (adhisțāna), "the condition of being an enjoyer" (bhoktŗbhāva), or as the "condition of being a seer" (draştŗtva), these are all to be construed as passive functions, for the Puruşa is totally inactive (akartŗbhāva), totally detached (mādhyasthya, udāsīna), and isolated (kevala, kaivalya) from prakrtit.

The mūla prakrrti together with its guña-s or qualities make up everything that is in the manifest world including both the psycho-mental and the physical dimensions of the world and man, the puruşa is nothing, or the presence of nothingness in the world. It is a kind of emptiness at the very heart of the world and man, but it is nothingness or emptiness, which reveals being of the world. The nature of Puruşa that by its very nature it appears as not what it is. That is, as pure consciousness it simply witnesses or sees. The Mūla Prakŗ̣ti on the other hand, is simply
undifferentiated, unconscious thing-ness, or that which is witnessed. The Prakrti is like the nonconscious milk which functions as nourishment for the calf says Sāņkhya kārikā of Iśvarakŗ̧̧̦ña. ( Karika No:20 \& 21)

> Tasmāt tatsamyogādacetanam cetanādiva lingam /
> gunakartrutve ca tathā karteva bhavatyudāsinah //

Therefore, it is from their association of that the Linga (cause), the non-intelligent body seems intelligent and though the agency really belongs to the Gunas, the constituents, the indifferent one appears as the agent.

$$
\begin{gathered}
\text { puruşasya darśanārtham kaivalyārtham tathā pradhānasya/ } \\
\text { pangvandhavadubhayorapi tatkrutassargah // }
\end{gathered}
$$

The association of the two, as of the lame man and a blind man, is for contemplation by the Spirit of the Nature and for the release of the Spirit. Creation proceeds from this union.

Therefore, through this union, the insentient evolute (prakrrti) appears as if it is intelligent; and similarly also from agency belonging to the guņa-s, the neutral spirit (puruşa) appears as if it were the agent. Further, as in DNA, exactly similar consideration applies in the concept of Veda also, i.e., the Veda is composed of four fundamental values known as Samhita (unified state), Rishi (knower), Devata (process) and Chandas (known). In the first mandala of Rg veda these values are expressed systematically as 192 suktas. These sequentially give rise to the forty (40) aspects of the Veda which the Vedic Scientists have demonstrated corresponding the structure and function to forty principle components of the human physiology.

To refer the same further, the symbolism of prakŗti and puruşa as cosmological is in Hindu art, namely temple building (Agni purāṇa) (61.19) notes that the flags of the temple are constituted by two parts, the streamer and rod, and suggests that the streamer is prakŗti and the rod is puruşa. They are perhaps identified with prakŗti and the puruşa because the streamer is agitated by the wind, while the rod is always immovable like puruşa. The main sanctum of the temple, the vimāna is called purușa while the frame of the temple is called prakŗti. The entire Veda and Vedic Literature is a sequential and precise elaboration of the relationship between silence and dynamism contained in the name Ŗk. This is visualized as a whirlpool, where infinite dynamism collapses or spirals onto a point just as the human physiology is the elaboration of the relationship between the silent and dynamic strands of the DNA molecule. The Vedas are the book of Science particularly, The Vedic language is a symbolic language, where words are only the indicator of the phenomena, having vast ocean of mathematical knowledge in the background. If one knows the related phenomena, then only he can give the answers related with the symbols.

# THE IMPLEMENTATION OF LINEAR PROGRAMMING PROBLEM IN CURTAILING THE DISTRESSING INCREASE OF POLLUTION AT THE CAPITAL METROPOLIS 

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#### Abstract

This article gives a brief study on THE IMPLEMENTATION OF LINEAR PROGRAMMING PROBLEM IN CURTAILING THE DISTRESSING INCREASE OF POLLUTION AT THE CAPITAL METROPOLIS which implements Linear Programming Problem and Big M method to solve the problem for reducing the air pollution in our capital city.


## 1. INTRODUCTION

According to the census 2011, India is the second most populated country with population of over 1.21 billion people. In a decade the acceleration in urbanization level has been from $27.81 \%$ to $31.16 \%$ [2]. Pollution is a major problem in all parts of the world. The mixture of solid particles, dust, and gases in air are the contributing factors of air pollution. The solid particles include car emissions, chemicals from factories, pollen and mold spores. The major part of air pollution in the cities is formed by a gas called ozone. People with heart or lung disease, older adults and children are at greater risk from air pollution. The major source of air pollution is from vehicles and heavy-duty trucks which emit a high level of particulate matter, ozone and other air pollutants. The Environmental Defence Fund(EDF) estimates that on-road vehicles cause one-third of the air pollution that produces smog in India, and transportation causes $27 \%$ of greenhouse gas emissions.[1] Some of the Major Air Pollutants and their impacts on human beings and the world are Particulate matter ( $P M_{10}$ and $P M_{2.5}$ ), Nitrogen dioxide $\left(\mathrm{NO}_{2}\right)$, Ozone $\left(\mathrm{O}_{3}\right)$, Sulphur dioxide $\left(\mathrm{SO}_{2}\right)$ and Carbon monoxide (CO).

Air pollution in Delhi is caused mainly by industry and vehicles. It is mainly observed that as many as 10,000 people a year also die prematurely in Delhi as a result of air pollutants. On 25 November 2019, the Supreme Court of India made statements at the pollution in Delhi announcing " Delhi has turn out to be worse than narak (hell)" [9]. In November 2017, the air pollutants spiked some distance beyond appropriate levels. Degrees of $P M_{2.5}$ and $P M_{10}$ particulate matter hit 999 micrograms according to cubic meter, even as the safe limits for the ones pollutants are 60 and a hundred respectively [10]. In 2010, based on the 12 months of the WHO survey, the common $P M_{10}$ degree in Delhi was $286 \mu \mathrm{~g} / \mathrm{m}^{3}$. In 2013, the $P M_{2.5}$ degree changed into $153 \mu \mathrm{~g} / \mathrm{m}^{3}$. These stages are considered very dangerous. The PM levels in Delhi have come to be worse since the WHO survey. In December-January 2015, in Delhi, a mean
$P M_{2.5}$ stage of $226 \mu \mathrm{~g} / \mathrm{m}^{3}$ become mentioned with the aid of us embassy monitors in Delhi. As of October 2017, specialists in numerous tracking stations have reportedly measured an air high-quality index (AQI) of 999.

Operations Research (OR) is a discipline that deals with the application of advanced analytical methods to help make better decisions [7]. Operations research is often concerned with determining the extreme values of some real-world objective: the maximum (of profit, performance, or yield) or minimum (of loss, risk, or cost). The Big M method is a method for solving problems in linear programming problem using the simplex algorithm. This method extends the power of the simplex algorithm to problems that contains "greater-than" constraints [8]. In this article a Linear Programming Problem is formed and Big M method is used to solve the problem for reducing the air pollution in our capital city.

## 2. IMPLEMENTATION OF SUSTAINABLE TRANSPORTATION SYSTEM IN INDIA

Sustainable transportation refers to any way of transportation that has low adversary effects on the environment. Sustainable transportation includes taking walks, biking, transit, carpooling, car sharing, and green motors such as electric taxis, electric cars, flexible gas motors, bio buses, hydrogen motors, partial 0 emission motors and so on [3].

In India, especially in New Delhi, the pollution level has touched the peak level in recent days. The air pollution is beyond the safer level. For a better quality of air, the air quality index must lie between 0 and 50 . But the AQI value has reached 700 which is 14 times more than the safer level. To control air pollution immediately, we must first be aware of the fact that $41 \%$ of the air pollution in our capital is caused by vehicular emissions [4].


Figure 4: AQI value in New Delhi [9]
All of us recognize that the Delhi metro is a rapid device serving Delhi and its satellite
television for cities of Ghaziabad, Faridabad, Gurgaon, Noida, Bahadurgarh and Ballabhgarh, in the national capital vicinity of India [6]. If we interconnect the primary cities in New Delhi via the use of railroads, especially near the working places that are at most of 600 m from the working locations with affordable price tickets most of the people will use rails instead of the usage of their private vehicles. This can reduce air pollutants. Thus, if the railways are less expensive, then it can be endorsed by all.

All the above measures are taken to control the boom of the pollutant stage inside the city. But the essential factors are to govern the air pollutants which have already reached dangerous level in the city. The smog degree in Delhi is mentioned as one of the worst levels of Air Quality Index in Delhi. To conquer this situation, one of the first-rate innovations is the smog-free towers which miles an eco-friendly technology and cleans $30.000 \mathrm{~m}^{3}$ according to an hour and uses a small quantity of strength [11].

It has been calculated that for constructing one smog-unfastened tower fees 38 lakhs and it does now not a large deal to assemble it in our town. This could optimize the pollution stage in high significance. They get rid of harmful and hypersensitivity-causing particulates from the air, leaving only sparkling air behind. The air cleaner does not just clean up smog, it could also be used to make first-rate jewelry. The nice carbon debris that the tower collectsmay be condensed to create tiny 'gemstones' that can be embedded in jewelry pieces like jewelry and cufflinks. Every one of the tiny stones is the equal of 1000 cubic meters of air. Even though plans are made to assemble the smog-unfastened towers in Delhi it cannot attain its complete swing. Therefore, taking proper steps to enforce such a form of towers in public places of our capital territory will successfully govern the scenario

## 3. LINEAR PROGRAMMINGB PROBLEM - GENERAL FORM: [8]

The general form for a linear programming problem is as follows: Objective function: $\operatorname{Max} / \operatorname{Min} f\left(X_{1}, X_{2}, \ldots X_{n}\right)=C_{1} X_{1}+C_{2} X_{2}+\ldots+C_{n} X_{n}$ such that Subject to constraints:

$$
\mathrm{a}_{\mathrm{i} 1} \mathrm{X}_{1}+\mathrm{a}_{\mathrm{i} 2} \mathrm{X}_{2}+\ldots+\mathrm{a}_{\mathrm{in}} \mathrm{X}_{\mathrm{n}} \leq \text { or } \geq \text { or }=\mathrm{b}_{\mathrm{i}}, \quad \mathrm{i}=1,2, \ldots \mathrm{~m}
$$

## Sign restrictions:

$$
\begin{array}{cc}
\left(X_{i} \geq 0\right) \text { or }\left(X_{i} \leq 0\right) & \text { or ( } X_{i} \text { urs), } \\
\text { where "urs" means unrestricted. } & \mathfrak{j}=1,2, \ldots n \\
\hline
\end{array}
$$

## ALGORITHM FOR SIMPLEX METHOD: [8]

For the solution of any L.P.P. by simplex algorithm, the existence of an initial basic feasible solution is always assumed. The algorithms for the computation of an optimum solution are as follows:

1. Check whether the objective function of the given L.P.P. is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximizing it by using the result Minimum (z) $=-\operatorname{Maximum}(-z)$
2. Check whether all $b_{i}(\mathrm{i}=1,2 \ldots \mathrm{~m})$ are non-negative. If any one of the $b_{i}$ is negative then multiply the corresponding inequation of the constraints by -1 , so as to get all $b_{i}$ ( $\mathrm{i}=1,2 \ldots \mathrm{~m}$ ) non-negative.
3. Convert all the inequalities of the constraints into equations by introducing slack and/or surplus variables in the constraints. Put the costs of these variables equal to zero.
4. Obtain an initial basic feasible solution to the problem in the form $X_{B}=$
$B^{(-1)} b$ and put it in the first column of the simplex table.
5. Compute the net evaluations $z_{i}-c_{j}(\mathrm{j}=1,2 \ldots \mathrm{n})$ by using the relation

$$
z_{i}-c_{j}=C_{B} y_{j}-c_{j}
$$

If all $q_{z}-c_{j} \geq 0$, then the initial basic feasible solution $X_{B}$ is an optimum basic feasible solution else proceed to the next step.
6. If there are more than one negative $z_{i}-c_{j}$, then choose the most negative of them. Let it be $w_{r}-c_{r}$, for $\mathrm{j}=\mathrm{r}$.
(i) If all $y_{i r} \leq 0(\mathrm{i}=1,2, \ldots \mathrm{~m})$ then there is an unbounded solution to the given solution.
(ii) If atleast one $y_{i r}>0(\mathrm{i}=1,2, \ldots \mathrm{~m})$ then the corresponding vector $y_{r}$ enters the basis $y_{B}$.
7. Compute the ratios $\widehat{X_{B i}}, y_{i r}>0,{ }_{y}=1,2, \ldots \mathrm{~m}$ and chogose the minimum of them. Let the minimum of these ratios be $\underline{\underline{X B} k}$. Then the vector $y_{k}$ will level the basis $y_{B}$. The common element $y_{k r}$, which is in the $k^{t / t}$ row and the $r^{t h}$ column is known as the leading element (or pivotal element) of the table.
8. Convert the leading element to unity by dividing its rows by the leading element itself and all other elements in its column to zeros by making use of the relations.

The algorithm is now implemented as follows:

## IMPLEMENTATION OF THE PROBLEM:

The air pollution components $P M_{2.5}$ and $P M_{10}$ in the first week of November 2019 in New Delhi reaches 346 and 392 and same as in second week reaches 288 and 342 respectively. For air quality to be good, the AQI value must be less than or equal to 50 . If the approximate death rate due to these particulate matters are 248(thousand) and 48.6 (thousand) respectively.

Find the solution to linear programming problem so that the casualities due to these pollutants are reduced.

## SOLUTION:

Pollution level

|  | $P M_{2.5}$ | $P M_{10}$ |
| :---: | :---: | :---: |
| $1^{\text {st }}$ week | 346 | 392 |
| $2^{\text {nd }}$ week | 288 | 342 |

Decision variables:
$x_{1}=$ Levels of $P M_{2.5}$ in New Delhi
$x_{2}=$ Levels of $P M_{10}$ in New Delhi

Objective function:
Minimize $Z=248 x_{1}+48.6 x_{2}$
Subject to the constraints:

$$
\begin{aligned}
& 346 x_{1}+392 x_{2} \geq 50 \\
& 288 x_{1}+342 x_{2} \geq 50
\end{aligned}
$$

and $\quad x_{1} \geq 0, x_{2} \geq 0$
By introducing the surplus and artificial variables, $S_{1}, S_{2}, A_{1}, A_{2} \geq 0$, the standard form of the above
L.P.P. can be represented as follows:

$$
\text { Maximize } Z^{*}=-248 x_{1}-48.6 x_{2}+0 S_{1}+0 S_{2}-M A_{1}-M A_{2}
$$

Subject to the constraints:

$$
\begin{aligned}
& 346 x_{1}+392 x_{2}-S_{1}+A_{1}=50 \\
& 288 x_{1}+342 x_{2}-S_{2}+A_{2}=50
\end{aligned}
$$

and $\quad x_{1}, x_{2}, S_{1}, S_{2}, A_{1}, A_{2} \geq 0$

Here $x_{1}$ and $x_{2}$ are non-basic variables and $A_{1}$ and $A_{2}$ are basic variables. Thus the initial basic feasible solution is

$$
A_{1}=50 \text { and } A_{2}=5
$$

Table 3: Initial iteration

|  |  | $C_{j}$ | -248 | -48.6 | 0 | 0 | -M | -M |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C_{B}$ | $y_{B}$ | $b_{i}$ | $x_{1}$ | $x_{2}$ | $S_{1}$ | $S_{2}$ | $A_{1}$ | $A_{2}$ | Ratio |
| -M | $A_{1}$ | 50 | 346 | 392 | $-1$ | 0 | 1 | 0 | $\frac{50}{392}$ |
| -M | $A_{2}$ | 50 | 288 | 342 | 0 | -1 | 0 | 1 | $\frac{50}{342}$ |
|  | $Z_{j}$ | -100M | -634M | -734M | M | M | -M | -M |  |
|  | $Z_{j}-C_{j}$ |  | $\begin{gathered} -634 \mathrm{M}+ \\ 248 \end{gathered}$ | $\begin{gathered} -734 \mathrm{M}+ \\ 48.6 \end{gathered}$ | M | M | 0 | 0 |  |

Table 4: First iteration

|  |  | $C_{j}$ | -248 | -48.6 | 0 | 0 | -M | -M |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C_{B}$ | $y_{B}$ | $b_{i}$ | $x_{1}$ | $x_{2}$ | $S_{1}$ | $S_{2}$ | $A_{1}$ | $A_{2}$ | Ratio |
| -48.6 | $x_{2}$ | $\begin{aligned} & \underline{25} \\ & 196 \end{aligned}$ | $\frac{173}{196}$ | 1 | $\frac{-1}{392}$ | 0 | $\frac{1}{392}$ | 0 | - |
| -M | $A_{2}$ | $\frac{625}{98}$ | $\frac{-1359}{98}$ | 0 | $\frac{171}{196}$ | -1 | $\frac{-171}{196}$ | 1 | $\frac{1250}{171}$ |
|  | $Z_{j}$ | $\begin{aligned} & \frac{-625 M}{98} \\ & -6.199 \end{aligned}$ | $\begin{aligned} & \frac{1359 M}{98} \\ & -42.8969 \end{aligned}$ | -48.6 | $\begin{aligned} & \hline-171 M \\ & \hline 196 \\ & +0.124 \end{aligned}$ | M | $\frac{171 M}{196}$ | -M |  |
|  | $Z_{j}-C_{j}$ |  | $\begin{aligned} & \frac{1359 M}{98} \\ & +205.1031 \end{aligned}$ | 0 | $\begin{aligned} & \frac{-171 M}{196} \\ & +0.124 \end{aligned}$ | M | $\frac{367 M}{196}$ | 0 |  |

Table 5: Second iteration

|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Since all $Z_{j}-C_{j} \geq 0$, the optimum solution is obtained.
The optimum solution is
$x_{1}=0$ and $x_{2}=0.1462$ Thus Maximize $Z^{*}=-248 x_{1}-48.6 x_{2}$
Maximize $Z^{*}=-7.1053$ (thousand)
Since Minimum $Z=-$ Maximum ( $-Z$ ), we have, Minimize $Z=$
$7.1053($ thousand $)=7105.3 \sim 7105$

## FINDINGS:

From the calculations it is found that the causalities due to $P M_{2.5}$ and $P M_{10}$ due to these pollutants can be reduced to 7105 approximately.

## CONCLUSION

Apart from mind-blowing theories and abstract ideologies, Mathematics has always played a vital role in solving several real-life problems pertaining in the world. These solutions are not mere computations using formulae, but they are standard results, that will help us in eradicating several issues in the environment. For the world to be a better place to live in, it is necessary that people take some basic measures so that they are free from all sort of mishaps. This is possible only if everyone realises their role in safe guarding the resources of the world. They must be aware of the eco-friendly measures which can be an efficient substituent for the pollutants. Though it is not possible to eradicate these pollutants in a single day, this can be achieved if they are substituted regularly thus making it completely eradicate in a span of few years.

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# PROGRESS OF HIGHER EDUCATION IN INDIA WITH SPECIAL REFERENCE TO SCHEMES AND POLICIES SINCE INDEPENDENCE N. UMA MAHESHWARI, 

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#### Abstract

India has excelled as a centre of learning from Ancient times. Ancient universities of repute, like the Nalanda, Takshashila attracted scholars from different corners of the world. The ancient system of education in the Vedic period was marked by the Brahmanical and the Buddhist systems of education. The Medieval era led to blending of cultures and to the advent of the Madrasa as an important centre of education. The British colonial rule in India grafted into the Indian society the system of education which was designed by the British for the maintenance of their imperial administration in India, called the Macaulay scheme of education This forced inheritance made India feel the need for reconstruction far before independence, but it took effect post-independence. Motivated by the able leadership of Prime Minister Jawaharlal Nehru, the Indian system of higher education started expanding and was nourished time and again by various public policies and formation of different commissions and committees, like the University Education Commission (1948-49), foundation of the University Grants Commission (UGC) in 1956, Kothari Commission (1964-66), formulation of the first National Policy on Education (1968), and so on, till today, when an initiative of the Government of India is on to usher in and implement a New Education Policy. Sincerity has always reflected on part of the Government of India at all times to improve the higher education system through apt policies. However, despite the efforts, issues of access, equity, quality have frequently jaundiced the system, thereby raising the question of effectiveness of the policies. This paper intends to study the question of how conducive and effective the policy environment has been when promoting and improving higher education in India, by critically analyzing the different policies that have sometimes nourished, sometimes afflicted the system over time. Being on the brink of constructing a New Education Policy, it is essential to identify, isolate and hence rectify the erroneous policies, and by taking a critical and analytical journey through the higher education policies since independence, this paper fruitfully attempts to provide a clear picture of the advantages and disadvantages of the different policies that have been prevalent in India at different times since independence.


Keywords: New Education Policy, Higher education policies, Macaulay scheme of commission, government of India education, university education commission, UGC, Kothari

## Introduction

Since ancient period, India has excelled as a centre of learning. Ancient universities of repute, like the Nalanda. Takshashila. Vikramshila and Vallabhi attracted scholars from different corners of the world. The ancient system of education in the Vedic period was marked by the Brabmanical and the Buddhist systems of education. The Medieval era led to blending of cultures and to the advent of the Madrasa as an important centre of education. "Till the I 8" century, India had three distinct traditions of advanced scholarship in the Hindu gurukulas, the Buddhist viharas and the Quranic madarsas." A major transformation came up in the Indian higher education through the initiatives of the British leaving both negative and positive impacts. The colonial system of education in India was developed in three stages: (a) the efforts of the East India Company (17651813), (b) the efforts of the British Parliament (1813-1853); and (c) the educational efforts under direct British rule (1854-1947). The first phase was marked by the foundation of the Calcutta Madrasha in 1781 by Warren Hastings, followed by the establishment of the Benaras Sanskrit College in 1791 by Jonathan Duncan. During this period, English education had been gaining popularity with the efforts of missionaries. Lord Wellesley established the Fort William College for the training of youth civilians in 1800, and ushered in western education by bringing English officials and Indian Pandits together. Shortly, Raja Ram Mohan Roy launched a movement in favour of western learning and liberal education and founded the Hindu College in 1817, which was renamed Presidency College in June, 1855. But the motive of the British was to graft into the Indian society the system of education which was designed by the British for the maintenance of their imperial administration in India. Consequently, Macaulay"s minute of February 1835 saw a rejection of the Orientalists and a bias in favour of spreading Western knowledge through the English language, thereby supporting the Anglicists. Both fortunately and unfortunately this decision has reverberated in Indian higher education through the nineteenth and twentieth centuries and has its echoes even in the twenty-first century.

In 1857 the East India Company was dissolved and the British Crown had established its political power over India. The first three modern universities were now established in Calcutta, Bombay and Madras. University education in British India experienced very slow growth. Almost 30 years later the fourth university - that of Allahabad - came up and it took another 30 years for the fifth and the sixth universities, University of Mysore and Banaras Hindu University, to be founded,

Apart from being slow in progress and geographically uneven, this western form of education totally wrecked the indigenous form of learning by infusing a new system and a new language. Sarcastically enough, neither London nor Cambridge could be replicated and thus the result was a heavily biased and elitist system.

This forced inheritance made India feel the need for reconstruction far before independence, but it took effect post-independence. Motivated by the able leadership of Prime Minister Jawaharlal Nehru, the Indian system of higher education started expanding and was nourished time and again by various public policies and formation of different commissions and committees, like the University Education Commission (1948-49), foundation of the University Grants Commission (UGC) in 1956, Kothari Commission (1964-66), formulation of the first National Policy on Education (1968), and so on, till today, when an initiative of the Government of India is on to usher in and implement a New Education Policy. Sincerity has always reflected on part of the Government of India at all times to improve the higher education system through apt policies. There has been unthinkable growth and expansion (as shown in Table 1) and today it has the status of being one of the largest educational systems in the world.

Table 1: Expansion of Universities

|  | $\mathbf{1 9 5 0 - 5 1}$ | $\mathbf{1 9 9 0} \mathbf{- 9 1}$ | $\mathbf{2 0 0 3 - 0 4}$ | $\mathbf{2 0 0 6 - 0 7}$ | $\mathbf{2 0 1 2 - 1 3}$ | $\mathbf{2 0 1 4 -}$ <br> $\mathbf{1 5}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Universities Level Institutions | 25 | 177 | 320 | 371 | 624 | 760 |
| Colleges | 700 | 7346 | 16885 | 18064 | 37204 | 38498 |
| Teachers (in thousand) | 15 | 272 | 457 | 488 | 951 | 1473 |
| Students enrolled (in million) | 0.1 | 4.9 | 9.95 | 11.20 | 21.50 | 34.20 |

Source : compiled from the various reports of the University Grants Commission \& AISHE
However, despite the efforts, issues of access, equity, quality have frequently jaundiced the system, thereby raising the question of effectiveness of the policies. Researchers have often been haunted by questions like whether it is the need of the hour and pragmatism or whether it is populism, public pressure and selfish interests that have driven higher education policies of India. In this backdrop the paper intends to carry out a study on the higher education policies that have prevailed in India since independence.

## Research questions and objectives

Being on the brink of constructing a New Education Policy, it is essential to identify, isolate and hence rectify the erroneous policies, and by taking a critical and analytical journey through the higher education policies since independence, this paper attempts to provide a clear picture of the
advantages and disadvantages of the different policies that have been prevalent in India at different times since independence. The research question is, How conducive has the policy environment been with respect to higher education in India since Independence. To seek the answer to this question, my objective is to compartmentalize the time since independence into two halves- one extending from independence and continuing till 1986 and another describing period since 1986 till 2015. The study will analyze the Government initiatives that have been undertaken in this regard and make inquiries into the scope of their application and assessment of their merit.

## Literature review

There is hardly any evidence of an extensive and comprehensive research focusing on the policies prevalent in the higher education sector of India since independence. However, there have been various papers and books which have, in their discussion, touched upon the policies prevalent in their times. Some of them have been presented below. Also, reports published by the UGC and the MHRD remain a prime source of the policies prevalent at the time the reports have been published. Schenkrnan, A. S., as early as 1954 , has made an enlightening study on the then prevailing higher education system of India. Mathur, A.B. (1992) describes the Indian University system as a functioning anarchy. Srivastava, M. (1994) argues that the nationalist education policy seeks to derive legitimacy from the British system of the Anglicist policymakers. Sharma, S. (2002) dwells on the history and development of higher education in India and throws light on some of the policies that have ruled the system time to time. Kumar, T.R. and Sharma, V. (2003) point out that the declining importance being assigned to the higher education sector, in stark contrast to developed countries, has created disparities that are increasing over time. Siugh, A. (2004) comments that the UGC, when it was established in 1956, was not made as powerful as originally envisaged, as the power vested in the centre by the Constitution was not given concrete expression and that the Educational policy was last reviewed in India in .1986 but the failure to implement its recommendations may not have been an accident: the specific line of authority laid down by the Constitution to fulfill the given mandate was not followed. Sahni, R. and Kale, S. (2004) discuss the present system of higher education and attempt to find the possible implications for India in being a signatory to GATS and conclude that in the absence of a coherent education policy, the effects of opening up could lead to a distorted function of education in our society. M. Anandakrishnan (2007), Thomas Joseph (2007) and Tilak (2007) in three separate short pieces reflect on the recommendations of the National Knowledge Commission (NKC). Agarwal, P. (2009) in his informative, up-to-date and analytical book about Indian higher education, talks about the changing policies of the system over time. Tilak
(2010) discusses that the Prohibition of Unfair Practices in Technical Educational Institutions, Medical Educational Institutions and Universities Bill 2010, is inadequate for tackling the host of corrupt and unfair practices. Hatekar, N. (2009) says that success of the new central universities and the proposed national universities will come about at the cost of the state universities, which cater to the majority of students in the country and that the postgraduate teaching and research in state universities will have to be taken over by the central government to ensure the survival of this important part of the higher education system. Kolhatkar, M.R. (2012) considers the interrelationship between education and federalism as a political system and in the process sheds light on some of the higher education policies since independence. Tilak (2013) 1231 Takes a look back at the development of higher education since the ancient age. Sharma, K.A. (2013)] takes a stride back at the establishment, growth and evolution of the UGC ett its sixtieth year. Pathak, B.K. (2014) takes a critical look at the Narayana Murthy Recommendations on higher education and comments while the committee seems to be concerned about the poor quality of higher education, its recommendations or formulae appear to treat higher educational institutions like factories and that there seems to be a mismatch between its recommendations and the objectives of the Twelfth Five-Year Plan. Padmanabhan, C. (2014) light on the Rashtriya Uchatar Shiksha Abhiyan (Rusa) of the Government of India to fund higher education and comments that there is a mismatch between the diagnosis and the prescription in the document setting out the agenda of the mission, that the diagnostic part reads like a well-versed critique of commercialisation and privatisation in higher education, but the solutions prescribed would result in a reinforced entry of the corporate sector into higher education. Jawli, N. (2015) discusses the recent major strides of the Government of India in higher education. Subramanian, T.S.R. (2016), the chairperson of the five- member committee entrusted by the MHRD, GOT, in late 2015 for drafting a new education policy, and which has submitted its report on 27th May, 2016, finds the education sector to be "disarray" and discusses the immediate needs for revamping and reforming this sector by eradicating social, economic, religious, regional gaps to accessing education and by enhancing quality of education imparted. Deshpande, S. (2016) criticizes the report of the Committee for evolution of the New Education Policy, 2016 to be based on a „blurred sense of the big picture".

## Methodology

An extensive research has been done on the policies prevalent and prevailing in the higher education sector in India and their impacts, by gathering data and information from various books, journals, websites, newspapers articles, reports. This research led to valuable insights, which have
helped enriching the study. The entire information has been presented in the following discussion in a we11-structured descriptive-cum-analytical manner that is easy to read and interesting to follow and that is supplemented with personal views and judgements. The sources of information and data have been suitably cited and properly referenced.

## An analysis on the Journey of Higher Education since Independence

The time period covering the years since independence till today has been subdivided into two major sections — one ranging from 1947 till 1986 and the other extending from 1986 till 2016. The reason for this categorization is that, 1986, as will be unfurled in the discussion that follows, proves to be a watershed year for the higher education system of India.
The period from 1947 till 1986
The 1947 draft constitution of independent India legally delegated all powers regarding education to the provincial governments which laid more stress on the objective of increasing access than quality. As per the recommendation of the Sarkar Committee (1945) higher technical institutes were formed based on the Massachusetts Institute of Technology in the four regions of India. This resulted in the setting up of the five Indian Institutes of Technology at Kharagpur (1950), Bombay (1958), Kanpur (1959), Madras (1960) and Delhi (1961). The All India Council for Technical Education was set up in 1945, to oversee all technical education (diploma, degree and post-graduate) in the country. Under the able leadership of Pandit Jawaharlal Nehru, the Government of India set up the University Education Commission (UEC) under the chairmanship of Dr. S. Radhakrishnan in 1948. The UEC discussed all aspects of university education, and based on its recommendation, the University Grants Commission (UGC) was set up in 1953 for the coordination of development and maintenance of standards in higher education. UGC became a statutory organization by the act of parliament in 1956. Since then, UGC has been effectively contributing to the Indian higher education system, framing appropriate policies needed to reform and revamp the higher education system. The Nehruvian Period (1947-1964) was more focused on large-scale industrialization which thereby gave impetus to growth of higher technical institutions, but with Indira Gandhi taking over in 1964, the focus shifted to poverty and rural issues and the same tone is seen to be reflected in education as well. Set up in 1964, under the chairmanship of D.S. Kothari, the Education Commission (Kothari Commission) submitted its report in 1966 which set in motion the National Policy on Education (NPE) in 1968, still considered to be a landmark event in the history of India. The NPE became the basis of reforms that helped strengthen higher education system in India. Another important development that followed was the 42 Amendment to
the Constitution, (as a part of the Centralization Agenda of Indira Gandhi during internal Emergency) which made Education a concurrent subject in Indian Constitution, that is, now education became a joint responsibility of the central and the state governments, while earlier it was solely in the hands of the state governments.

As may be noticed above, the shift in focus from agriculture to manufacturing in the Second Plan led to a parallel shift in emphasis from elementary education to higher and higher technical education. This trend continued for quite some time, till the mid-1980s when the bias against school education was recognized. Thus came the watershed year 1986, when PM Rajiv Gandhi-led Government of India decided to launch long pending revision of the 1968 National Policy on Education in order to prepare India to face challenges of the 21 st century.

## The period since 1986 till 2015

The National Policy on Higher Education (1986) translated the vision of Radhakrishnan Commission and Kothari Commission in five main goals thr higher education, which include Greater Access, Equal Access (or Equity), Quality and Excellence, Relevance and Value Based Education (Kuppusamy, S, 2009). The NPE of 1986 revamped the higher education system by its recommendations of expansion of Higher Educational Institutions (HEIs), development of autonomous colleges, redesigning of courses, enhancing quality research, training of teachers, increasing coordination between national and state level bodies, fostering mobility between institutions. In 1992, the policy was revised by a committee under Janardhana Reddy, recommending planned development of higher education through different measures. The Action Plan of 1992 included schemes which were directed towards expansion of intake capacity in general, and that of the disadvantaged groups such as the poor, SC, ST, minorities, girls, the physically challenged persons, and those in the educationally backward regions, in particular. The Schemes / Programmes were designed to improve the quality through strengthening academic and physical infrastructure, to promote excellence in those institutions which have exhibited potential for excellence, and to develop curriculum to inculcate right values among the youth (Vrat, Prem, 2006). However, it must be pointed out that from 1986 onwards, the GOT emphasized more on elementary education than higher education, thus marking the beginning of a long period of two-and-a half decades of neglect of higher education. Despite the weakening of public expenditure on higher education, growth continued through the emergence of private institutions. Almost from zero in the 1980s, private institutions numbered to 90 in 2011 to 261 privately managed universities in 2014-15. The neoliberal policies of the 1990s have accelerated it.

Meanwhile, the sector of higher education in India continued toddling, sometimes nourished by and sometimes inflicted by several committees and commissions. The UGC Model Act, 2003 recommended for the mobilization of financial resources to become self-sufficient through different schemes like sponsored R\&D projects by companies, consultancy services, etc. C. N. Rao Committee (2005) imposed tough norms on foreign universities trying to open institutions in India. It was at the beginning of the 11th Five Year Plan, that the realization of prolonged neglect of higher education dawned on the policy-makers. The Approach paper to the FYP figures out that only $10 \%$ of the addressable global IT/ITES (Information Technology IT—enabled services) market has been realized. II has also recommended full exploitation of private sector initiatives in higher learning through public private partnership (PPP) (Sharma, 2006). The National Knowledge Commission (2006) recommends an expansion of higher education to at least $15 \%$ by 2015 and an increase in government assistance to at least $1.5 \%$ of GDP for higher education, along with proposing an establishment of an independent regulatory authority for higher education (IRAHE). Yet another Important committee was the Yashpal Committee (2009) which has recommended the formation of National Commission for Higher Education and Research (NCHER) as a single autonomous body which will subsume all regulatory bodies in higher education in India.

## Recent Progress in higher education

After the abolition of the Planning commission with the coming of the new Government in 2014, the Ministry of Human Resource and Development (MHRD) has taken some path-breaking initiatives in the field of higher education to make our young India competent enough with necessary skills. Understanding the drastic changes that the world and India have undergone, the GOl has felt the need for a New Education Policy in tune with the present challenges and demands. It has been democratic in its process to construct the NEP by a multi-level consultative process involving all stakeholders, recording their opinions on My Government, a platform for citizen engagement towards good governance in India. The GOI appointed a five-member committee under the chairmanship of T.S.R. Subramanian to evolve a draft NEP, and it submitted its report in May, 2016. The GOI is now engaged in studying it. The draft policy has not been free from criticism. The main areas of intervention of the draft policy are: (i) access and participation, (ii) quality of education, (iii) curriculum and examination reforms, (iv) teacher development and management and (v) skill development and employability. Doubts have already cropped up on the effectiveness of the draft policy, particularly in the higher education sector. Apart from this, the current GOT has allocated a $13 \%$ increase in the funds for higher education in the budget. Also the
government aims to establish new HEIs, including two lIMs (one in Jammu \& Kashmir and another in. Andhra Pradesh), two IITs along with live new AIIMS. There is emphasis on reducing regional disparity with states such as Jammu \& Kashmir, Bihar, Himachal Pradesh, Tamil Nadu and Assam getting AIIMS. Other than these institutes, there are also plans to set up new National Institutes of Pharmaceutical Education and Research, one Institute of Sciences and Educational Research, a Centre for Film Production, Animation and Gaming, and the like.

The MHRD has also initiated the Swayam Programme, a Massive Open Online Course (MOOC) platform to provide online education to the citizens of India. In order to coalesce „Digital India" programme with higher education, the GOT has decided to set up a National e-Library, thus making it possible for the nation to access rich educational materials. Another beneficial step by the government has been the launch of Pradhan Mantri Vidyalakshmi Karyakram - a fully IT based financial aid authority to help administer and monitor all educational loans and scholarships, so that there is no hindrance to students applying for financial aid. Pandit Madan Mohan Malviya Mission for teacher training has been launched to enhance quality of teaching. The first inter- University centre for teacher education has already been established at Banaras Hindu University by the UGC as part of the mission. There is also plan to introduce choice based credit system in varsities. Also, Unnat Bharat Abhiyan, a mission to promote transfer of technologies from the laboratory to the land has been initiated to connect remote villages with HEIs.

## Conclusion

As a concluding remark, it must be mentioned that the rich tradition of excellence in higher education that was initiated in the ancient era, has continued over time in India, and postindependence India has witnessed tremendous effort by the Government of India and also the State governments to sustain this richness in higher education in the country. The period since 1947 to 1986 was a period of massive improvement in higher education. 1986 onwards, for quite a long period there was a slump in the progress in higher education, though this period experienced massive privatization in the field of higher education in India. But before the pendulum could swing too far, higher education has again become one of the most important agenda for the Government of India, which is now actively involved in bringing about colossal transformation of the system through effective reforms, and the New Education Policy would hopefully be successful in this attempt.

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# ASCERTAINING THE SHORTEST PATH FROM FIVEROADS TO GOVERNMENT HOSPITAL, SALEM BY USING DYNAMIC PROGRAMING PROBLEM 

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#### Abstract

This paper gives a brief study on "Ascertaining the Shortest Path from Five Roads to Government Hospital, Salem by using Dynamic Programming Problem". Also, the Problem of finding the shortest path using the route network consisting of several paths and finally some suggestions and conclusions are given.

\subsection*{1.1 Introduction}

The shortest path problem concentrates on finding the path with minimum distance, time or cost from a source node to the destination node, and it is a fundamental problem in networks and is widely applied in transportation, routing, communication, computer networks and scheduling. So the shortest path problem has been studied extensively in the fields of operations research, transportation engineering, computer science and so on. In real life applications, the arc lengths could be uncertain and to determine the exact value of these arc lengths is very difficult or sometimes difficult for decision maker. In such a situation fuzzy shortest path problem (FSPP) seems to be more realistic, where the arc lengths are characterized by fuzzy numbers. While determining a shortest path in a fuzzy environment we required ranking of fuzzy numbers.


### 1.2 Preliminaries

In this section, some elementary concepts and definition in fuzzy set theory used through out this project are described.
Definition 1.2.1. [1]
A diagraph is a graph each of whose edges are directed. Hence, an Acyclic Diagraph is a directed graph without cycle.

Definition 1.2.2. [5]
A fuzzy set ' A ' of the real line R with membership function $\mu_{A}: \mathrm{X} \rightarrow[0,1]$ is called fuzzy number if,
(i) 'A' must be normal and convex fuzzy set.
(ii) The support of 'A' must be bounded.
(iii) 'A' must be closed interval for every $\alpha \in[0,1]$.

## Definition 1.2.3. [1]

Let $\mathrm{A}=\left(m_{1}, \gamma_{1}, \delta_{1}\right) L R$ and $\mathrm{B}=\left(m_{2}, \gamma_{2}, \delta_{2}\right) L R$ be any two LR triangularfuzzy numbers. Then the fuzzy sum of these two numbers is given by

$$
\mathrm{A}+\mathrm{B}=\left(m_{1}, \gamma_{1}, \delta_{1}\right)+\left(m_{2}, \gamma_{2}, \delta_{2}\right) L R=\left(m_{1}+m_{2}, \gamma_{1}+\gamma_{2}, \delta_{1}+\delta_{2}\right) .
$$

Definition 1.2.4. [1]
Let $\mathrm{A}=\left(l w_{1}, l p_{1}, r p_{1}, r w_{1}\right)$ and $\mathrm{B}=\left(l w_{2}, l p_{2}, r p_{2}, r w_{2}\right)$ be any two $\pi_{2}$ shaped fuzzy numbers. Assume the A and B are real numbers, then the Additionof A and B is given by A $+\mathrm{B}=\left(l w_{1}+l w_{2}, l p_{1}+l p_{2}, r p_{1}+r p_{2,1}+r w_{2}\right)$.
Definition 1.2.5. [1]
Let $L_{1}=\left(l w_{1}, l p_{1}, r p_{1}, r w_{1}\right)$ and $L_{2}=\left(l w_{2}, l p_{2}, r p_{2}, r w_{2}\right)$, be any two $\pi_{2}$ shaped fuzzy numbers, then the minimum operation between $L_{1}$ and $L_{2}$ is denotedby

$$
L_{m_{i}}\left(L_{1}, L_{2}\right)=\left\{\max \left(l w_{1}, l w_{2}\right), \min \left(l p_{1}, l p_{2}\right), \min \left(r p_{1}, r p_{2}\right), \min \left(r w_{1}, r w_{2}\right)\right\} .
$$

Definition 1.2.6. [4]
Dynamic Programming Problem (DP) is an algorithmic technique for solving an optimization problem by breaking it down into simpler subproblems and utilizing the fact that the optimal solution to the overall problem depends upon the optimal solution to its subproblems.
Definition 1.2.7. [1]
A network is a set of objects (called nodes or vertices) that are connectedtogether. The connection between the nodes are called edges or links. If the edges in a network are directed, ie., pointing in only one direction, thenetwork is called a directed network or a directed graph. If the all edges are bidirectional, or undirected, the network is an undirectednetwork or undirected graph.
Definition 1.2.8. [2]
The Single-Source shortest path problem in which we have to find the shortestpaths from a source vertex v to all other vertices in the graph.

Definition 1.2.9. [2]
The Single-Destination shortest path problem, in which we have to find the shortest from all the vertices in the directed graph to a single destination vertex v . This can be reduced to the singlesource shortest path problem by reversing the arcs in the directed graph.

## 2. FINDING THE SHORTEST PATH USING THE ROUTENETWORK

In this section, an attempt is made to find the shortest path between the two places Five Roads and Government Hospital of Salem city, Tamilnadu in two ways namely, Algorithm for fuzzy
shortest path problem with respect to acceptability index and Recursive nature of Dynamic Programming (DP) computations. Finally, the conclusion is given by comparing the results obtained in the Algorithm with respect to acceptability index with the results of the method Recursive nature of Dynamic Programming (DP) computations.

### 2.1. ALGORITHM FOR FUZZY SHORTEST PATH PROBLEM BASED ON ACCEPTABILITY INDEX

In this section, it is aimed to find the shortest path from five roads to reach Government Hospital, Salem and it is achieved based on the Acceptability index for the Algorithm taken.

## Algorithm [3]

Step 1: Construct a Network $G=(V, E)$ where $V$ is the set of vertices and $E$ is the setof edges. Here $G$ is an acyclic diagraph.
Step 2: Calculate all possible Paths $P_{i}$, from the source vertex $s$ to the destination vertex $d$ and compute the corresponding path lengths $L_{i}=\left(l w_{i}, l_{i}, \mathrm{rp}_{\mathrm{i}}, \mathrm{rw}_{\mathrm{i}}\right)$ for $\mathrm{i}=1,2 . \quad \mathrm{N}$.

Step 3: Calculate the Fuzzy Shortest Length $L_{\text {min }}$ and the set $L_{\text {min }}=(l w, l p, r p, r w)$.
Step 4: Calculate the Acceptability Index $\operatorname{AI}\left(\mathrm{L}_{\min }<\mathrm{L}_{\mathrm{i}}\right)$ between $\mathrm{L}_{\text {min }}$ and $\mathrm{L}_{\mathrm{i}}$ for $\mathrm{i}=1,2 . . . . . . \mathrm{n}$, and then Ranking is given to the paths based on the Acceptability Index.
Step 5: Identify the Shortest Path with the highest Acceptability Index AI ( $\left.\mathrm{L}_{\min }<\mathrm{L}_{\mathrm{i}}\right)_{\text {. }}$.

## PROBLEM

In case of emergency to any patient this dissertation will help in finding the best route to reach Government Hospital from five roads as soon as possible. The possible place between Five Roads and Government Hospital of Salem city are listed below and they are assigned node numbers for calculations purpose.

| Node No | Name of the Place |
| :--- | :--- |
| 1 | Five Roads (5 Roads) |
| 2 | Hasthampatty |
| 3 | Four Roads |
| 4 | Three Roads (3 Roads) |
| 5 | Collectorate |
| 6 | Pal Market Bridge |
| 7 | Government Hospital (GH) |

Tabel 1 : Name of the place with node number which are used in network

| Initial node | End node | Distance in km |
| :---: | :---: | :---: |
| 1 | 2 | 2.6 |
| 1 | 3 | 2.7 |
| 1 | 4 | 1.5 |
| 2 | 5 | 3.6 |
| 3 | 5 | 1.2 |
| 3 | 6 | 2.1 |
| 4 | 7 | 2.0 |
| 5 | 7 | 0.5 |
| 6 | 7 | 1.2 |

Tabel 2 : Distance from one place to another place


## Step 1:

Constructed a Network with 7 vertices and 8 edges


Route Network

The length of each arc mentioned in theabove network are calculated as follows:
Let $\mu_{\mathrm{A}}$ be the membership function associated with arc $\mathrm{A}(1-2)$ then $\mu_{A}=(1.5,1.9,2.4,2.6)$.
Let $\mu_{\mathrm{B}}$ be the membership function associated with arc $\mathrm{B}(1-3)$ then $\mu_{B}=(1.5,1.8,2.5,2.7)$.
Let $\mu_{C}$ be the membership function associated with $\operatorname{arc} \mathrm{C}(1-4)$ then $\mu_{C}=(0.5,0.8,1.2,1.5)$.
Similarly the remaining membership values are computed:

## Step 2:

Calculating all possible paths $P_{\mathrm{i}}$, from the source ' s ' to the destination vertex ' d ' in the Route Network and computed the corresponding path lengths $L_{i}=\left(\mathrm{lw}_{\mathrm{i}}, \mathrm{lp}_{\mathrm{i}}, \mathrm{rp}_{\mathrm{i}}, \mathrm{rw}_{\mathrm{i}}\right)$ for $\mathrm{i}=1,2, \ldots \ldots, \mathrm{n}$ as follows:

Path $\mathbf{P}_{1}:$ 1-2-5-7 with the path length $\mathbf{L}_{1}=\mu_{A}+\mu_{D}+\mu_{H}$

$$
\begin{aligned}
& =(1.5,1.9,2.4,2.6)+(3.2,4.5,5.5,5.8)+(3.9,4.1,4.2,4.4) \\
& =(8.6,10.5,12.1,12.8)=\left(\mathrm{lw}_{1}, \mathrm{lp}_{1}, \mathrm{rp}_{1}, \mathrm{rw}_{1}\right)
\end{aligned}
$$

Path $P_{2}$ :1-3-5-7 with the path length $L_{2}=\left(\mathrm{lw}_{2}, \mathrm{lp}_{2}, \mathrm{rp}_{2}, \mathrm{rw}_{2}\right)$
Path $P_{3}$ :1-3-6-7 with the path length $L_{3}=\left(\mathrm{lw}_{3}, \mathrm{lp}_{3}, \mathrm{rp}_{3}, \mathrm{rw}_{3}\right)$
Path $P_{4}$ :1-4-6-7 with the path length $L_{4}=\left(\mathrm{lw}_{4}, \mathrm{lp}_{4}, \mathrm{rp}_{4}, \mathrm{rw}_{4}\right)$

Step 3:
Calculating the Fuzzy Shortest Length $L_{\min }$ as follows and setting $L_{\text {min }}=(\mathrm{lw}, \mathrm{lp}, \mathrm{rp}, \mathrm{rw})$
$L_{\text {min }}=\left(L_{1}, L_{2}, L_{3}, L_{4}\right)=\left\{\max \left(\mathrm{lw}_{1}, l_{w_{2}}, \mathrm{lw}_{3}, \mathrm{lw}_{4}\right), \min \left(\mathrm{lp}_{1}, \mathrm{lp}_{2}, \mathrm{lp}_{3}, \mathrm{lp}_{4}\right), \min \left(\mathrm{rp}_{1}, \mathrm{rp}_{2}, \mathrm{rp}_{3}, \mathrm{rp}_{4}\right), \min \left(\mathrm{rw}_{1}\right.\right.$, $\left.\left.\mathrm{rw}_{2}, \mathrm{rw}_{3}, \mathrm{rw}_{4}\right)\right\}=\{\max (8.6,8.2,8.3,6.2), \min (10.5,9.1,9.4,7.2), \min (12.1,10.2,10.6,8.3)$, $\min (12.8,10.9,11.7,9.7)\}$
$\mathrm{L}_{\text {min }}=(8.6,7.2,8.3,9.7)=(\mathrm{lw}, \mathrm{lp}, \mathrm{rp}, \mathrm{rw})$
The Fuzzy shortest length is $\mathrm{L}_{\min }=(8.6,7.2,8.3,9.7)$
Step 4:
Calculated the Acceptability Index AI $\left(L_{\min }<L_{\mathrm{i}}\right)$ between $L_{\min }$ and $L_{\mathrm{I}}$ and the ranking is given to the paths based on the Acceptability Index.

Path $\mathrm{P}_{1}$ : 1-2-5-7 $\left(L_{\min }<L_{1}\right)=0.8926$
Path $\mathrm{P}_{2}: 1-3-5-7\left(L_{\min }<L_{2}\right)=0.9521$
Path $\mathrm{P}_{3}$ : 1-3-6-7 $\left(L_{\text {min }}<L_{2}\right)=0.9424$
Path $\mathrm{P}_{4}: 1-4-6-7\left(L_{\min }<L_{2}\right)=0.8688$

Results of the network based on the Acceptability Index are given in the table asbelow:

| Paths | $\mathbf{A l}\left(L_{\text {min }}<L_{\mathrm{i}}\right)$ | Ranking |
| :---: | :---: | :---: |
| $P_{1}: 1-2-5-7$ | 0.8926 | 3 |
| $P_{2}: 1-3-5-7$ | 0.9521 | 1 |
| $P_{1}: 1-3-6-7$ | 0.9424 | 2 |
| $P_{1}: 1-4-6-7$ | 0.8688 | 4 |

Table 3 : Network Based On Acceptability Index

## Step 5:



From the above discussion it is identified that the shortest path between 5 Roads (1) and Government Hospital (7) is 1-3-5-7 (i.e., 5 Roads - 4 Roads - Collectorate - GH). The Shortest Path with the highest Acceptability Index AI $\left(\mathrm{L}_{\min }<\mathrm{L}_{\mathrm{i}}\right)=0.9521$ is 1-3-5-7.

### 2.2. RECURSIVE NATURE OF DYNAMIC PROGRAMMING (DP) COMPUTATIONS

This section deals with finding the shortest route between the chosen two places of the Salem city by using the method Recursive nature of dynamic programming computation.

The main idea of DP is to decompose the problem into (more manageable) subproblems. Computations are then carried out recursively where the optimum solution of one subproblem is used as an input to the next subproblem. The optimum solution for the entire problem is at hand when the last subproblem is solved. The manner in which the recursive computations are carried out depends on how the original problem is decomposed. In particular, the subproblems are normally linkedby common constraints. The feasibility of these common constraints is maintained at all Iterations.

### 2.2.1. Basic Properties of DP computations

(i) The computation at each stage are a function of the feasible route of that stage, and only that stage.
(ii) A current stage is linked to the immediately preceding stage only (without regard to earlier stages) based on the shortest-distance summary of the immediate preceding stage.

## PROBLEM 1: (Shortest Route Problem)

Suppose that we want to select the shortest highway route between two cities, the network in the Figure 10 provides the possible routes between the starting city at node 1 and the destination city at node 7 . The routes passes through intermediate cities designed by node 2 to node 6 . Solve the problem by enumerating all the routes between node 1 and node 7 (there are four such routes). To solve the problem by DP, first decomposed the Route network into stages as delineated by the vertical dashed lines in Figure 11. Next, carry out the computations for each stage separately.


Figure 10 : Route Network.
The general idea for determining the shortest route is to compute the shortest (cumulative) distance to all the terminal nodes of a stage and then used these distance as input data to the immediately succeeding stage. Starting from node 1 ,stage 1 reaches three end nodes namely node 2 , node 3 and node 4 and its computations are simple,

## Stage 1 Summary

Shortest distance from node 1 to node $2=7$ miles (from node 1) Shortest distance from node 1 to node $3=8$ miles (from node 1)Shortest distance from node 1 to node $4=5$ miles (from node 1)


Decomposition of Shortest Route Problem into stages
Next, stage 2 has two end nodes namely node 5 and node 6 . Considering node 5 first,
we see from the Figure 11 that node 5 can be reached from three nodes namely node 2, node 3 and node 4 by thee different routes $(2,5),(3,5)$, and $(4,5)$. This information, together with the shortest distances to node 2, node 3 and node 4 determines the shortest (cumulative) distance to node 5 as

Shortest distance to node 5=12 (From node 4)
Node 6 can be reached from the node 3 and node 4 only. ThusShortest distance to node $6=$ 17 (From node 3)

## Stage 2 Summary

Shortest distance from node 1 to node $5=12$ miles (from node 4 ) Shortest distance from node 1 to node $6=17$ miles (from node 3 )
Shortest distance to node $7=21$ (From node 5)
The last step is to consider stage3. The determination node 7 can be reached from either node 5 or node 6 . Using the summary results from stage 2 and the distances from node 5 and node 6 to node 7, we get

## Stage 3 Summary

Shortest distance from node 1 to node $7=21$ miles (from node 5)
Stage 3 summary shows that the shortest distance between node 1 and node 7 is 21 miles. To determine the optimal route, start at stage 3 summary, where node 7 links to node 5 , stage 2 summary links node 4 to node 5 and stage 1 summary links node 4 to node 1 . Thus, the shortest route is $1-4-5-7$.


## PROBLEM 2:

The network in the Figure 13 provides the possible routes between the starting place node 1(Five Roads, Salem) and the destination place at node 7(Government Hospital, Salem) of the Salem city. Solved the problem by enumerating all the routes between node 1 and node 7 (there are four such routes). To solve the problem by DP, first decomposed the Route network into stages as delineated by the vertical dashed lines in Figure 14. Next carried out the computations for each stage separately.


Figure 13 : Route Network
The general idea for determining the shortest route is to compute the shortest (cumulative) distance to all the terminal nodes of a stage and then used these distance as input data to the immediately succeeding stage. Starting from node 1 , stage 1 reaches three end nodes namely node 2 , node 3 and node 4 and its computations are simple.

## Stage 1 Summary

Shortest distance from node 1 to node $2=2.6 \mathrm{~km}$ (from node 1)Shortest distance from node 1 to node $3=2.7 \mathrm{~km}$ (from node 1 ) Shortest distance from node 1 to node $4=1.5 \mathrm{~km}$ (from node 1)


Decomposition of Shortest Route Problem into stages
Next, stage 2 has two end nodes namely node 5 and node 6 . Considering node 5 first, we see from the Figure 14 that node 5 can be reached from two nodes namely node 2, and node 3 by two different routes $(2,5)$, and ( 3,5 ),. This information, together with the shortest distances to node 2 , node 3 and node 4 determines the shortest (cumulative) distance to node $5=3.9$ (From 3). Node 6 can be reached from the node 3 and node 4 only. Thus Shortest distance to node $6=3.5$ (From node 4)

## Stage 2 Summary

Shortest distance from node 1 to node $5=3.9 \mathrm{~km}$ (from node 3)Shortest distance from node 1 to node $6=3.5 \mathrm{~km}$ (from node 4)

Shortest distance to node $7=4.4$ (From node 5)

The last step is to consider stage3. The determination node 7 can be reached from either node 5 or node 6 . Using the summary results from stage 2 and the distances from node 5 and node 6 to node 7, we get


## Stage 3 Summary

Shortest distance from node 1 to node $7=4.4 \mathrm{~km}$ (from node 5)
Stage 3 summary shows that the shortest distance between node 1 and node 7 is 4.4 km . Thus, the shortest route is $1-3-5-7$. (i.e., 5 Roads -4 Roads - Collectorate $-G H)$.

## Suggestions :

- It is becoming difficult for the emergency services to find best route to any destination in order to save lives in real time. Finding the shortest path helps in emergency cases.
- People in doing travel activities have an impact on high economic growth. This can cause the level of congestion to increase due to the wrong route selection resulting in the built up of vehicles on certain roads resulting in an ineffective travel.
- Shortest path helps to avoid traffic and help to reach the destination ontime.


## Conclusion:

In this dissertation we used an algorithm for solving the shortest path problem on a network with fuzzy arc lengths, where the shortest path is identified using the concept of ranking function. The main advantage of this algorithm is that the ranking given to the paths will be helpful for the decision makers as they make decision in choosing the best of all possible path alternatives. Verification is also done by using the Recursive nature of dynamic programming computations. Thus the findings of this dissertation will be helpful for the Salem people.

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# MOLECULAR DOCKING EVALUATION OF 8-CHLOROQUINOLINE2-CARBALDEHYDE FOR NEURODEGENERATIVE DISORDER 

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#### Abstract

The title compound a quinoline derivative 8-Chloroquinoline 2-Carbaldehyde is optimized by using DFT/B3LYP method 6-31G (d,p) basis set. The geometrical parameters have been obtained using same basis set. The interpretations of vibrational assignments have been calculated by VEDA program. The ${ }^{1} \mathrm{H}$ and ${ }^{13} \mathrm{C}$ NMR chemical shifts are calculated by GIAO method. The vibrational optical polarization characteristics were studied by VCD spectrum. The electronic and charge transfer properties have been explained on the basis of highest occupied molecular orbitals (HOMOs), lowest unoccupied molecular orbitals (LUMOs) and UV absorption spectrum. Natural Bond Orbital (NBO) analysis has been performed for analyzing charge delocalization. In addition, Mulliken charges and MEP are performed in the molecule to identify the reactive sites. The values of dipole moment, polarizability and hyperpolarizability have been used to calculate nonlinear activity. The molecular docking studies were performed for the title molecule with different proteins by using AutoDock software.


Keywords: DFT; Vibrational assignments; MEP; HOMO-LUMO; NBO; Molecular docking. a* Corresponding Author - sudhasenthil07@gmail.com

### 1.1 Introduction

Quinoline derivatives have wide applications such as life saving drugs, optical switches in nonlinear optics, sensors in electrochemistry and in the field of inorganic chemistry [1, 2]. Nitrogen and oxygen containing heterocyclic compounds have received considerable attention due to their wide range of pharmacological activity [3]. Quinoline is a heterocyclic aromatic organic compound. It has several anti-malarial derivatives, like quiline, chloroquine, amodiaquine, and primaquine [4]. The current interest in the development of new antimicrobial agents can be partially ascribed both to the increasing emergence of bacterial
resistance to antibiotic therapy and to newly emerging pathogens [5, 6]. In synthetic medicinal chemistry the quinoline motif is widely exploited revealing a spectrum of activity covering antimalarial [7], anticancer [8], antifungal, antibacterial, antiprotozoic, antibiotic [9] and antiHIV [10] effects. Quinoline compounds are known to be effective antimicrobial compounds [11]. 8-chlorquinoline 2-carbaldehyde is an organic compound with formula $\mathrm{C}_{10} \mathrm{H}_{6} \mathrm{ClNO}$ and molecular weight is $191.614 \mathrm{~g} / \mathrm{mol}$. These aldehydes have wide range of ligand-protein activity. 8-Chloroquinoline 2-Carbaldehyde a derivative of quinoline [12] is treated for both Parkinson's disease [13] and schizophrenia [14].

Parkinson's disease is the second most common neurodegenerative disorder and the most common movement disorder [15]. Characteristics of Parkinson's disease are progressive loss of muscle control, which leads to trembling of the limbs and head while at rest, stiffness, slowness and impaired balance. As symptoms it may become difficult to walk, talk and complete simple tasks [16].

Schizophrenia is a mental disorder characterized by abnormal behavior, strange speech and a decreased ability to understand reality [17]. Other symptoms include false belief, unclear or confused thinking and reduced social engagement. People with schizophrenia often have additional mental health problems such as anxiety and depression [18]. The 8CQ2C is docked with two different proteins like 2V60 and 4MRW.

The title compound is optimized by DFT/B3LYP method. The molecular structural parameters like bond length and bond angle, vibrational assignments, HOMO - LUMO and UV analysis are studied. ${ }^{1} \mathrm{H}-\mathrm{NMR}, 13 \mathrm{C}-\mathrm{NMR}$ and VCD spectra are studied. Also, Mulliken, MEP and NBO calculations are studied to identify the reactive sites. In addition hyperpolarizability calculations are found to validate for NLO activity. Molecular docking investigation has been performed to find out the hydrogen bond lengths, binding energy and drug activity of the molecule.

### 1.2 Computational details

The computational calculations are carried out by DFT method using Gaussian 09 [19] program and Gauss view visualization software [20]. 8CQ2C molecule has been completely optimized by the DFT/B3LYP method with 6-31 G ( $\mathrm{d}, \mathrm{p}$ ) basis set. The vibrational assignments were obtained by using Veda4 software program [21]. The electronic properties such as UV absorption, HOMO (Highest Occupied Molecular Orbital) and LUMO (Lowest Unoccupied Molecular Orbital) energies were calculated using DFT method. In addition, Mulliken charges, the dipole moment and nonlinear optical (NLO) activity, such as the first hyperpolarizability, MEP analysis of the title molecule are computed. The NMR chemical shift

H and C were carried out using GIAO method in the combination of DFT/B3LYP method with 6-31 G ( $\mathrm{d}, \mathrm{p}$ ) basis set. The molecular docking were obtained by the Autodock Tools version 1.5.6 software package [22] and the docking results are viewed and analyzed using pymol [23], and Discovery studio [24] visualization software.

## Results and Discussion

### 1.3 Molecular geometry

The bond parameters (bond length and bond angles) of the 8CQ2C molecules are listed in Table 1 using DFT/B3LYP method. The optimized molecular structure was obtained from Gaussian 09 and viewed by Gauss View 5.0 programs. The optimized structure is shown in Fig 1. The global minimum energy $\mathrm{E}=-974.8020$ Hartrees. The molecular structure of 8CQ2C belongs to C1 point group symmetry. The title compound having nine C-C, six C-H, one $\mathrm{C}=\mathrm{O}$, two $\mathrm{C}-\mathrm{N}$ and one $\mathrm{C}-\mathrm{Cl}$ bond lengths and total number of atoms 19. All the bond lengths and angles are in the normal ranges [25].

The general bond lengths of $\mathrm{C}=\mathrm{C}$ are obtained from the range 1.371 to $1.394 \AA$ [26]. The aromatic ring $\mathrm{C}=\mathrm{C}$ bond length $\mathrm{C} 1-\mathrm{C} 2, \mathrm{C} 5-\mathrm{C} 6, \mathrm{C} 8=\mathrm{C} 11$ falls on the range $1.372 \AA$ which agree with general range. The general values of C-C bond lengths are obtained from 1.339 $1.417 \AA$ [27]. The bond lengths of C1-C6, C3-C4, C2-C3, C4-C5, C4-C8 $=1.421 \AA$ are due to single bond. The general value of $\mathrm{C}-\mathrm{N}$ bond length is obtained from 1.325 to $1.437 \AA$. The bond length C12=N16 is obtained from $1.422 \AA$ is due to the attached electronegative atom. The maximum bond length is $\mathrm{C} 2-\mathrm{C} 115=1.76 \AA$ which it is attached to electronegative atom. The general value of $\mathrm{C}-\mathrm{H}$ fall in the range $0.93 \AA$. In the present case bond lengths $\mathrm{C} 1-\mathrm{H} 7$, C6-H10 and C17-H18 are increased from the range $1.1 \AA$. The increasing band length C17$\mathrm{O} 19=1.43 \AA$ is attached with double bond of electronegative atom.

The calculated approximate bond angles of C5-C4-C8, C2-C3-N16 are exactly 121 in the benzene ring. The bond angles C2-C1-C6, C2-C1-C3, C2-C1-H7, C1-C2-C115, C4-C5-C6, C6-C5-H9, C1-C6-C5, C5-C6-H10, C4-C8-C11, C8-C11-H14, N16-C12-N16 and C3-N16C 12 all are same values for 120 orespectively. Hence, the homonuclear bond lengths (C1-C2, C5-C6, C1-C6, C3-C4, C2-C3, C4-C5, C4-C8 and C11-C12) are higher than the heteronuclear bond lengths (C12-N12, C1-H7, C6-H10 and C17-H18). The reason is same charges are repulsive and opposite charges are attractive.


Fig 1. Optimized structure of 8CQ2C
Table 1. Geometrical parameters from DFT/B3LYP method.

| Bond Length | Theoretical bond <br> Length ( $\AA$ ) | Bond Angle | Theoretical bond <br> angle (deg) |
| :---: | :---: | :---: | :---: |
| C1-C2 | 1.372 | C2-C1-C6 | 120.376 |
| C1-C6 | 1.416 | C2-C1-H7 | 120.647 |
| C1-H7 | 1.100 | C6-C1-H7 | 118.97 |
| C2-C3 | 1.422 | C1-C2-C3 | 120.509 |
| C2-C115 | 1.760 | C1-C2-C115 | 120.883 |
| C3-C4 | 1.418 | C3-C2-C115 | 118.606 |
| C3-N16 | 1.422 | C2-C3-C4 | 119.095 |
| C4-C5 | 1.421 | C2-C3-N16 | 121.700 |
| C4-C8 | 1.422 | C4-C3-N16 | 119.203 |
| C5-C6 | 1.373 | C3-C4-C5 | 119.210 |
| C5-H5 | 1.100 | C3-C4-C8 | 119.059 |
| C6-H10 | 1.100 | C5-C4-C8 | 121.730 |
| C8-C11 | 1.372 | C4-C5-C6 | 120.455 |
| C8-H13 | 1.100 | C6-C5-H9 | 118.644 |
| C11-C12 | 1.416 | C1-C6-C5 | 120.899 |
| C11-H14 | 1.100 | C1-C6-H10 | 120.351 |
| C12-N16 | 1.372 | C5-C6-H10 | 119.002 |
| C12-C17 | 1.540 | C4-C8-C11 | 120.645 |
| C17-H18 | 1.070 |  | 120.549 |


| C17-O19 | 1.430 | C4-C8-H13 | 118.591 |
| :--- | :--- | :---: | :---: |
|  |  | C11-C8-H13 | 120.859 |
|  |  | C8-C11-C12 | 120.366 |
|  |  | C8-C11-H14 | 120.649 |
|  |  | C12-C11-H14 | 118.983 |
|  |  | C11-C12-N16 | 120.323 |
|  |  | C11-C12-C17 | 118.991 |
|  |  | C16-C12-C17 | 120.685 |
|  |  | C12-C17-H18 | 120.496 |
|  |  | C12-C17-O19 | 109.471 |
|  |  | H18-C17-O19 | 109.471 |

### 1.4 Vibrational Assignments

8CQ2C consists of total number of 19 atoms are existing 51 normal modes of vibrations. These vibrational frequencies are computed by DFT/B3LYP method 6-31G ( $\mathrm{d}, \mathrm{p}$ ) basis set. The calculated vibrational frequencies (Unscaled and Scaled), IR intensity, Raman activity are given in Table 2. The theoretical spectrum of FT-IR and FT-Raman are shown in
Fig 2.

## C-H Vibrations

Heterocyclic aromatic compound and its derivatives are structurally very close to benzene. The $\mathrm{C}-\mathrm{H}$ stretching vibrations for hetero aromatic molecule appear in the region from 3100-3000 $\mathrm{cm}^{-1}$ [28, 29]. The C-H vibrations for 8CQ2C were observed at 3088, 3081, 3065, 3050, $3048 \mathrm{~cm}^{-1}$ with \%PED is reported. The general value for in plane bending of $\mathrm{C}-\mathrm{H}$ is the range of $1000-1300 \mathrm{~cm}^{-1}$ [30]. In the present compound C-H in plane bending are observed at 1190, 1169, 1129 and $1095 \mathrm{~cm}^{-1}$ with including \% PED conforming CH bending.

## $\mathbf{C}=\mathbf{O}$ Vibration

The $\mathrm{C}=\mathrm{O}$ stretching vibration is highly affected for intermolecular hydrogen atom. Hence, hydrogen bonding due to decrease the double bond character of the carbonyl group and shifting absorption band to lower frequency. The $\mathrm{C}=\mathrm{O}$ stretching vibration is very strong and sharp band appearing in the region $1850-1550 \mathrm{~cm}^{-1}$ [31]. In presence of hydrogen atom near to the carbonyl group and it is simulated at $1723 \mathrm{~cm}^{-1}$ with $91 \%$ of PED. The theoretical wavenumber $1723 \mathrm{~cm}^{-1}$ also confirm that Gauss View animation option for given $\mathrm{C} 17=\mathrm{O} 19$.

## $\mathrm{C}-\mathrm{C}$ and $\mathrm{C}=\mathrm{C}$ Vibrations

The ring C-C and C=C stretching vibrations usually occur in the regions $1650-1430 \mathrm{~cm}^{-1}$ and 1380-1280 $\mathrm{cm}^{-1}$, respectively. The actual positions of these modes are determined not so much by the nature of the substituent but rather by the form of the substitution around the ring [32]. According to C-C stretching vibrations are appears in region 1585, 1575, 1532 and $1476 \mathrm{~cm}^{-1}$ agree well with general range. The $\mathrm{C}=\mathrm{C}$ stretching vibrations were established at 1339, 1284 and $1222 \mathrm{~cm}^{-1}$ which coincide with literature values. The in-plane and out-of-plane bends are reported at $652-509 \mathrm{~cm}^{-1}$ and $477-282 \mathrm{~cm}^{-1}$ respectively. The in-plane bending of $\beta C C C$ is appear in the range $652 \mathrm{~cm}^{-1}$ with the PED percentage is $33 \%$. The out-of plane $\delta C C C C$ bend was observed at 419 and $204 \mathrm{~cm}^{-1}$.

## C-Cl Vibration

The C-Cl stretching vibrations give generally strong bands in the region $800-600 \mathrm{~cm}^{-1}$. In the present peak at $652 \mathrm{~cm}^{-1}$ is agreed with literature value. The $\mathrm{C}-\mathrm{Cl}$ deformation modes are appeared in the region $460-175 \mathrm{~cm}^{-1}$. In the present molecule peaks are appeared at 443 , 410 and $228 \mathrm{~cm}^{-1}$ with PED percentage is 20,42 and $39 \%$ respectively.

## $\mathbf{C}=\mathbf{N}$ and $\mathbf{C - N}$ vibrations

The C-N stretching frequency is a very tough task since it falls in a composite region of the vibrational spectrum, i.e., mixing of several bands is possible in this region [33]. The $\mathrm{C}=\mathrm{N}$ and $\mathrm{C}-\mathrm{N}$ stretching modes appear around $1600-1500 \mathrm{~cm}^{-1}$ and 1300-1290 $\mathrm{cm}^{-1}$ [34] respectively. The C-N stretching vibration was observed at $1284 \mathrm{~cm}^{-1}$ which, is well matched with the literature value. The $\mathrm{C}=\mathrm{N}$ stretching vibration is appeared in the region $1427 \mathrm{~cm}^{-1}$.



Fig 2. Theoretical FT-IR and FT-Raman spectra of 8CQ2C
Table 2. Theoretical vibrational assignments of 8CQ2C with DFT/B3LYP method 631G (d, p) basis set.

| S1.No | Calculated frequencies ( $\mathrm{cm}^{-1}$ ) |  | IR intensity | Raman activity | \% Potential Energy <br> Distribution (PED) |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Unscaled frequency | Scaled frequency |  |  |  |
| 1 | 3230 | 3088 | 1.9915 | 122.5108 | $\nu \mathrm{CH}(100)$ |
| 2 | 3223 | 3081 | 3.8047 | 190.6865 | $\nu \mathrm{CH}(96)$ |
| 3 | 3206 | 3065 | 15.0989 | 168.0079 | $\nu \mathrm{CH}(93)$ |
| 4 | 3190 | 3050 | 7.1215 | 133.8051 | $\nu \mathrm{CH}(87)$ |
| 5 | 3188 | 3048 | 1.8775 | 12.2100 | $\nu \mathrm{CH}(87)$ |
| 6 | 2967 | 2836 | 81.6869 | 113.9178 | $\nu \mathrm{CH}(100)$ |
| 7 | 1804 | 1723 | 254.0920 | 187.5745 | $\nu \mathrm{OC}(91)$ |
| 8 | 1658 | 1585 | 7.8472 | 58.4751 | $\nu \mathrm{CC}(42)$ |
| 9 | 1648 | 1575 | 2.9679 | 95.9017 | $\nu \mathrm{CC}(55)$ |
| 10 | 1603 | 1532 | 9.1013 | 29.6118 | $\nu \mathrm{CC}(49)$ |
| 11 | 1544 | 1476 | 13.1408 | 25.2022 | $\nu \mathrm{CC}(21) ; \beta \mathrm{HCC}(27)$ |
| 12 | 1493 | 1427 | 37.6452 | 14.7409 | vNC(36); $\beta$ HCC(16) |
| 13 | 1465 | 1401 | 12.7348 | 146.5212 | $\beta \mathrm{HCC}(50)$ |
| 14 | 1401 | 1339 | 9.2703 | 225.6608 | $\nu \mathrm{CC}(62) ; \beta \mathrm{CNC}(10)$ |
| 15 | 1391 | 1330 | 0.2543 | 30.4168 | $\beta \mathrm{HCO}(71)$ |
| 16 | 1364 | 1304 | 9.9375 | 2.4671 | $\nu \mathrm{CC}(41)$ |


| 17 | 1343 | 1284 | 27.8245 | 45.3592 | vNC(16); vCC(17) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | 1278 | 1222 | 29.0341 | 33.7513 | vNC(10); vCC(21); $\beta \mathrm{HCC}(13)$ |
| 19 | 1245 | 1190 | 10.3566 | 4.2636 | $\nu \mathrm{CC}(29)$; $\beta \mathrm{HCC}(40)$ |
| 20 | 1223 | 1169 | 22.4116 | 25.1403 | $\beta \mathrm{CNC}(10) ; \beta \mathrm{HCC}(47)$ |
| 21 | 1181 | 1129 | 4.1477 | 9.6040 | $\beta \mathrm{HCC}(60)$ |
| 22 | 1145 | 1095 | 4.5390 | 16.3995 | $\nu \mathrm{CC}(10) ; \beta \mathrm{HCC}(62)$ |
| 23 | 1090 | 1042 | 4.7019 | 9.6352 | $\nu \mathrm{CC}(59) ; \beta \mathrm{HCC}(17)$ |
| 24 | 1032 | 987 | 0.2764 | 6.4745 | $\begin{aligned} & \tau \mathrm{HCCC}(61) ; \\ & \tau \mathrm{HCCC}(11) \end{aligned}$ |
| 25 | 1008 | 964 | 0.2619 | 0.0587 | $\tau$ СССС(77) |
| 26 | 1007 | 963 | 38.7542 | 2.1605 | $\begin{gathered} \nu \mathrm{ClC}(25) ; \beta \mathrm{CNC}(11) ; \\ \beta \mathrm{CCC}(31) \end{gathered}$ |
| 27 | 987 | 944 | 0.7726 | 0.3712 | $\begin{aligned} & \hline \text { चHCCC(71); } \\ & \delta C C C C(11) \end{aligned}$ |
| 28 | 919 | 879 | 0.4471 | 2.9488 | $\tau \mathrm{HCCC}(84)$ |
| 29 | 904 | 864 | 27.7668 | 0.2652 | $\nu \mathrm{CC}(23) ; \beta \mathrm{CCC}(21)$ |
| 30 | 867 | 829 | 39.8479 | 1.6734 | $\tau$ НССС(77) |
| 31 | 852 | 815 | 11.2384 | 23.6542 | $\begin{gathered} v \mathrm{CC}(12) ; \beta \mathrm{CCC}(14) ; \\ \beta \mathrm{CCN}(12) \end{gathered}$ |
| 32 | 819 | 783 | 1.2290 | 2.3140 | $\begin{gathered} \tau \mathrm{HCCC}(16) ; \\ \tau \mathrm{CCCC}(44) \end{gathered}$ |
| 33 | 780 | 746 | 26.8303 | 2.7301 | $\begin{gathered} \tau \mathrm{HCCC}(73) ; \\ \tau \mathrm{CCCC}(15) \end{gathered}$ |
| 34 | 769 | 735 | 54.1753 | 2.2194 | $\nu C C(12) ; \beta \mathrm{CCN}(27)$ |
| 35 | 682 | 652 | 26.7380 | 8.9252 | $\nu \mathrm{CCl}(14) ; \beta \mathrm{CCC}(33)$ |
| 36 | 671 | 641 | 0.2195 | 0.0725 | $\begin{gathered} \delta \operatorname{CCCC}(40) ; \\ \tau \operatorname{CCCN}(40) ; \\ \tau \operatorname{CCCC}(21) \end{gathered}$ |
| 37 | 627 | 599 | 4.3908 | 0.8504 | $\begin{gathered} \nu \mathrm{CC}(13) ; \beta \mathrm{CCO}(32) ; \\ \beta \mathrm{CCN}(13) \end{gathered}$ |
| 38 | 569 | 543 | 0.0049 | 2.4302 | $\begin{aligned} & \tau \mathrm{HCCC}(13) ; \\ & \delta C C C C(60) \end{aligned}$ |


| 39 | 536 | 512 | 0.2083 | 9.8838 | $\beta$ CCC(55) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 40 | 514 | 419 | 0.3966 | 0.5207 | $\begin{aligned} & \hline \delta \operatorname{CCCC}(12) ; \\ & \tau \operatorname{CCCN}(23) ; \\ & \tau \operatorname{CCCC}(35) \end{aligned}$ |
| 41 | 463 | 443 | 5.2396 | 9.5045 | $\beta \mathrm{ClC}(15) ; \beta \mathrm{CCC}(20)$ |
| 42 | 431 | 412 | 0.2816 | 3.3354 | $\begin{aligned} & \hline \text { бCCCC(10); } \\ & \tau \operatorname{CNCC}(64) \end{aligned}$ |
| 43 | 429 | 410 | 5.8515 | 5.6801 | $\beta \mathrm{CCl}(51) ; \beta \mathrm{CCC}(42)$ |
| 44 | 361 | 345 | 1.2694 | 5.1779 | $\nu \mathrm{CCl}(14) ; \beta \mathrm{CCC}(14) ;$ $\beta \mathrm{CCO}(16) ; \beta \mathrm{CCN}(21)$ |
| 45 | 312 | 298 | 1.5142 | 0.7750 | $\begin{aligned} & \hline \tau \mathrm{HCCC}(11) ; \\ & \tau \mathrm{CCCN}(58) \end{aligned}$ |
| 46 | 239 | 228 | 7.9312 | 2.8326 | $\beta \mathrm{CCCl}(39)$ |
| 47 | 213 | 204 | 9.5952 | 0.0534 | $\begin{gathered} \hline \delta \operatorname{CCC}(16) ; \\ \tau \operatorname{CCCN}(40) \end{gathered}$ |
| 48 | 192 | 183 | 3.8015 | 0.3850 | 8CCCC(60) |
| 49 | 142 | 136 | 1.8011 | 1.3247 | $\beta \mathrm{CCCl}(64)$ |
| 50 | 113 | 108 | 0.9963 | 3.3120 | $\begin{aligned} & \hline \tau \text { CCC(45); } \\ & \delta С С С С(11) \end{aligned}$ |
| 51 | 78 | 75 | 0.7571 | 2.4633 | $\begin{aligned} & \hline \tau \operatorname{CCC}(11) ; \\ & \delta С С С С(75) \end{aligned}$ |

$\nu$ - Stretching, $\beta$ - in - plane bending, $\delta$-out of plane bending, $\tau$ - torsion

### 1.5 NMR spectral analysis

The isotropic chemical shifts are frequently used in identification of reactive organic as well as ionic species. It is recognized that accurate predictions of molecular geometries are essential for reliable calculations of magnetic properties [35]. The theoretical spectra were computed by gauge-independent atomic orbital (GIAO) [36] functional in combination with B3LYP method 6-31 G (d, p) basis set. The theoretical ${ }^{1} \mathrm{H}$ and ${ }^{13} \mathrm{C}$ NMR chemical shift values are presented in Table 3.

The computed ${ }^{1} \mathrm{H}$ NMR chemical shift values are $10.4,8.4,8.2,8.0,7.8$ and 7.7 ppm ( $18-\mathrm{H}, 14-\mathrm{H}, 13-\mathrm{H}, 7-\mathrm{H}, 9-\mathrm{H}$ and $10-\mathrm{H}$ ) respectively. Since, the proton numbered H 18 has the maximum chemical shift value ( 10.4 ppm ), as it is near the electronegative O19 atom. The protons $14-\mathrm{H}, 13-\mathrm{H}, 7-\mathrm{H}, 9-\mathrm{H}$ and $10-\mathrm{H}$ with chemical shift values are 8.4, 8.2, 8.2 8.0, 7.8 and
7.7 ppm respectively. Whereas proton chemical shifts are due to shielding (or upfield). In the organic molecules, ${ }^{13} \mathrm{C}$ NMR chemical shifts are usually lying in the region of $10-200 \mathrm{ppm}$ [37]. In the present ${ }^{13} \mathrm{C}$ NMR chemical shift values are $176.9,138.7,122.5,116.0,112.3$ and $105.5 \mathrm{ppm}(17-\mathrm{C}, 12-\mathrm{C}, 8-\mathrm{C}, 1-\mathrm{C}, 5-\mathrm{C}$ and $11-\mathrm{C})$ respectively. The maximum chemical shift value of carbon atom $17-\mathrm{C}$ having 176.9 ppm due to deshieled (downfield) and minimum chemical shift is $11-\mathrm{C}$ with 105.5 ppm . The theoretical spectra of ${ }^{13} \mathrm{C}$ NMR and ${ }^{1} \mathrm{H}$ chemical shift is shown in Fig 3.


Fig 3. Theoretical spectra of ${ }^{13} \mathrm{C}$ NMR and ${ }^{1} \mathrm{H}$ NMR chemical shift of 8 CQ 2 C .
Table 3. Theoretical chemical shift values of ${ }^{13} \mathrm{C}$ NMR and ${ }^{1} \mathrm{H}$ NMR

| Atom | Chemical Shift (ppm) | Atom | Chemical Shift (ppm) |
| :---: | :---: | :---: | :---: |
| $17-\mathrm{C}$ | 176.9 | $18-\mathrm{H}$ | 10.4 |
| $12-\mathrm{C}$ | 138.7 | $14-\mathrm{H}$ | 8.4 |
| 8-C | 122.5 | $13-\mathrm{H}$ | 8.2 |
| $1-\mathrm{C}$ | 116.0 | $7-\mathrm{H}$ | 8.0 |
| 5-C | 112.3 | 9-H | 7.8 |
| $11-\mathrm{C}$ | 105.5 | $10-\mathrm{H}$ | 7.7 |

### 1.6 VCD spectral analysis

Vibrational circular dichroism (VCD) is similar to the electro-magneto-optic effect (Zeeman Effect) and it is directly related to IR of vibrational optical activity which detects differences in attenuation of left and right circularly polarized light passing through the compound. It is the extension of circular dichroism spectroscopy into the IR and near infrared region [38]. The VCD analysis were calculated by using DFT/B3LYP method with 6-31G (d,p) basis set. The theoretical VCD gives two-dimensional structural information because VCD is sensitive to the mutual orientation of distinct ligand groups in a molecule. It is also used for the identification of absolute configurations of the organic compounds [39]. The vibrational difference with respect to the left and right circularly polarized light radiations and the result is combination of an emission and absorption spectra associated with biologically and optically significant of organic molecules.

In the present case, the VCD spectrum was originate from zero to $4000 \mathrm{~cm}^{-1}$ and the intensive peak started with $400 \mathrm{~cm}^{-1}$. The intense peaks are represented to the absorption and emission was predicted in both positive and negative phase due to the left and right polarization. The absorption intensity is usually unequal in both up and down of the VCD spectrum. The vibrational polarization bands belong to middle IR region which corresponding to the $\mathrm{C}-\mathrm{C}, \mathrm{C}=\mathrm{C}, \mathrm{C}-\mathrm{N}, \mathrm{C}=\mathrm{N}, \mathrm{C}-\mathrm{Cl}$ and $\mathrm{C}=\mathrm{O}$ stretching respectively. These various vibrational modes are used to the optical and biological applications. The VCD spectrum is shown in Fig
4.


Fig 4. Theoretical VCD spectrum.

### 1.7 UV- Visible analysis

The theoretical UV- Visible spectrum were obtained by DFT/B3LYP method with 6$31 \mathrm{G}(\mathrm{d}, \mathrm{p})$ basis set. The UV-Visible analysis was carried out different electronic transitions of the 8CQ2C. The calculated absorption maximum along with optical parameters such as excitation energy, excitation wavelength, and oscillator strength are given in the Table 4. The theoretical calculations predicted two peaks at 246.23 nm and 238.73 nm with corresponding
excitation energies are 5.03 eV and 5.19 eV respectively. The maximum absorption peak at 246.23 nm are due to $\pi \rightarrow \pi^{*}$ transition in the title compound. Theoretical UV- Visible spectrum is shown in Fig 5.


Fig 5. Theoretical UV- visible spectrum
Table 4. Theoretical electronic absorption spectrum values of 8CQ2C.

| Excited <br> states | Wavelength ( $\boldsymbol{\lambda}_{\max }$ ) <br> in nm | Excitation <br> energy (eV) | Oscillator <br> strength (f) | Assignment |
| :---: | :---: | :---: | :---: | :---: |
| S1 | 246.23 | 5.0353 | 0.1208 | $\pi \rightarrow \pi^{*}$ |
| S2 | 238.73 | 5.1935 | 0.0586 | $\pi \rightarrow \pi^{*}$ |

### 1.8 Frontier molecular orbital energy analysis

HOMO means the highest occupied molecular orbital and LUMO means the lowest unoccupied molecular orbital. HOMO and LUMO are important parameters in defining the reactivity of chemical species [40-43]. The energy of HOMO indicates nucleophilicity and LUMO indicates electrophilicity [44]. HOMO-LUMO energy gap reflects the kinetic stability of the molecule [45]. The HOMO shows the various prominent donor orbitals and the LUMO shows that of prominent acceptor orbitals. HOMO-1 and LUMO+1, represents the respective donor and acceptor levels one energy state below and above these levels respectively [46]. The computed energy gaps and other quantum descriptors like electronegativity $(\chi)$, chemical hardness $(\eta)$, softness $(S)$, chemical potential ( $\mu$ ), and electrophilicity index $(\omega)$ are given in the title compound. The HOMO and LUMO orbitals of 8CQ2C are shown in Fig 6.

The electronic properties of the molecule are calculated from the total energies and the Koopman's theorem [47]. The ionization potential ( $\mathrm{IP}=-\mathrm{E}_{\text {номо }}$ ), electron affinity ( $\mathrm{EA}=$ - $\mathrm{E}_{\text {LUMO }}$ ), electrophilicity index $\left(\omega=\mu^{2} / 2 \eta\right)$, electronegativity $(\chi=(I+A) / 2)$, chemical hardness $(\eta=(I-A) / 2)$ and softness $(S=1 / 2 \eta)$ are listed in Table 5. The calculated energy values of HOMO and LUMO are -0.32668 and 0.03221 eV respectively. The energy gap between

HOMO and LUMO is 0.29 eV . The values of IP, EA, $\chi$ are $0.3266 \mathrm{eV}, 0.0322 \mathrm{eV}$ and 0.1472 eV . The chemical hardness is $\eta=0.1799 \mathrm{eV}$ and softness is $\mathrm{S}=5.5574 \mathrm{eV}$ were calculated in the title compound. The softness value of 8CQ2C indicates that it belongs to soft material category. The title compound of 8CQ2C is a negative ionization potential and it is stable molecule. The lowering of frontier molecular orbital energy gap confirms the charge transfer within the molecule which reflects its kinetic stability and substantiates its bioactivity.


Fig 6. The HOMO and LUMO orbitals of 8CQ2C.
Table 5. The Frontier Molecule Orbital values of 8CQ2C.

| FMOs | 8CQ2C |
| :--- | :---: |
| E $_{\text {номо }}(\mathrm{eV})$ | -0.3266 |
| $\mathrm{E}_{\text {Luмо }}(\mathrm{eV})$ | -0.0322 |
| $\mathrm{E}_{\text {номо }}$ - $\mathrm{E}_{\text {Luмо }}$ gap (eV) | 0.2944 |
| Ionization Potential (I) | 0.3266 |
| Electron affinity (A) | 0.0322 |
| Electronegativity ( $\chi$ ) | 0.1472 |
| Chemical hardness ( $\eta$ ) | 0.1799 |
| Softness (S) | 5.5574 |
| Electrophilicity index $(\omega)$ | 0.0601 |

### 1.9 Molecular Electrostatic Potential (MEP) analysis

MEP is very useful for docking analysis in the two species like protein and ligand interact mainly through their potentials. MEP surface helps to predict the reactivity of a wide variety of chemical systems in both electrophilic and nucleophilic reactions, the study of biological recognition processes and hydrogen bonding interactions [48]. The electrostatic potential increases in the order red $<$ orange $<$ yellow $<$ green $<$ blue [49]. The different values of the electrostatic potential at the surface are represented by different colors; red
represents regions of most electro negative electrostatic potential, blue represents regions of most positive electrostatic potential and green represents regions of zero potential. MEP for the title compound is shown in Fig 7. The oxygen atom is present in 8CQ2C is surrounded by higher electron density given by deep red in the presence of double bond of carbon atom. The hydrogen atoms are attached in the carbon atoms and it is positive electronegative potential and the color representation is blue. The benzene ring is also zero potential in given color is green. The electrostatic potential is largely responsible for the binding of a substrate to its receptor binding sites since the receptor and the corresponding ligands recognize each other at their molecular surface [50, 51]. Hence, only the electrostatic potential of oxygen atom is interact with two amino acids like SER A and TYR A in the active side of the ligand.


Fig 7. Molecular Electrostatic Potential of 8CQ2C

### 1.10 Natural bonding orbital (NBO) analysis

NBO analysis is performed to determine the electronic transitions from filled orbital of one system to unfilled orbital of another system. The second order fock matrix method is carried out to the interactions of donor-acceptor in the NBO analysis [52]. It is mainly used to measure the delocalization of electron density and such a kind of hyper conjugation of electronic orbitals. The higher the $\mathrm{E}(2)$ values, the molecular interaction of donor - acceptor are more intense and greater is the stability of entire molecule. The analysis of various donors and acceptors are indicates only two types of donors $\pi \& \sigma$ and two types of acceptors $\pi^{*} \& \sigma^{*}$ respectively.

The Delocalization of electron density amid occupied Lewis-type (bond or lone pair) NBO orbitals and properly unoccupied (antibond or Rydgberg) non-Lewis NBO orbitals resemble to a stabilizing donor-acceptor interaction. The second order perturbation of stabilization energies $\mathrm{E}(2)$ between bonding and antibonding has been given in the Table 6.

The maximum stabilization energy is $23.67 \mathrm{kJmol}^{-1}$ and it is obtained for the interactions between $\pi$ (C8-C11) $\rightarrow \pi^{*}$ (C12-N16). In the other interactions are $\mathrm{LP}(2) \mathrm{O} 19 \rightarrow \pi^{*}$ (C17$\mathrm{H} 18), \pi(\mathrm{C} 12-\mathrm{N} 16) \rightarrow \pi^{*}(\mathrm{C} 3-\mathrm{C} 4), \pi(\mathrm{C} 3-\mathrm{C} 4) \rightarrow \pi^{*}(\mathrm{C} 12-\mathrm{N} 16), \pi(\mathrm{C} 3-\mathrm{C} 4) \rightarrow \pi^{*}(\mathrm{C} 1-\mathrm{C} 2)$ and $\pi$ (C8-C11) $\rightarrow \pi^{*}$ (C3-C4) which leads to strong delocalization of 20.78, 20.04, 16.44, 16.05 and 16.02 respectively. The very important interactions of Lewis bond and non-Lewis bond orbitals with the C115, N16 and O19 lone pairs. The maximum intermolecular charge transfer are occurs in the $\pi$ bonding and $\pi^{*}$ antibonding orbitals thus the entire title compound have a $\pi$ conjugated system and it is more stability. The more reactive side is carbonyl group ( $\mathrm{C} 17=\mathrm{O} 19$ ) and it also interacts with good way of protein-ligand interactions.
Table 6. NBO transition data of 8CQ2C calculated using second order perturbation method of Fock matrix.

| Donor <br> (i) | ED(e) | Acceptor (j) | ED(e) | $\begin{gathered} \mathrm{E}(2)^{\mathrm{a}} \\ \mathrm{Kal} / \mathrm{Mol} \end{gathered}$ | $\begin{gathered} \mathrm{E}(\mathbf{i})-\mathrm{E}(\mathbf{j})^{\mathrm{b}} \\ \text { a.u } \end{gathered}$ | $\mathrm{F}(\mathrm{i}, \mathrm{j})^{\mathrm{c}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\pi$ (C3-C4) | 1.96632 | $\pi^{*}$ (C1-C2) | 0.29982 | 16.05 | 0.27 | 0.062 |
| $\pi$ (C3-C4) | 1.51545 | $\pi^{*}$ (C12-N16) | 0.34209 | 16.44 | 0.26 | 0.061 |
| $\pi$ (C8-C11) | 1.98090 | $\pi^{*}$ (C3-C4) | 0.46074 | 16.02 | 0.28 | 0.063 |
| $\pi$ (C8-C11) | 1.70976 | $\pi^{*}$ (C12-N16) | 0.01796 | 23.67 | 0.28 | 0.073 |
| $\pi$ (C12-N16) | 1.98528 | $\pi^{*}$ (C3-C4) | 0.04219 | 20.04 | 0.32 | 0.076 |
| LP(3)Cl15 | 1.91820 | $\pi^{*}$ (C1-C2) | 0.02281 | 13.88 | 0.33 | 0.064 |
| LP(1)N16 | 1.91688 | $\pi^{*}$ (C11-C12) | 0.03637 | 11.13 | 0.87 | 0.089 |
| $\mathrm{LP}(2) \mathrm{O} 19$ | 1.88672 | $\pi^{*}(\mathrm{C} 17-\mathrm{H} 18)$ | 0.05905 | 20.78 | 0.69 | 0.108 |

ED, Electron density,
${ }^{a} \mathrm{E}^{(2)}$ means energy of hyper conjugative interaction (stabilization energy),
${ }^{\mathrm{b}}$ Energy difference between donor and acceptor i and j NBO orbitals,
${ }^{c} F(i, j)$ Fork matrix element between $i$ and $j$ NBO orbitals.

### 1.11 Mulliken atomic charges

The Mulliken population analysis gives the individual charge on each atoms in the molecule and it is the several characteristics of the molecular structure. The mulliken atomic charges are shown in Fig 8. Among the carbon atoms, all the carbon atoms except C3, C4, C6, C12 and C17 are all of negative charges. The presence of nitrogen and oxygen atoms are more electronegative and it makes the two double bond of carbon atoms C12 and C17 more negative. Hence N16 and O19 are more negative which can attract the more atoms from the
neighbor. The C115 atom is one of positive charge attached to the electronegative atom C2 because this atom makes the double bond C2 and C1 atoms are the electro negativity. All the hydrogen atoms are positively charged because they donate electrons to the nearby carbon atoms [53] and all the carbon and nitrogen atoms are negatively charged. The atomic charges are given in the Table 7. Hence, the sums of Mulliken charges of all atoms are calculated to be zero and confirmed the charge neutrality.


Fig 8. Mulliken atomic charges.
Table 7. Mulliken atomic charges of 8CQ2C

| Atomic Number | Natural atomic charges |
| :---: | :---: |
| C1 | -0.082 |
| C2 | -0.121 |
| C3 | 0.257 |
| C4 | 0.132 |
| C5 | -0.131 |
| C6 | 0.081 |
| H7 | 0.118 |
| C8 | -0.087 |
| H9 | 0.100 |
| H10 | 0.106 |
| C11 | -0.098 |
| C12 | 0.218 |
| H13 | 0.107 |
| H14 | 0.127 |
| C115 | 0.022 |
| N16 | -0.550 |
| C17 | 0.274 |


| H18 | 0.094 |
| :--- | :--- |
| O19 | -0.404 |

### 1.12 Non-Linear Optical (NLO) Property

The NLO activity provides the key functions for optical modulation and switching, frequency shifting and optical logic for the developing technologies in areas such as communication, signal processing and optical inter connections [54,55]. Organic molecules able to manipulate photonic signals efficiently are of importance in technologies such as optical communication, optical computing, and dynamic image processing [56,57]. The polarizability, hyperpolarizability and dipole moment also calculated by using DFT/B3LYP method with 6-31 G (d,p) basis set. To find out the dipole moment ( $\mu$ ), polarizability ( $\alpha$ ), and hyper polarizability ( $\beta$ ) are defined as [58] using $x$, $y$ and $z$ components,
Dipole moment is

$$
\mu=\left(\mu^{2} x+\mu^{2} y+\mu^{2} z\right)^{1 / 2}
$$

Polarizability is

$$
\alpha_{0}=(\alpha x x+\alpha y y+\alpha z z) / 3
$$

Hyperpolarizability is

$$
\beta=\left(\beta^{2} x+\beta^{2} y+\beta^{2} z\right)^{1 / 2}
$$

Where

$$
\begin{aligned}
& \beta x=(\beta x x x+\beta x y y+\beta x z z) \\
& \beta y=(\beta y y y+\beta y z z+\beta y x x) \\
& \beta z=(\beta z z z+\beta z x x+\beta z y y) \\
& \beta=\left[(\beta x x x+\beta x y y+\beta x z)^{2}+(\beta y y y+\beta y z z+\beta y x x)^{2}+(\beta z z z+\beta z x x+\beta z y y)^{2}\right]^{1 / 2}
\end{aligned}
$$

The polarizability and hyperpolarizability values are given by Gaussian 09 and it is reported in atomic units (a.u), the calculated values have been converted into electrostatic units (e.s.u) ( $\alpha ; 1$ a.u. $=0.1482 \times 10-24$ e.s.u; $\beta ; 1$ a.u. $=8.3693 \times 10-33$ e.s.u) [59]. The values of dipole moment is $\mu=4.885 \mathrm{D}$ and hyperpolarizability is $\beta=6.7349 \times 10^{-30}$ esu. The title compound is 18 times greater than that of the standard NLO material Urea $\left(0.3728 \times 10^{-30}\right.$ esu). Hence, 8CQ2C also has NLO property. The calculated $\beta$ components and $\beta_{\text {tot }}$ value of 8CQ2C are given in Table 8.

Table 8. Calculated $\beta$ components and $\beta_{\text {tot }}$ value of 8CQ2C.

| $\beta$ Components | 8CQ2C |
| :--- | :--- |


| $\beta_{\mathrm{xxx}}$ | 521.3285 |
| :--- | :--- |
| $\beta_{\mathrm{xxy}}$ | 224.6322 |
| $\beta_{\mathrm{xyy}}$ | 13.76503 |
| $\beta_{\mathrm{yyy}}$ | 220.9898 |
| $\beta_{\mathrm{xxz}}$ | 14.46556 |
| $\beta_{\mathrm{xyz}}$ | 8.52040 |
| $\beta_{\mathrm{yyz}}$ | -5.54994 |
| $\beta_{\mathrm{xxz}}$ | -0.02040 |
| $\beta_{y z z}$ | -0.39329 |
| $\beta_{z z z}$ | 0.03289 |
| $\beta_{\text {Total }}(\mathrm{esu})$ | 6.73495 X 10 |

### 1.13 Molecular docking analysis

The molecular docking method is used to find out the ligand binding site to a receptor and predict the binding orientations. Now a days, computational investigations act as emerging tools for studying a number of molecular parameters [60, 61]. In the present work, the molecular docking analysis was carried out for the 8CQ2C ligand with more common targeted proteins associated with the Parkinson's and schizophrenia diseases. Parkinson's disease is a neurodegenerative disorder and Schizophrenia is a brain disorder but it also life -long disease that cannot be cured but can be controlled with proper treatment. Recently, many researchers have been focusing the molecular docking studies on the mentioned targeted proteins [62-65].

To find the two proteins like 2 V 60 and 4 MRW and these structures of the targeted proteins were obtained from the RCSB PDB format [66]. Autodock Tools version 1.5.6 is recently have been used as a convenient tool to get insights of the molecular mechanism of ligand-protein interactions and to bind the receptor of 3D structure. Discovery studio visualizer were utilize for the evaluation of hydrogen bonds in the ligand-protein interaction. The ligand PDB file was created by using the optimized molecular structure of the 8CQ2C molecule. The AutoDock Tools graphical user interface [67] was used to prepare the target proteins for docking. The ligand and water molecules present in the targeted proteins were removed. The polar hydrogen bond and Kollman charges were added in the targeted proteins.

The intermolecular interactions were studied for the 8CQ2C with different proteins like 2V60 and 4MRW. As a result of docking is find out the ten conformers were obtained and
their binding energies with inhibitory constants were listed in given Table 9. The binding energies with inhibitory constants of 2 V 60 and 4 MRW are found to be -6.76 and -5.7 k cal $\mathrm{mol}^{-1}$ with 11.1 and 6.91 um respectively. 3D model interaction of title compound with proteins shown in Fig 9. 2D model of interaction between amino acid residues is shown in Fig 11 and it is predicted the green dotted lines are represent the hydrogen bonds, the dark yellow dotted lines re represent the electrostatic bonds and pink dotted lines are represent the hydrophobic bonds. The amino acids are mainly involved in the ligand and protein interactions. The docking of 8CQ2C interact with different amino acids like Serine A: 59 with 2.10 A ${ }^{\circ}$ (Symbol SER A: 59) and Tyrosine A:60 with 2.25 A $^{\circ}$ (Symbol TYR A:60) in 2V60 protein. For another amino acid like Tyrosine A: 524 with $2.61 \mathrm{~A}^{\circ}$ (Symbol TYR A: 524) were interacted with 8CQ2C and 4MRW protein. These interactions are depends upon the nature of the functional groups present in the ligand. The docked confirmation of the active site of 8CQ2C is shown in Fig 10. These results indicate that the 8CQ2C ligand possesses the lowest binding energy and inhibition constant for the targeted proteins. Hence, these docking results will be useful for developing the effective treatment of Parkinson's and Schizophrenia diseases.

(a) 2V60 receptor

(b) 4MRW receptor

Fig 9. 3D Model of ligand 8CQ2C with (a) 2V60 and (b) 4MRW receptors

(a) 2V60 receptor

(b) 4MRW receptor

Fig 10. The schematic of the docked confirmation of the active site of 8CQ2C with (a) 2V60 and (b) 4MRW receptors.

(a) 2V60 receptor

(b) 4MRW receptor

Fig 11. 2D model of interaction between amino acid residues with 8CQ2C and 2V60 \& 4MRW.

Table 9. The possible conformers of 8CQ2C docked with 2V60 and 4MRW receptors.

| Conformer | Binding Energy $\left(\mathrm{k} \mathrm{cal} \mathrm{mol}^{-1}\right)$ |  | Inhibitory constant $(\mu \mathrm{M})$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 2 V 60 | 4 MRW | 2 V 60 | 4 MRW |


| 1. | -6.76 | -5.7 | 11.1 | 66.58 |
| :---: | :---: | :---: | :---: | :--- |
| 2. | -6.76 | -5.7 | 11.11 | 66.42 |
| 3. | -6.76 | -5.59 | 11.16 | 79.3 |
| 4. | -6.75 | -5.59 | 11.24 | 80.36 |
| 5. | -6.76 | -5.69 | 11.07 | 67.27 |
| 6. | -6.75 | -5.7 | 11.33 | 66.91 |
| 7. | -6.76 | -5.69 | 11.13 | 67.34 |
| 8. | -6.76 | -5.68 | 11.04 | 68.41 |
| 9. | -6.75 | -5.6 | 11.2 | 79.09 |
| 10. | -6.76 | -5.7 | 11.12 | 66.69 |

### 1.14 Conclusion

In the present work, 8CQ2C were optimized by using DFT/B3LYP method with 6-31 $\mathrm{G}(\mathrm{d}, \mathrm{p})$ basis set. The optimized structure provides various bond lengths and bond angles coinciding with the literature values and also calculated the global minimum energy $\mathrm{E}=$ 974.8020 Hartrees in title compound. The various vibrational frequencies are calculated on the basis of PED by using VEDA program. The theoretical NMR provides the ${ }^{1} \mathrm{H}$ NMR and ${ }^{1} \mathrm{C}$ NMR chemical shift values are reported. The theoretical VCD spectrum was simulated and also predicted to both obtain optical and biological activity of the title compound. The UV-Vis analysis was simulated theoretically and the maximum absorption peak is 246.23 nm is associated with $\pi \rightarrow \pi^{*}$ transition. The band gap energy is 0.29 eV and the HOMO-LUMO provides the energy difference between the charge transfer interactions in the molecule. The possible electrophilic and nucleophilic reactive sites were predicted by MEP. In the negative potential sites are on oxygen, chlorine and nitrogen atoms as well as the positive potential sites are hydrogen atoms in the title compound. Stabilization of title compound is carried out by hyper conjugation interactions and charge delocalization has been calculated by NBO analysis. Mulliken charge distribution has been calculated. The dipole moment, polarizability and hyperpolarizability reveal that title compound has considerable NLO activity and hence can be used for material science applications. The docking result clearly provides that the two different proteins interact with ligand. The most active side is C17=O19 of 8CQ2C ligand interactions takes for different amino acids like SER A: 59, TYR A: 60 and TYR A: 524 with given resolutions $2.10 \mathrm{~A}^{\circ}, 2.25 \mathrm{~A}^{\circ}$ and $2.61 \mathrm{~A}^{\circ}$ respectively. Hence, 8 CQ 2 C may be used in the treatment of Parkinson's and Schizophrenia diseases but in-vito \& in-vivo studies have to been carried out before implementation.

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# THE DUAL BETWEEN MAN AND ROBOT IN THE ERADICATION OF WEEDSBY INCORPORATING BIG M METHOD 

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#### Abstract

In this paper, with the aid of Big-M method in Linear Programming Problem, an attempt has been done to find out which method among Manual method and Robotic method is more suitable and beneficial in eradicating the weeds in tomato fields, thereby providing a long lasting solution for a problem prevailing for several years in agricultural fields.


## 1. INTRODUCTION

Agriculture is the backbone of our country. Though the world is now becoming technologically sound, well - equipped with multi-storey buildings, making door delivery of all types of goods and all varieties of food possible, it is necessary to understand that we cannot obtain food from these web resources and equipped buildings. As for farming in Salem district, about $70 \%$ of the population is engaged in Agriculture. In relation to Agricultural Department Administration, Salem district is divided into 5 agricultural divisions namely Salem, Attur, Sankagiri, Mettur and Omalur and Tomatoes are one of the leading vegetable cropsproduced in our Salem district. In 2020, over 37,500 pounds of tomatoes were produced per acre within three months. To achieve this feat 5,000 tomato plants were required. Linear programming problem [1] was first introduced by Leonid Kantorovich [2] in 1939. He developed the earliest linear programming problems that were used by the army during WWII in order to reduce the costs of the army and increase the efficiency in the battlefield.

Though several plants were cultivated for production of tomatoes, unwanted weeds plants occupied half of the tomato field. To minimize the weed plants in the tomato field, farmers strive hard to eradicate them by plucking them manually or spraying chemicals in the agricultural field. Parallelly, real-time intelligent "Robotic weed control machine" which was developed for selectively spraying chemicals to unwanted weed plants in early stage is also used to minimize weeds. The precision algorithm is used in the Robotic weed control machine to classify the grass like weeds from tomato plants and applying a chemical spray only to the target weeds while travelling at a speed of $0.45 \mathrm{~m} / \mathrm{s}$. Our ultimate aim is to compare manual methods and robotic methods to find out which is the best method to eradicate weeds in farming fields.

### 1.1. MATERIALS AND METHODS OF ROBOTIC WEED CONTROL MACHINE [3]

The Materials and Methods that are used in robotic weed control system consisted ofthree separate subsystems:

1. The machine vision subsystem,
2. The spray actuator subsystem,
3. The displacement sensing subsystem.


Figure 3 [3]: Schematic showing a side view of the robotic weed control system as mounted on the tractor-drawn toolbar.

### 1.2. ALGORITHM FOR IMAGE PROCESSING SYSTEM [4]

The algorithm starts with image acquisition. The next step is the processing of the image captured. The image is subjected to morphological modifications like thresholding, erosion and dilation to detect the presence of the plants in the Region of Interest (ROI), and in determining whether it is a weed or the plantation crop and final step is the directedspray of the herbicide on the weeds in the ROI
Step 1: Capturing of images.
Step 2: Conversion of RGB image to binary image.
Step 3: An image segmentation is conducted to divide the image into two classes i.e.plant and background.

Step 4: Applying Erosion on image using structuring element.
Step 5: Applying Dilation on the eroded image with structuring element.
Step 6: Summation of pixels by dividing images into parts.
Step 7: Compare sum and threshold and identify plant as narrow or broad leaves.

### 1.3. MATHEMATICAL ANALYSIS FOR IMAGE PROCESSING SYSEM [4]

The proposed mathematical Analysis is given below.
$\mathrm{I}=$ input images $\mathrm{O}=$ output images
$\mathrm{F}=$ functions to be appliedI $=\{\mathrm{I} 1, \mathrm{I} 2, \mathrm{I} 3\}$
$\mathrm{F}=\{\mathrm{F} 1, \mathrm{~F} 2, \mathrm{~F} 3, \mathrm{~F} 4\}$
Capturing of the ImageOutput $1=\mathrm{F} 1\{\mathrm{I} 1\}$
$>\quad$ To convert the RGB image into Grayscale imageF2 $=0.229 * \mathrm{R}+0.587 * \mathrm{G}+0.114 * \mathrm{~B}$ Output2 $=$ F2 2 I2 $\}$
$>$ To convert the Grayscale image into Binary image F3=im2bw
> To apply morphological operations
$>\quad$ Apply Erosion on Binary image $\mathrm{A}=$ imerode $(\mathrm{A}, \mathrm{B})$
> Apply Dilation on Eroded Image $\mathrm{D}=$ imdilate (A,se)
> To calculate sum and threshold value

$$
\text { Sum }=\sum_{i=0}^{N} \sum_{i=0}^{M}
$$

Sum $=\operatorname{sum}($ sum(Eroded image) $)$
$\mathrm{N}=$ number of broad leaves
$\mathrm{M}=$ number of narrowleaves

## 2. PRELIMINARIES

In this section, the basic concepts required for analysis are studied.
Definition 1.2.1 [5] Operation Research is the application of the scientific methods, techniques and tools to problems involving the operations of system so as to provide in control of the operations with optimum solution to the problem.

Definition 1.2.2 [5] Linear programming is a method to achieve the best outcome in a mathematical model whose requirements are represented by linear relationships. Linear programming is a special case of mathematical programming.

Definition 1.2.3 [5] Let $Z$ be a linear function on $\mathrm{R}^{\mathrm{n}}$ defined by
(a) $Z=c_{1 x_{1}}+c_{2} x_{2}+\cdots+c_{n} x_{n} w h e r e c_{j} s$ are constants. Let $\left(a_{i j}\right)$ be an $m \times n$ realmatrixand $\left\{b_{1}\right.$, $\left.\mathrm{b}_{2}, \ldots, \mathrm{~b}_{\mathrm{m}}\right\}$ be a set of constants such that
(b) $a_{111} x_{1}+a_{12 x_{2}}+\cdots+a_{1 n} x_{n}(\geq,=, \leq)$
$\mathrm{b}_{1} \mathrm{a}_{21} \mathrm{x}_{1}+\mathrm{a}_{22 \mathrm{x} 2}+\cdots+\mathrm{a}_{2 \mathrm{n}} \mathrm{x}_{\mathrm{n}}(\geq,=, \leq) \mathrm{b}_{2}$
$\vdots$
$a_{m 1} x_{1}+a_{m 2} x_{2}+\cdots+a_{m n x} x_{n}(\geq,=, \leq) b_{n}$
and finally let
(c) $\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots, \mathrm{x}_{\mathrm{n}} \geq 0$

The problem of determining an n -tuple ( $\mathrm{x} 1, \mathrm{x} 2, \ldots, \mathrm{x}_{\mathrm{n}}$ ) which makes a minimum (or minimum) and satisfies (b) and (c) is called the General linear programming problem.
Definition 1.2.4 [5] The linear function $Z=c_{1 \times 1}+c_{2} \mathrm{x}_{2}+\cdots+\mathrm{c}_{\mathrm{n}} \mathrm{X}_{\mathrm{n}}$ which is to be minimized (or maximized) is called Objective function of the General L.P.P.
(a) Definition 1.2.5 [5] The inequalities $a_{11 x_{1}}+a_{12} x_{2}+\cdots+a_{1 n} x_{n}(\geq,=, \leq)$ $\mathrm{b}_{1} \mathrm{a}_{21 \mathrm{x} 1}+\mathrm{a}_{22 \mathrm{x} 2}+\cdots+\mathrm{a}_{2 \mathrm{n} \mathrm{x}_{\mathrm{n}}}(\geq,=, \leq) \mathrm{b}_{2}$ $\vdots$
$a_{\mathrm{m} 1 \mathrm{x} 1}+\mathrm{am}_{\mathrm{m} 2 \mathrm{x} 2}+\cdots+\mathrm{amnx}_{\mathrm{n}}(\geq,=, \leq) \mathrm{b}_{\mathrm{n}}$
are called the Constraints of the General L.P.P
Definition 1.2.6 [5] The set of inequalities $\mathrm{x} 1, \mathrm{x} 2, \ldots, \mathrm{x}_{\mathrm{n}} \geq 0$ is usually known as the set of Non-negative restrictions of the General L.P.P
Definition 1.2.7 [5] An n-tuple ( $\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots, \mathrm{x}_{\mathrm{n}}$ ) of real numbers which satisfies the constrains of a General L.P.P is called a Solution to the General L.P.P

Definition 1.2.8 [5] Any solution to a General L.P.P which also satisfies the non-negative restrictions of the problem is called a Feasible Solution to the General L.P.P

Definition 1.2.9 [5] Any feasible solution which optimizes minimizes (or maximizes) the objective of a General L.P.P is called an Optimum Solution to the General L.P.P
Definition 1.2.10 [5] The Big-M method is a method of solving linear programming problems using the simplex algorithm to problems that contain "greater-than" constrains.

Definition 1.2.11 [5] A feasible solution to an L.P.P. which is also a basic solution to the problem is called a Basic Feasible Solution to the L.P.P.

## 3. COMPARISON OF ERADCIATION OF WEEDS BY ROBOTIC MEANS AND MANUAL METHOD IN TOMATO FIELDS USING BIG M METHOD.

Removing of unwanted weed plants by farmersin the field would be more expensive and only minimum number of weed plants are removed by framers in particular time. In order to minimize the expenditure and removing maximum number of weed plants in particular time, we use the real-time intelligent "Robotic weed control machine". In this section, we compare the significance of robotic weed control machine and the manual effort of humans in removing weeds from tomato fields using "Big-M Method" (Method of penalties) to find out the best between the two.

The most common types of weeds prevalent in the tomato fields are Mauritian grass, Siam weeds and Spiny pigweed. Thus secondary data has been taken to find out how much of
these weeds are present in several fields and with the aid of Big-M method, minimization of these weeds in each field is determined.

## ALGORITHM FOR MATHEMATICAL FORMULATION OF THEPROBLEM [8]

The formulation of the Linear Programming Problem is explained below:
Step-1: Study the given situation to find the key decisions to be made.

Step-3: State the feasible alternative which generally are: $x_{j} \geq 0$, for all $j$.
Step-4: Identify the constraints in the problem and express them as linear inequalities or equations, LHS of which are linear functions of the decision variables.
Step-5: Identify the objective function and express it as a linear function of the decision variables.

## ALGORITHM OF BIG-M METHOD [5]

The Big-Method is an alternative method of solving a linear programming problem involving artificial variables. In this method we assign a very high penalty (say M) to the artificial variables in the objective function. The procedure is as follows.

Step-1: Write the given L.P.P into its standard form and check whether there were exists a starting basic feasible solution.
Step-1a: If there is a ready starting basic feasible solution. Move on to step-3.
Step-1b: If there does not exists a ready starting basic feasible solution. move on to step-2.
Step-2: Add artificial variables to the left side of each equation that has no obvious starting basic variables. Assign a very high penalty (say M) to these variables in objective function.
Step-3: Apply simplex method to the modified L.P.P. Following cases may arise at the last iteration.

Step-3a: At least one artificial variable is present in the basis with zero value. In such a case the current basic feasible solution is degenerate.
Step-3b: At least one artificial variable is present in the basis with a positive value.
In such a case, the given problem is said to have a pseudo-optimum basic feasible solution. For the solution of any L.P.P by simplex algorithm, the existence of an initial basic feasible solution is always assumed. The steps for the computation of an optimum solution are as follows.

Step-1: Check whether the objective function of the given L.P.P is to be maximized. If it is to be minimized then we convert into a problem of maximizing it by using the result. Min $z=-$ Max (-z).
Step-2: Check whether all $b_{i}(i=1,2 \ldots . . \mathrm{m})$ are non-negative. If any one of the $b_{i}$ is negative
then multiply the corresponding equation of the constraint by -1 , so as to get all $b_{i}$ (i=1,2,.....m)non-negative.

Step-3: Convert all the equations of the constrains into equations by introducing slack or surplus variable in the constrains. Put the costs of the variables equal to zero.
Step-4: Obtain a basic feasible solution to the problem in the form $\mathrm{xB}_{\mathrm{B}}=\mathrm{B}^{-1} \mathrm{~b}$ and put in the first column of the simplex table.

Step-5: Compute the net evaluations $\mathrm{z}_{\mathrm{j}} \mathrm{C}_{\mathrm{j}}(\mathrm{j}=1,2 \ldots \ldots . \mathrm{m})$ by using the relation $\mathrm{Z}_{\mathrm{j}}-\mathrm{C}_{\mathrm{j}}=\mathrm{C}_{\mathrm{Byj}}-\mathrm{c}_{\mathrm{j}}$ where $y_{j}=B^{-1} a_{j}$. Examine the sign of $Z_{j}-C_{j}$
Step-5a: If all $\left(Z_{j}-C_{j}\right) \square 0$ then the initial basic feasible solution $C_{B}$ is an optimum basic feasible solution.

Step-5b: If at least one $\left(\mathrm{Z}_{\mathrm{j}}-\mathrm{C}_{\mathrm{i}}\right)<0$, proceed on to the next step.
Step-6: If there are more than one negative $Z_{j}-C_{j}$ then choose the most negative of them.
Let it be $\mathrm{z}_{\mathrm{r}}$ - c there is an unbounded solution to the given problems.
Step-7: Compute the ratios $\left\{\mathrm{X}_{\mathrm{Bi}} \mid \mathrm{Y}_{\mathrm{ir},} \mathrm{Y}_{\mathrm{ir}}>0, \mathrm{i}=1,2, \ldots . . \mathrm{m}\right\}$ and choose the minimum of them. Let element ykr, which is in the k-th row and the r-th column is known as theleading element (pivot element) of the table.

Step-8: Convert the leading element to unity by dividing its row by the leading element itself and all other elements in its column to zero.

Step-9: Go to step-5 and repeat the computational procedure until either an optimum solution is obtained or there is an indication of an unbounded solution.

## COMPUTATION TO FIND OUT THE NUMBER OF WEEDS ERADICATED BY MACHINE.

The following table gives the number of weeds eradicated by machines in several field based on [6]

| No of trials <br> machinetravel in <br> field | Mauritian <br> grass <br> (weed-1) | Siam weeds <br> (weed-2) | Spiny <br> pigweed <br> (weed-3) |
| :---: | :---: | :---: | :---: |
| 1 | 83 | 65 | 52 |
| 2 | 93 | 67 | 35 |
| 3 | 115 | 108 | 52 |
| 4 | 39 | 34 | 27 |


| 5 | 212 | 188 | 113 |
| :---: | :---: | :---: | :---: |
| 6 | 136 | 55 | 28 |
| 7 | 118 | 138 | 79 |
| Total no of weeds | 796 | 655 | 386 |

## Steps 1,2 and 3:

Decision Variables: Let $x_{i}(i=1,2,3,4,5,6,7)$ represent the number of times the machine moves on the field.

Objective function: The objective is to find out how many weeds are eradicated by the method of spraying chemicals in agricultural fields.
i.e., Maximize $z=93 x_{1}+62 x_{2}+14 x_{3}+14 x_{4}+79 x_{5}+60 x_{6}+71 x_{7}$.
subject to constrains: Number of unremoved weeds in the fields.
$83 \mathrm{x}_{1}+93 \mathrm{x}_{2}+115 \mathrm{x}_{3}+39 \mathrm{x}_{4}+212 \mathrm{x}_{5}+136 \mathrm{x}_{6}+118 \mathrm{x}_{7}=796$ (Mauritian grass)
$65 \mathrm{x}_{1}+67 \mathrm{x}_{2}+108 \mathrm{x}_{3}+34 \mathrm{x}_{4}+188 \mathrm{x}_{5}+55 \mathrm{x}_{6}+138 \mathrm{x}_{7}=655$ (Siam weeds)
$52 \mathrm{x}_{1}+35 \mathrm{x}_{2}+52 \mathrm{x}_{3}+27 \mathrm{x}_{4}+113 \mathrm{x}_{5}+28 \mathrm{x}_{6}+79 \mathrm{x}_{7}=386$ (spiny pigweed)

$$
\text { and } x_{i}=0(i=1,2,3,4,5,6,7)
$$

## FORMULATION OF BIG-M PROBLEM FOR IDENTIFYING THE NUMBER OF WEEDS ERADICATED BYMACHINE

Write the given L.P.P in the standard form.
Maximize $z=93 x_{1}+62 x_{2}+14 x_{3}+14 x_{4}+79 x_{5}+60 x_{6}+71 x_{7}-M A_{1}-M A_{2}-$ MA $_{3}$
Subject to constrains:
$83 x_{1}+93 x_{2}+115 x_{3}+39 x_{4}+212 x_{5}+136 x_{6}+118 x_{7}+A_{1}=796$
$65 \mathrm{x}_{1}+67 \mathrm{x}_{2}+108 \mathrm{x}_{3}+34 \mathrm{x}_{4}+188 \mathrm{x}_{5}+55 \mathrm{x}_{6}+138 \mathrm{x}_{7}+\mathrm{A}_{2}=655$
$52 \mathrm{x}_{1}+35 \mathrm{x}_{2}+52 \mathrm{x}_{3}+27 \mathrm{x}_{4}+113 \mathrm{x}_{5}+28 \mathrm{x}_{6}+79 \mathrm{x}_{7}+\mathrm{A}_{3}=386$
and $\mathrm{x}_{\mathrm{i}}=0(\mathrm{i}=1,2,3,4,5,6,7)$
INITIAL ITERATION

|  | G | : | 93 | 62 | 72 | 14 | 79 | 60 | 71 | -M | -M | -M | Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{\mathrm{B}}$ | $Y_{B}$ | $\mathrm{X}_{\mathrm{B}}$ | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | X4 | $\mathrm{X}_{5}$ | $\mathrm{X}_{6}$ | $\mathrm{X}_{7}$ | $\mathrm{A}_{1}$ | $\mathrm{A}_{2}$ | $\mathrm{A}_{3}$ |  |
| -M | $\mathrm{A}_{1}$ | 796 | 83 | 93 | 115 | 39 | 212 | 136 | 118 | 1 | 0 | 0 | 3.75 |
| -M | $\mathrm{A}_{2}$ | 655 | 65 | 67 | 108 | 34 | 188 | 55 | 138 | 0 | 1 | 0 | 3.48 |
| -M | $\mathrm{A}_{3}$ | 386 | 52 | 35 | 52 | 27 | 113 | 28 | 79 | 0 | 0 | 1 | 3.42 |
|  | Zi | -1837M | -200M | -195M | -275M | -100M | -513M | -219M | -335M | -M | -M | -M |  |
|  | $\mathrm{Zi} \mathrm{Cam}_{\mathrm{i}}$ |  | $\begin{aligned} & -200 \mathrm{M} \\ & -93 \end{aligned}$ | $\begin{aligned} & -195 \mathrm{M} \\ & -67 \end{aligned}$ | $\begin{aligned} & -275 \mathrm{M} \\ & -72 \end{aligned}$ | $\begin{array}{\|l} \hline-100 \mathrm{M} \\ -14 \end{array}$ | $\begin{aligned} & -513 M \\ & -79 \end{aligned}$ | $\begin{aligned} & -219 \mathrm{M} \\ & -60 \end{aligned}$ | $\begin{aligned} & -335 \mathrm{M} \\ & -71 \end{aligned}$ | 0 | 0 | 0 |  |

- Since all $\left(Z_{j}-C_{j}\right) \leq 0$ the current solution is not optimal
- The most negative of $\left(Z_{j}-C_{j}\right)$ corresponds to $X_{5}$ and so $X_{5}$ enters the basis.
- The leaving variable correspond to the minimum value of $\mathrm{R}_{3}$, which is $A_{3}$.
- Here pivot element is 113

New Pivot Equation $=$ old $A_{3}$ Equation $\square$ Pivot Element

| Pivot equation | 3.42 | 0.46 | 0.31 | 0.46 | 0.24 | 1 | 0.25 | 0.70 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.01 |  |  |  |  |  |  |  |  |  |  |

New $A_{1}$ equation $=$ old $A_{1}$ equation - (212) (pivot equation)
New $A_{1}$ equation

| Old <br> $A_{1}$ | 796 | 83 | 93 | 115 | 39 | 212 | 136 | 136 | 118 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| -212 | 3.42 | 0.46 | 0.31 | 0.46 | 0.24 | 1 | 0.25 | 0.70 | 0 | 0 | 0.01 |
| New $A_{1}$ | 70.96 | -14.52 | 27.28 | 17.48 | -11.88 | 0 | 83 | -30.40 | 1 | 0 | 2.12 |

New $A_{2}$ equation $=$ old $A_{2}$ equation - (188) (pivot equation)
New $A_{2}$ equation

| Old $A_{2}$ | 655 | 65 | 67 | 108 | 34 | 188 | 55 | 138 | 0 | 1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $-\mathbf{1 8 8}$ | 3.42 | 0.46 | 0.31 | 0.46 | 0.24 | 1 | 0.25 | 0.70 | 0 | 0 | 0.01 |
| new $A_{2}$ | 12.04 | -21.48 | 8.72 | 21.52 | -11.12 | 0 | 8 | 2.64 | 0 | 0 | 1.88 |

## FIRST ITERATION

|  | $\mathrm{C}_{\mathrm{i}}$ | $:$ | 93 | 62 | 72 | 14 | 79 | 60 | 71 | -M | -M | -M | Ratio |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{C}_{\mathrm{B}}$ | $\mathrm{Y}_{\mathrm{B}}$ | $\mathrm{X}_{\mathrm{B}}$ | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{X}_{5}$ | $\mathrm{X}_{6}$ | $\mathrm{X}_{7}$ | $\mathrm{~A}_{1}$ | $\mathrm{~A}_{2}$ | $\mathrm{~A}_{3}$ |  |
| -M | $\mathrm{A}_{1}$ | 70.96 | -14.52 | 27.28 | 17.48 | -11.88 | 0 | 83 | -30.4 | 1 | 0 | 2.12 | 0.85 |
| -M | $\mathrm{A}_{2}$ | 12.04 | -21.48 | 8.72 | 21.52 | -11.12 | 0 | 8 | 2.64 | 0 | 1 | 1.88 | 1.51 |
| 79 | $\mathrm{X}_{5}$ | 3.42 | 0.46 | 0.31 | 0.46 | 0.24 | 1 | 0.25 | 0.70 | 0 | 0 | 0.01 | 13.68 |
|  | $\mathrm{Z}_{\mathrm{K}}$ | -83 M | 36 M | -36 M | -39 M | -23 M | 79 | -91 M | 27.82 | -M | -M | -4 M |  |
|  |  | +27 | +36.34 | +24.49 | +36.34 | +18.96 |  | +19.75 | +55.3 |  |  | +0.79 |  |
|  | $\mathrm{Z}_{\mathrm{i}} \mathrm{C}_{\mathrm{i}}$ |  | 36 M | -36 M | -39 M | -23 M | 0 | -91 M | 27.82 M | 0 | 0 | -3 M |  |
|  |  |  | -56.66 | -37.51 | -35.66 | +4.96 |  | -40.25 | -15.70 |  |  | +0.76 |  |

- Since all $\left(Z_{j}-C_{j}\right) \leq 0$ the current solution is not optimal
- The most negative of $\left(Z_{j}-C_{j}\right)$ corresponds to $X_{6}$ and so $X_{6}$ enters the basis.
- The leaving variable corresponds to the minimum value of $R_{1}$, which is $A_{1}$.
- Here pivot element is 83


## New Pivot Equation $=$ Old $\boldsymbol{A}_{1}$ Equation $\div$ Pivot Element

| pivot equation | 0.85 | -0.17 | 0.33 | 0.21 | -0.14 | 0 | 1 | -0.37 | 0.01 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

New $A_{2}$ equation $=$ old $A_{2}$ equation - (8) (pivot equation) New $A_{2}$ equation

| Old $A_{2}$ | 12.04 | -21.48 | 8.72 | 21.52 | -11.12 | 0 | 8 | 2.64 | 0 | 1 | 1.88 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| -8 | 0.85 | -0.17 | 0.33 | 0.21 | -0.14 | 0 | 1 | -0.37 | 0.01 | 0 | 0.03 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| New $A_{2}$ | 5.24 | -20.12 | 6.08 | 19.84 | -10 | 0 | 0 | 5.60 | 0.08 | 1 | 1.64 |

New $X_{5}$ equation $=$ old $X_{5}$ equation - (0.25) (pivot equation)
New $X_{5}$ equation

| Old $\mathrm{X}_{5}$ | 3.42 | 0.46 | 0.31 | 0.46 | 0.24 | 1 | 0.7 | 0 | 0 | 0 | 0.01 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{- 0 . 2 5}$ | 0.85 | -0.17 | 0.33 | 0.21 | -0.41 | 0 | 1 | -0.37 | 0.01 | 0 | 0.03 |
| New $\mathrm{X}_{5}$ | 3.21 | 0.50 | 0.23 | 0.41 | 0.27 | 1 | 0 | 0.79 | 0 | 0 | 0 |

## Proceed similarly, we get FIFTH ITERATION

|  | $\mathrm{c}_{\mathrm{j}}$ | $:$ | 93 | 62 | 72 | 14 | 79 | 60 | 71 | -M | -M | -M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{c}_{\mathrm{B}}$ | $\mathrm{y}_{\mathrm{B}}$ | $\mathrm{x}_{\mathrm{B}}$ | $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ | $\mathrm{x}_{3}$ | $\mathrm{x}_{4}$ | $\mathrm{x}_{5}$ | $\mathrm{x}_{6}$ | $\mathrm{x}_{7}$ | $\mathrm{~A}_{1}$ | $\mathrm{~A}_{2}$ | $\mathrm{~A}_{3}$ |
| 60 | $\mathrm{x}_{2}$ | 2.42 | 0 | 1 | 0 | -0.27 | -0.23 | 3.85 | -1.81 | 0.04 | -0.04 | 0.04 |
| 72 | $\mathrm{x}_{3}$ | 3.61 | 0 | 0 | 1 | 0.31 | 1.52 | -1.69 | 2.05 | -0.02 | 0.04 | 0.02 |
| 93 | $\mathrm{x}_{1}$ | 4.03 | 1 | 0 | 0 | 0.72 | 1.44 | -0.54 | 1.21 | -0.01 | -0.02 | -0.05 |
|  | $\mathrm{z}_{\mathrm{j}}$ | 784.75 | 93 | 62 | 72 | 72.54 | 229.1 | 66.8 | 147.91 | 0.11 | -1.46 | -0.73 |
|  | $\mathrm{z}_{\mathrm{j}}-\mathrm{c}_{\mathrm{j}}$ |  | 0 | 0 | 0 | 58.54 | 150.1 | 6.8 | 76.91 | $0.11+\mathrm{M}$ | -1.46 | -0.73 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

Since all $\left(Z_{j}-C_{j}\right) \geq 0$, the current solution is optimal
$X_{1}=4.03, X_{2}=2.42, X_{3}=3.61$ and Maximize $Z=784.75$
Thus, it is clear that the weeds are eradicated to the maximum extent in utmost three trials when they are eradicated by robotic machines.

## COMPUTATION FOR THE ERADICATION OF WEEDS BY MANUALMETHOD

The following table gives the number of weeds eradicated by farmers

| No. of times <br> farmers travel in <br> fields | Mauritian <br> grass <br> (weed-1) | Siam <br> weeds <br> (weed-2) | Spiny <br> pigweed <br> (Weed-3) |
| :---: | :--- | :--- | :--- |
| 1 | 90 | 72 | 75 |
| 2 | 82 | 64 | 79 |
| 3 | 102 | 92 | 113 |
| 4 | 33 | 28 | 36 |
| 5 | 209 | 83 | 279 |
| 6 | 116 | 54 | 60 |
| 7 | 152 | 92 | 113 |
| Total no. of weeds | 784 | 775 |  |

## MATHEMATICAL FORMULATION FOR THE GIVEN DATA

Here we determine the number of weeds removed by chemical spray and give the data for the number of weeds removed by farmers in the place of decision variables:

Decision Variables: Let $\mathrm{x}_{\mathrm{i}}(\mathrm{i}=1,2,3,4,5,6,7)$ represents the number of farmers in the field.
Objective function: The objective is to determine how many weeds have been eradicated byfarmers in the fields by manual work.

Thus from the above table, we have the Objective function and the Constraint equation asfollows:

Maximize $\mathrm{z}=56 \mathrm{x}_{1}+32 \mathrm{x}_{2}+40 \mathrm{x}_{3}+17 \mathrm{x}_{4}+21 \mathrm{x}_{5}+49 \mathrm{x}_{6}+49 \mathrm{x}_{7}$
Subject toconstrains:
$90 \mathrm{x}_{1}+82 \mathrm{x}_{2}+102 \mathrm{x}_{3}+33 \mathrm{x}_{4}+209 \mathrm{x}_{5}+116 \mathrm{x}_{6}+152 \mathrm{x}_{7}=784$ (Mauritian grass)
$72 \mathrm{x}_{1}+72 \mathrm{x}_{2}+64 \mathrm{x}_{3}+92 \mathrm{x}_{4}+83 \mathrm{x}_{5}+54 \mathrm{x}_{6}+92 \mathrm{x}_{7}=48$ (Siam weeds)
$75 x_{1}+79 x_{2}+113 x_{3}+36 x_{4}+113 x_{5}+60 x_{6}+113 x_{7}=775 \quad$ (spiny pigweed)
and $\mathrm{x}_{\mathrm{i}}=0(\mathrm{i}=1,2,3,4,5,6,7)$

## COMPUTATION OF SOLUTION USING BIG-M METHOD

Based on the algorithm given in the previous section we now solve the Linear Programming Problem and find the solution.

The standard form the given LPP is
Maximize $z=56 x_{1}+32 x_{2}+40 x_{3}+17 x_{4}+21 x_{5}+49 x_{6}+49 x_{7}-\mathrm{MA}_{1}-\mathrm{MA}_{2}-\mathrm{MA}_{3}$
Subject to constrains

$$
\begin{aligned}
& 90 x_{1}+82 x_{2}+102 x_{3}+33 x_{4}+209 x_{5}+116 x_{6}+152 x_{7}+A_{1}=784 \\
& 72 x_{1}+72 x_{2}+64 x_{3}+92 x_{4}+83 x_{5}+54 x_{6}+92 x_{7}+A_{2}=485 \\
& 75 x_{1}+79 x_{2}+113 x_{3}+36 x_{4}+113 x_{5}+60 x_{6}+113 x_{7}+A_{3}=775 \\
& \text { and } x_{1}=0(i=1,2,3,4,5,6,7)
\end{aligned}
$$

|  | $\mathrm{C}_{\mathrm{i}}$ | $:$ | 56 | 32 | 40 | 17 | 21 | 49 | 49 | -M | -M | -M | Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{\mathrm{B}}$ | $\mathrm{Y}_{\mathrm{B}}$ | $\mathrm{X}_{\mathrm{B}}$ | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{X}_{5}$ | $\mathrm{X}_{6}$ | $\mathrm{X}_{7}$ | $\mathrm{~A}_{1}$ | $\mathrm{~A}_{2}$ | $\mathrm{~A}_{3}$ |  |
| -M | $\mathrm{A}_{1}$ | 784 | 90 | 82 | 102 | 33 | 209 | 116 | 152 | 1 | 0 | 0 | 3.75 |
| -M | $\mathrm{A}_{2}$ | 485 | 72 | 64 | 92 | 28 | 83 | 54 | 92 | 0 | 1 | 0 | 5.84 |
| -M | $\mathrm{A}_{3}$ | 775 | 75 | 79 | 113 | 36 | 279 | 60 | 113 | 0 | 0 | 1 | 2.77 |
|  | $\mathrm{Z}_{\mathrm{i}}$ | -2044 M | -237 M | -225 M | -307 M | -97 M | -571 M | -230 M | -357 M | -M | -M | -M |  |
|  | $\mathrm{Z}_{\mathrm{i}} \mathrm{C}_{\mathrm{i}}$ |  | -237 M <br> -56 | -225 M <br> -32 | -307 M <br> -40 | -97 M <br> -17 | -571 M <br> -21 | -230 M <br> -49 | -357 M <br> -49 | 0 | 0 | 0 |  |

## INITIAL ITERATION

Since all $\left(Z_{j}-C_{j}\right) \leq 0$, the current solution is not optimal,

- The most negative of $\left(Z_{j}-C_{j}\right)$ corresponds to $X_{5}$ and so $X_{5}$ enters the basis.
- The leaving variable corresponds to the minimum value of $\mathrm{R}_{3}$, which is $A_{3}$.
- Here pivot element is 279

New Pivot Equation $=$ Old $A_{3}$ Equation $\div$ Pivot Element

| pivot equation | 2.78 | 0.27 | 0.28 | 0.40 | 0.12 | 1 | 0.21 | 0.40 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

New $A_{1}$ equation =old $A_{1}$ equation - (209) (pivot equation)

| Old $A_{1}$ | 784 | 90 | 82 | 102 | 33 | 209 | 116 | 152 | 1 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{2 0 9}$ | 2.78 | 0.27 | 0.28 | 0.40 | 0.12 | 1 | 0.21 | 0.40 | 0 | 0 | 0 |
| New $A_{1}$ | 202.98 | 33.57 | 23.48 | 18.4 | 7.92 | 0 | 72.11 | 8.4 | 1 | 0 | 0 |

New $A_{2}$ equation = old $A_{2}$ equation - (188) (pivot equation)
New $A_{2}$ equation

| Old $A_{2}$ | 485 | 72 | 64 | 92 | 28 | 83 | 54 | 92 | 0 | 1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| -83 | 2.78 | 0.27 | 0.28 | 0.40 | 0.12 | 1 | 0.21 | 0.40 | 0 | 0 | 0 |
| new $A_{2}$ | 254.26 | 49.59 | 40.76 | 58.8 | 18.04 | 0 | 36.57 | 58.8 | 0 | 1 | 0 |

## FIRST ITERATION

|  | $\mathrm{Ci}_{\mathrm{i}}$ | : | 56 | 32 | 40 | 17 | 21 | 49 | 49 | -M | -M | -M | Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{\mathrm{B}}$ | $Y_{B}$ | $\mathrm{X}_{\mathrm{B}}$ | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | X4 | $\mathrm{X}_{5}$ | $\mathrm{X}_{6}$ | $\mathrm{X}_{7}$ | $\mathrm{A}_{1}$ | $\mathrm{A}_{2}$ | $\mathrm{A}_{3}$ |  |
| -M | $\mathrm{A}_{1}$ | 202.98 | 33.57 | 23.48 | 18.4 | 7.92 | 0 | 72.11 | 8.4 | 1 | 0 | 0 | 2.81 |
| -M | $\mathrm{A}_{2}$ | 254.26 | 49.59 | 40.76 | 58.8 | 18.04 | 0 | 36.57 | 58.8 | 0 | 1 | 0 | 6.95 |
| 21 | $\mathrm{X}_{5}$ | 2.78 | 0.27 | 0.28 | 0.40 | 0.12 | 1 | 0.21 | 0.40 | 0 | 0 | 0 | 13.20 |
|  | Zi | $\begin{gathered} 457.24 \mathrm{M} \\ +58.38 \end{gathered}$ | $\begin{gathered} 83.06 \mathrm{M} \\ +5.88 \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-64.24 \mathrm{M} \\ & +5.88 \\ & \hline \end{aligned}$ | $\begin{gathered} \hline-71.20 \mathrm{M} \\ 8.40 \\ \hline \end{gathered}$ | $\begin{gathered} -25.96 \mathrm{M} \\ +2.52 \\ \hline \end{gathered}$ | 21 | $\begin{gathered} -108.68 \mathrm{M} \\ +4.41 \\ \hline \end{gathered}$ | $\begin{gathered} -67.20 \mathrm{M} \\ +8.40 \\ \hline \end{gathered}$ | -M | -M | 0 |  |
|  | $\mathrm{Z}_{\mathrm{i}}-\mathrm{C}_{\mathrm{i}}$ |  | $\begin{gathered} 83.06 \mathrm{M} \\ -50.33 \\ \hline \end{gathered}$ | $\begin{gathered} \hline-64.24 \mathrm{M} \\ -26.12 \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-77.2 \mathrm{M} \\ & -31.60 \\ & \hline \end{aligned}$ | $\begin{gathered} \hline-25.96 \mathrm{M} \\ -14.48 \\ \hline \end{gathered}$ | 0 | $\begin{gathered} \hline-108.68 \mathrm{M} \\ -44.59 \\ \hline \end{gathered}$ | $\begin{gathered} \hline-67.20 \mathrm{M} \\ -40.60 \\ \hline \end{gathered}$ | 0 | 0 | M |  |

- Since all $\left(Z_{j}-C_{j}\right) \leq 0$, the current solution is not optimal,
- The most negative of $\left(Z_{j}-C_{j}\right)$ corresponds to $X_{6}$ and so $X_{6}$ enters the basis.
- The leaving variable corresponds to the minimum value of $R_{1}$, which is $A_{1}$.
- Here pivot element is 72.11

New Pivot Equation=Old $A_{1}$ Equation $\div$ pivot Element

| pivot equation | 2.81 | 0.47 | 0.33 | 0.26 | 0.26 | 0.11 | 1 | 0.12 | 0.01 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## New $A_{2}$ equation $=$ old $A_{2}$ equation - (36.57) (pivot equation)

New $A_{2}$ equation

| Old $A_{1}$ | 254.26 | 49.56 | 40.76 | 58.80 | 18.04 | 0 | 36.57 | 58.80 | 0 | 1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| -36.57 | 2.81 | 0.47 | 0.33 | 0.26 | 0.11 | 0 | 1 | 0.12 | 0.01 | 0 | 0 |
| New $A_{1}$ | 151.50 | 32.40 | 28.69 | 49.29 | 14.02 | 0 | 0 | 54.41 | -0.37 | 1 | 0 |

New $X_{5}$ equation $=$ old $X_{5}$ equation - ( 0.21 ) (pivot equation)
New $X_{5}$ equation

| Old $X_{5}$ | 2.78 | 0.27 | 0.28 | 0.40 | 0.12 | 1 | 0.21 | 0.40 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| -0.21 | 2.81 | 0.47 | 0.33 | 0.26 | 0.11 | 0 | 1 | 0.12 | 0.01 | 0 | 0 |
| new $X_{5}$ | 2.19 | 0.17 | 0.21 | 0.35 | 0.10 | 1 | 0 | 0.37 | -0.03 | 0 | 0 |

Proceed similarly, we get

## FOURTH ITERATION

|  | $\mathrm{C}_{\text {j }}$ | : | 56 | 32 | 40 | 17 | 21 | 49 | 49 | -M | -M | -M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{\mathrm{B}}$ | $Y_{B}$ | $\mathrm{X}_{\mathrm{B}}$ | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | X4 | $\mathrm{X}_{5}$ | $\mathrm{X}_{6}$ | $\mathrm{X}_{7}$ | $\mathrm{A}_{1}$ | $\mathrm{A}_{2}$ | $\mathrm{A}_{3}$ |
| 49 | $\mathrm{X}_{6}$ | 0.63 | 0 | -0.08 | -0.46 | 0 | 0 | 1 | 0.67 | 0.02 | 0.01 | 0 |
| 56 | $\mathrm{X}_{1}$ | 4.63 | 1 | 0.88 | 1.52 | 0.43 | 0 | 0 | 1.67 | -0.02 | 0.03 | 0 |
| 21 | $\mathrm{X}_{5}$ | 1.39 | 0 | 0.09 | 0.09 | 0.02 | 1 | 0 | 0.08 | -0.03 | -0.01 | 0 |
|  | $\mathrm{Z}_{\mathrm{i}}$ | 319.34 | 56 | 47.25 | 64.47 | 24.5 | 21 | 49 | 128.03 | -0.77 | 2.26 | 0 |
|  | $\mathrm{Zi} \mathrm{CiO}_{\mathrm{i}}$ |  | 0 | 15.25 | 24.47 | 7.50 | 0 | 0 | 79.03 | $\begin{aligned} & -0.77 \\ & +M \end{aligned}$ | $\begin{aligned} & 2.26 \\ & +M \end{aligned}$ | M |

Since all $\left(Z_{j}-C_{j}\right) \geq 0$, this is the required optimal solution.
$X_{1}=4.63, X_{5}=1.39, X_{6}=0.63$ and Maximize $Z=319.346$
Thus, in manual method it takes atleast six attempts for the farmers to eradicate the weeds in their fields.

## CONCLUSION

Growth of weeds incurs a huge loss to the farmers and hence it is necessary that they remove them periodically so as to get profit for their hard work. Though it is necessary to go for alternate methods that can drastically reduce the burden of manual eradication of weeds, itis also necessary to be sure of the authenticity of the innovations and developed equipments, which is achieved with the aid of Big - M Method of the Linear Programming Problem as it strongly establishes the superiority of the robotic weed controller machine over laborious manualmethods.

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# Stagnation Point Flow of Micropolar Fluid over a Stretching / Shrinking Sheet 

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#### Abstract

In this article, the stagnation point flow of a micropolar fluid on a stretching/shrinking sheet has been discussed subject to the assumption of velocity slip. The similarity transformation is used to transform the modeled Partial Differential Equations (PDEs) into a system of Ordinary Differential Equations (ODEs). The numerical results have been found by the shooting method. Finally, the numerical results are presented with discussion of the effects of different physical parameters.


Keywords: Stretching/shrinking sheet, MHD, Micropolar Fluid, Adams Bash fourth Method.

## 1. Introduction

Stagnation point refers to the location in the flow field when the fluid velocity is zero.In the subject of fluid dynamics, the study of viscous, incompressible fluid passing through a permeable plate or sheet is crucial. Because of its wide range of applications in the manufacturing sectors, research on the stagnation point flow of an incompressible fluid across a permeable sheet has gained prominence in recent decades.

Some of the most common uses include fan-cooled electrical devices, atomic receptacles cooling for the length of an emergency power outage, solar receivers, andso on. Hiemenz [1] was the first to examine two-dimensional (2D) stagnation point flow, while Eckert [2] expanded this problem by include the energy equation to obtainan accurate answer. As a result, Mahapatra and Gupta [3], Ishak et al. [4], and Hayat et al. [5] investigated the effects of heat transmission in stagnation point across a permeable plate.

The effect of slip condition gives an interesting result for different fluids. Sharma et al. [6] investigated the slip effect of the heat transfer due to stretching sheet on a CuOwater nanofluid. A new model effect of second order slip velocity was introduced by Wu [7]. Wang et al. [8] extended the article of Wu [7] by considering the slip effect ofstagnation point flow on a heated vertical plate. Fang et al. [9] investigated the second order velocity slip effect on the viscous flow due to a stretching sheet. Nandeppanaveret al. [10] discussed the heat transfer and second order slip flow due to a stretching sheet. Deissler [11], Rosca and Pop [12] and Turkyilmazoglu [13] investigated the second order velocity slip effect, under different physical conditions.

Many researchers found interested in the study of the micropolar fluid for the different geometries. Erigen [14] was the first one who investigated the micropolar fluid. Ariman et al. [15] theoretically investigated the micropolar fluids and their applications. Ishak et al. [16] discussed the
stagnation point flow of a micropolar fluid in two-dimensional boundary layer flow of mixed convection on a stretching sheet. Bhargava et al. [17] numerically investigated the solutions of micro-polar transport due to a non-linear stretching sheet. Rees and Pop [18] theoretically discussed free convection from a vertical at plate in a micropolar fluid. Nazar et al. [19].

In this article, a review study of Sharma et al. [20] has been presented and then theflow analysis has been extended by considering the additional effects.

## 2. Mathematical Modeling

Consider a steady, two-dimensional stagnation point flow of an incompressible micropolar fluid on a stretching/shrinking sheet with the assumption of slip velocity effect. Assume that $u_{e}(x)=a x$ be the free stream velocity and $u_{w}(x)=b x$ be the stretching/shrinking velocity respectively, where $a$ and $b$ are some real constants.


Figure 1: Geometry of the problem.
For stretching sheet $b>0$ and for shrinking sheet $b<0$. The mathematical modelof the flow, presented by Sharma et al. [20] is as follows:

$$
\begin{align*}
& \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0  \tag{1}\\
& \frac{\partial u}{\partial x} u+\frac{\partial u}{\partial y} v=u_{e} \frac{\partial u_{e}}{\partial x}+\left(\frac{\mu+k}{\rho}\right) \frac{\partial^{2} u}{\partial y^{2}}+\frac{k}{\rho} \frac{\partial N}{\partial y}  \tag{2}\\
& \rho j\left(\frac{\partial N}{\partial x} u+\frac{\partial N}{\partial y} v\right)=\left(\mu+\frac{k}{2}\right) j \frac{\partial^{2} N}{\partial y^{2}}-k\left(2 N+\frac{\partial u}{\partial y}\right), \tag{3}
\end{align*}
$$

where the velocity components has been represented by $u$ and $v$ respectively. Dynamic viscosity is denoted by $\mu$, microrotation viscosity by $k$, fluid density by $\rho$, micro inertia density by $j$
and component of microrotation is denoted by $N$. The boundary conditions of the above equations are given as

$$
\left.\begin{array}{l}
v=0, \quad u_{w}(x)+u_{s l i p}=, \quad N=-n \frac{\partial u^{-}}{\partial y} \text { at } y=0,  \tag{4}\\
u=u_{e}(x), \quad N \rightarrow 0 \quad \text { as } \quad y \rightarrow \infty,
\end{array}\right\}
$$

Where $u_{e}(x), u_{s l i p}$ and $u_{w}(x)$ represent the free steam velocity, slip velocity and stretching / shrinking velocity. The stream function identically satisfies the continuity equation. Mathematically,

$$
\begin{equation*}
u=\frac{\partial \psi}{\partial y}, \quad v=\frac{\partial \psi}{\partial x} . \tag{5}
\end{equation*}
$$

Now, introduce the following similarity variables from [20],

$$
\left.\begin{array}{l}
\psi=\sqrt{v x u_{e}(x)} f(\eta)=\sqrt{a v} x f(\eta), \\
\eta=\sqrt{\frac{u_{e}(x)}{v x}} y=\sqrt{\frac{a}{v}} y,  \tag{6}\\
N=u_{e}(x) \sqrt{\frac{u_{e}(x)}{v x}}, \quad h(\eta)=a \sqrt{\frac{a}{v}} x h(\eta),
\end{array}\right\}
$$

where the stream function is represented by $\Psi$ and the kinematic viscosity isrepresented by $\mathcal{V}$.
Thus, the dimensionless form of the mathematical model of the present problem is:

$$
\begin{align*}
& \left(1-f^{\prime 2}\right)+(1+K) f^{\prime \prime \prime}+f f^{\prime \prime}+K h^{\prime}=0  \tag{7}\\
& \left(1+\frac{K}{2}\right) h^{\prime \prime}+f h^{\prime}=f^{\prime} h+K\left(2 h+f^{\prime \prime}\right) \tag{8}
\end{align*}
$$

Along with BC's

$$
\begin{align*}
& f(0)=0, f^{\prime}(0)=\varepsilon+\lambda f^{\prime \prime}(0)+\delta f^{\prime \prime \prime}(0), h(0)=-n f^{\prime \prime}(0),  \tag{9}\\
& f^{\prime}(\eta) \rightarrow 1, h(\eta) \rightarrow 0, \text { as } \eta \rightarrow \infty, \tag{10}
\end{align*}
$$

In the above equations, the micropolar parameter by $K=\frac{k}{\mu}=\frac{k}{v \rho}$ the stretching/shrinking rate has been represented by $\varepsilon=\frac{b}{a}$ the first order slip represented by $\lambda=A \sqrt{\frac{a}{v}}$ and the second order slip by $\delta=B \frac{a}{v}$ where $A$ and $B$ have the following formulations [7]

$$
\left.\begin{array}{l}
A=\frac{2}{3}\left(\frac{3-\alpha l^{3}}{\alpha}-\frac{3}{2} \frac{1-l^{2}}{K_{n}}\right) \lambda, \\
B=-\frac{1}{4}\left[l^{4}+\frac{2}{K_{n}^{2}}\left(1+l^{2}\right)\right] \lambda^{2} \tag{11}
\end{array}\right\}
$$

## 3. Solution Methodology

The numerical solution of the mathematical model in the form of non-linear differential equations $(7-8)$ along with the boundary conditions $(9-10)$ was reported by Sharma et al. [20]. They opted the finite-difference method for the numerical solution of the above model. In the present section, shooting method has been proposed to reproduce the same solution. The Adams Moulton method of order four and the Newton's method for solving the non-linear algebraic equations, are themain components of the shooting method. Let us re-write equation (7-8) as:

$$
\begin{align*}
& f^{\prime \prime \prime}=-\frac{1}{(1+K)}\left[f f^{\prime \prime}+\left(1-f^{\prime 2}\right)+K h^{\prime}\right]  \tag{12}\\
& h^{\prime \prime}=\frac{2}{(2+K)}\left[f h^{\prime}-f^{\prime} h-K\left(2 h+f^{\prime \prime}\right)\right] \tag{13}
\end{align*}
$$

Use the notations to construct a system of first order ODEs:

$$
\begin{equation*}
f=y_{1}, f^{\prime}=y_{2}, f^{\prime \prime}=y_{3}, h=y_{4}, h^{\prime}=y_{5} \tag{14}
\end{equation*}
$$

By using the notations (14), we have the following IVP:

$$
\begin{array}{lr}
\left.\begin{array}{ll}
y_{1}^{\prime}=y_{2}, & y_{1}(0)=0, \\
y_{2}^{\prime}=y_{3}, & y_{2}(0)=s, \\
y_{3}^{1}=-\frac{1}{(1+K)}\left[y_{1} y_{3}+\left(1-y_{2}^{2}\right)+K y_{5}\right], & \\
y_{3}(0)=\frac{1}{\lambda}\left[s-\varepsilon+\delta\left(\frac{1}{1+K}\right)\left(\left[\left(1-s^{2}\right)+K t\right]\right)\right], \\
y_{4}^{\prime}=y_{5}, & y_{4}(0)=-\frac{n}{\lambda}\left[s-\varepsilon+\delta\left(\frac{1}{1+K}\right)\left(\left[\left(1-s^{2}\right)+K t\right]\right)\right], \\
y_{5}^{1}=\frac{2}{(2+K)}\left[-y_{1} y_{5}+y_{2} y_{4}+K\left(2 y_{4}+y_{3}\right)\right], & \left.y_{5}(0)\right)=t .
\end{array}\right\}, ~ ?, ~
\end{array}
$$

In order to ${ }_{1}$ get the approximate $_{4}$ numeŗical results, the problem's domain is considered to be bounded i.e., $\left[0, \eta_{\infty}\right]$ here $\eta_{\infty}$ is chosen to be an appropriate finite positive real number so that the variation in the result for $\eta>\eta_{\infty}$ is ignorable. In (15), themissing initial conditions $s$ and $t$ are to be chosen such that

$$
\begin{equation*}
y_{2}\left(\eta_{\infty}, s, t\right)-1=0, y_{4}\left(\eta_{\infty}, s, t\right)=0 . \tag{16}
\end{equation*}
$$

To start the iterative process, choose $s=s_{0}$, and $t=t_{0}$. To the values of $s, t$ Newton's iterative scheme has been used

$$
\binom{s_{n+1}}{t_{n+1}}=\binom{s_{n}}{t_{n}}-\left(\begin{array}{ll}
\frac{\partial y_{2}}{\partial s} & \frac{\partial y_{2}}{\partial t}  \tag{17}\\
\frac{\partial y_{4}}{\partial s} & \frac{\partial y_{4}}{\partial s}
\end{array}\right)_{\left(s_{n}, t_{n}\right)}^{-1} \quad\binom{y_{2}\left(\left(\eta_{\infty}, s_{n}, t_{n}\right)-1\right)}{y_{4}\left(\left(\eta_{\infty}, s_{n}, t_{n}\right)\right)}
$$

To implement the Newton's scheme, consider the following notations:

$$
\begin{aligned}
& \frac{\partial y_{1}}{\partial s}=y_{6}, \frac{\partial y_{2}}{\partial s}=y_{7}, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . \frac{\partial y_{5}}{\partial s}=y_{10} \text {, }
\end{aligned}
$$

Differentiating equations (15), first w.r.t. $s$ and then w.r.t. $t$, we get the followingfifteen first order ODEs along with the associated initial conditions.

$$
\left.\begin{array}{lr}
y_{6}^{\prime}=y_{7}, & y_{6}(0)=0, \\
y_{7}^{\prime}=y_{8}, & y_{7}(0)=1, \\
y_{3}^{1}=-\frac{1}{(1+K)}\left[y_{6} y_{3}+y_{1} y_{8}-2 y_{2} y_{7}+K y_{10}\right], & y_{8}(0)=\frac{1}{\lambda}\left[1-\left(\frac{2 \delta s}{1+K}\right)\right], \\
y_{9}^{\prime}=y_{10}, & y_{9}(0)=-\frac{n}{\lambda}\left[1-\frac{2 \delta s}{1+K}\right], \\
\left.y_{10}^{1}=\frac{2}{(2+K)}\left[-y_{6} y_{5}+y_{7} y_{4}+y_{2} y_{9}-y_{1} y_{10}+K\left(2 y_{9}+y_{8}\right)\right], y_{10}(0)\right)=0, \\
y_{11}^{\prime}=y_{12}, & \left.y_{11}(0)\right)=0, \\
y_{12}^{\prime}=y_{13}, & \left.y_{12}(0)\right)=0, \\
y_{13}^{1}=-\frac{1}{(1+K)}\left[y_{11} y_{13}+y_{1} y_{13}-2 y_{2} y_{12}+K y_{15}\right], & y_{13}(0)=\frac{1}{\lambda}\left[\left(\frac{2 \delta s}{1+K}\right)\right], \\
y_{14}^{\prime}=y_{15}, & y_{14}(0)=-\frac{n}{\lambda}\left[\frac{2 \delta s}{1+K}\right], \\
\left.y_{15}^{1}=\frac{2}{(2+K)}\left[-y_{11} y_{5}+y_{12} y_{4}+y_{2} y_{14}-y_{1} y_{15}+K\left(2 y_{14}+y_{13}\right)\right], y_{15}(0)\right)=1 \tag{18}
\end{array}\right\}
$$

Next, the IVP in the form of fifteen first order ODEs given in (15) and (18) is solvedby the forth order Adams Moulton method and the Newton's method. If for a sufficiently small $\varepsilon^{*}$,

$$
\begin{equation*}
\max \left\{\left|y_{2}\left(\eta_{\infty}, s_{n}, t_{n}\right)-1\right|,\left|y_{4}\left(\eta_{\infty}, s_{n}, t_{n}\right)-1\right|\right\}>\varepsilon^{*} \tag{19}
\end{equation*}
$$

the guessed values of $s$ and $t$ are updated by the Newton's iterative scheme:

$$
\binom{s_{n+1}}{t_{n+1}}=\binom{s_{n}}{t_{n}}-\left(\begin{array}{ll}
y_{7} & y_{12}  \tag{20}\\
y_{9} & y_{14}
\end{array}\right)_{\left(s_{n}, t_{n}\right)}^{-1}\binom{y_{2}\left(\left(\eta_{\infty}, s_{n}, t_{n}\right)-1\right)}{y_{4}\left(\left(\eta_{\infty}, s_{n}, t_{n}\right)\right)}
$$

The iterative process is repeated until, the following criteria is met.

$$
\begin{equation*}
\max \left\{\left|y_{2}\left(\eta_{\infty}, s_{n}, t_{n}\right)-1\right|,\left|y_{4}\left(\eta_{\infty}, s_{n}, t_{n}\right)-1\right|\right\}<\varepsilon^{*} \tag{21}
\end{equation*}
$$

## 3. Results and Discussion

The main objective of the present section is to study the effect of different physical parameters like $K$ (micro-polar parameter), $\lambda$ (the first order slip parameter), $\varepsilon$ (the stretching/shrinking rate), $\delta$ (the second order slip parameter) on the velocity and micro-rotation profiles. The present results have been compared with the previous results of Wang [21] and Bachok et al. [22] for different values of the stretching/shrinking rate $\square$ in Table 1. Wang [21] and Bachok et al. [22] have discussed the stagnation point flow towards a stretching/shrinking sheet.

Figure 2 demonstrates the impact of first order slip $\lambda$ on the velocity profile for different physical parameters. By increasing the values of $\lambda$, the velocity profile is decreased. Hence the boundary layer thickness is increased.

Table 1: Comparison of $f^{\prime \prime}(0)$ for different values of $\varepsilon$. When $\lambda=0, \delta=0, K=0$, and $n=0.5$.

| Values of $\varepsilon$ | Wang [21] | Bachok et al. [22] | [20] | Current results |
| :---: | :---: | :---: | :---: | :---: |
| 2.0 | -1.88731 | -1.8873066 | -1.88730667 | -1.88730627 |
| 1.0 | 0 | 0 | 0 | 0 |
| 0.5 | 0.713300 | 0.7132949 | 0.71329496 | 0.71525570 |
| 0.0 | 1.232588 | 1.2325877 | 1.23258765 | 1.23257700 |
| -0.25 | 1.402240 | 1.4022408 | 1.40224081 | 1.40224872 |
| -0.5 | 1.495670 | 1.4956698 | 1.49566977 | 1.49566265 |
| -1.0 | 1.328820 | 1.3288170 | 1.32881688 | 1.32881259 |
| -1.2 | 0.554300 | 0.9324730 | 0.93247336 | 0.93247167 |
| -1.2465 | -- | 0.5842956 | 0.58428274 | 0.58428643 |

The impact of the first order slip $\lambda$ on the velocity profile is presented in Figure 2. By increasing the values of the $\lambda$, the velocity profile is increased. Physically, when slip occurs, the velocity of flow near the sheet is no longer equal to the stretching velocityof the sheet.

The variations in the micro-rotation profile for the $\lambda$ are demonstrated in Figure 3and 4. An opposite flow behavior is determined with the first and second solution. The thickness of boundary layer is deceased in the first solution and increases in the second solution.

Figures 5 and 6 demonstrate the impact of the second order slip parameter $\delta$ on the velocity profile. Figure 5 indicates that by increasing $\delta$, the velocity profile is increased. Figure 6 represents that by increasing $\delta$, the velocity profile is reduced.

The variations in the microrotation profile for different values of the second order velocity $\operatorname{slip} \delta$ are demonstrated in Figures 7 and 8. It shows that the microrotation profile is initially increased as $\delta$ is increased for the first solution and microrotation profile is decreased as $\square$ is increased for the second solution.

The variations in the velocity profile for micropolar parameter $K$ are demonstrated in Figures 9 and 10. By increasing the values of the micropolar fluid $K$, the velocity field is reduced in both the first and the second solution. It is evident from these Figure 9 and 10 that all curves approach the far field boundary conditions asymptotically.

The variations in the microrotation profile for micropolar parameter $K$ are demonstrated in Figures 11 and 12. From these graphs, it can be observed that increasing the micropolar $K$, the velocity field is reduced in the lower half of the surface whereas it is enhanced in the upper half. The velocity is going to reduce initially with the mounting values of the micropolar $K$. The boundary layer thickness is increased in both the first and the second solution.


Figure 2: Impact of $\lambda=0.05,0.1,0.15$ on $f^{\prime}(\eta)$.


Figure 3: Impact of $\lambda=0.05,0.1,0.15$ on $h$.


Figure 4: Impact of $\mathcal{S}=0.05,0.10,0.15$ on $f^{\prime}(\eta)$.


Figure 5: Impact of $\delta=0.05,0.10,0.15$ on $f^{\prime}(\eta)$.


Figure 6: Impact of $\delta=-0.05,-0.10,-0.15$ on $f^{\prime}(\eta)$.


Figure 7: Impact of $\delta=0.05,0.10,0.15$ on $h(\eta)$.


Figure 8: Impact of $\delta=-0.05,-0.10,-0.15$ on $f^{\prime}(\eta)$.


Figure 9: Impact of $K=0.05,0.10,0.15$ on $f^{\prime}(\eta)$.


Figure 10: Impact of $K=-0.05,-0.10,-0.15$ on $f^{\prime}(\eta)$.


Figure 11 : Impact of $K=0.05,0.10,0.15$ on $h(\eta)$ -


## 4. Conclusion

Conclusions which are obtained: Increasing the suction parameter, the velocity and microrotation profiles are increased. Due to an increase in the shrinking parameter, the velocity and micro- rotation profiles are decreased. Increasing the micropolar parameter, the velocity and micro-rotation profiles are decreased.

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# A VIEW ON THE ANALYSIS OF POLYCYSTIC KIDNEY DISEASES BY USING FUZZY MAMDANI MODEL 

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#### Abstract

A fuzzy logic based fuzzy MAMDANI model is developed in Polycystic Kidney Diseases and the results are produced graphically using MATLAB. Using this MAMDANI model, the level of creatinine can be predicted for the people affected by Polycystic Kidney Diseases and can be diagnosed.


Keywords: Linguistic variable, Mamdani model, Chronic stress, Surface viewer andRule viewer.

## 1 Introduction

Polycystic kidney disease (PKD) is an inherited disorder in which clusters of cysts develop primarily within kidneys, causing kidneys to enlarge and lose function over time. Cysts are noncancerous round sacs containing fluid. The cysts vary in size and they can grow very large. Having many cysts or large cysts can damage kidneys. Polycystic kidney disease can also cause cysts to develop in liver and elsewhere in the body. The disease can cause serious complications, including high blood pressure and kidney failure. PKD varies greatly in its severity, and some complications are preventable. Lifestyle changes and treatments might help to reduce from kidneys damage.
Complications associated with Polycystic Kidney Disease include:

- High blood pressure.
- Loss of kidney function.
- Chronic pain.
- Growth of cysts in the liver.
- Development of an aneurysm in the brain.
- Heart valve abnormalities.

Eventhough Polycystic Kidney Disease (PKD) is an inherited disorder, there are some measures to prevent the kidney from getting damaged. Blood pressure, Urea level and Potassium level should be maintained properly by frequent checkup. L.A. Zadeh [12] has introduced the concept called Fuzzy set. It can be used in wide range of domains where information are
incomplete and imprecise. A fuzzy number is aquantity, whose values are imprecise and it gives single valued numbers. Rajarajeswari et al. [6] presented a new operation on hexagonal fuzzy numbers. Ranking fuzzy number is used mainly in decision-making, data analysis, artificial intelligence and even in var- ious fields of the operation research. In fuzzy environment, ranking fuzzy numbers are very important in decision making procedure. The most important two types of fuzzy inference method are Mamdani and Sugeno fuzzy inference methods, Mamdani fuzzy inference is the most commonly seen inference method. This method was introduced by Mamdani and Assilian in 1975.

In this paper, a fuzzy logic based fuzzy MAMDANI model is developed in Polycystic Kidney Diseases and the results are produced graphically using MATLAB. Using this MAMDANI model, the level of creatinine can be predicted for the people affected by Polycystic Kidney Diseases and can be diagnosed.

## 2 Preliminaries

In this section, the basic concepts required to study this paper are investigated. Also, some basic concepts of fuzzy sets have been recalled. Related results and definitions are studied from various research articles.

Definition 2.1. [11] Let $R$ be the set of all real numbers. We assume a fuzzy number $\tilde{A}$ that can be expressed for all $x \in R$ in the form

$$
\mu_{\tilde{A}}(X)=\left\{\begin{array}{rcc}
\mu_{\tilde{A_{L}}}(X) & \text { for } & a \leq x \leq b \\
W & \text { for } & b \leq x \leq c \\
\mu_{\tilde{A_{K}}}(X) & \text { for } & c \leq x \leq d \\
0 & \text { otherwise }
\end{array}\right.
$$

where $0 \leq w \leq 1$ is a constant, $a, b, c, d$ are real numbers, such that $a<b \leq c<d, \mu_{\tilde{A}_{L}}(X):[a, b] \rightarrow$ $[0, w], \mu_{\tilde{A}_{R}}(X):[c, d] \rightarrow[0, w]$ are two strictly monotonic and continuous functions from $R$ to the closed interval $[0, w]$. Since $\mu_{\tilde{A}_{L}}(X)$ is continuous and strictly increasing, the inverse function $\mu_{\tilde{A}_{R}}(X)$ exists. Similarly $\mu_{\tilde{A}_{R}}(X)$ is continuous and strictly decreasing the inverse function of $\mu_{\tilde{A}_{R}}(X)$ also exist. The inverse functions of $\mu_{\tilde{A}_{L}}(X)$ and $\mu_{\tilde{A}_{K}}(X)$ can be denoted by $\mu_{\tilde{A}_{L}^{-1}}(X)$ and $\mu_{\tilde{A}_{R}^{-1}}(X)$ respectively. $\mu_{\tilde{A}_{L}^{-1}}(X)$ and $\mu_{\tilde{A}_{R}^{-1}}(X)$ are continuous on $[0, w]$ that means both $\int_{0}^{w} \mu_{\tilde{A}_{L}^{-1}}(X)$ and $\int_{0}^{w} \mu_{\tilde{A}_{R}^{-1}}(X)$.
Definition 2.2. [8] A fuzzy set A of the real line R with membership functions $\mu_{A}$ : $X \rightarrow[0,1]$ is called fuzzy number if,
(i) A must be normal and convex fuzzy set
(ii) The support of A must be bounded.
(iii) $a_{A}$ must be closed interval for every $a \in[0,1]$.

Definition 2.3. [6] A trapezoidal fuzzy number denoted by $\tilde{A}$ is defined as $\left(a_{1}, a_{2}, a_{3}, a_{4}\right)$ where the membership function

$$
\mu=\left\{\begin{array}{cc}
0 & x \leq a_{1} \\
\left(\frac{\mathrm{x}-\mathrm{a}_{1}}{\mathrm{a}_{2}-\mathrm{a}_{1}}\right) & a_{1} \leq a_{2} \\
1 & a_{2} \leq a_{3} \\
\left(\frac{\mathrm{a}_{4}-\mathrm{x}}{\mathrm{a}_{4}-\mathrm{a}_{3}}\right) & a_{3} \leq a_{4} \\
0 & \mathrm{x} \geq \mathrm{a}_{4}
\end{array}\right.
$$

Definition 2.4. [1] If $X$ is a collection of objects generically by $x$, then a fuzzy set $\tilde{A}$ in $X$ is a set of ordered pairs. $\mu_{A}(\mathrm{x})$ is called the membership function or grade of membership of that maps to the membership space.
Definition 2.5. [10] Let $\mathrm{R}: F(A) \quad \mathrm{R}$ where $F(A)$ be the set of all Trapezoidal Fuzzy Numbers and $R$ be the set of real numbers. The ranking of trapezoidal fuzzy numbers $\tilde{A}$ is defined and denoted as

$$
R(\tilde{A})=\frac{\mathrm{a}_{1}+\mathrm{a}_{2}}{2}+\frac{1}{4}\left(a_{4}-a_{3}\right)
$$

Definition 2.6. [6] Trapezoidal function is defined by a lower limit'a', an upper limit 'd', a lower support limit 'b', and a upper support limit ' c ', where $a \leq b \leq c \leq d$

$$
\mu=\left\{\begin{array}{cc}
0 & (x<a) \operatorname{or}(x>d) \\
\frac{\mathrm{x}-\mathrm{a}}{\mathrm{~b}-\mathrm{a}} & a \leq x \leq b \\
1 & b \leq x \leq c \\
\frac{\mathrm{~d}-\mathrm{x}}{\mathrm{~d}-\mathrm{c}} & c \leq x \leq d
\end{array}\right.
$$

Definition 2.7. [5] The concept of the fuzzy number plays a fundamental role in formu- lating Quantitative fuzzy variables. These are variables whose states are fuzzy numbers to represent linguistic concepts, such as very high, high, medium, and so on, as in- terpreted in a particular context, the resulting constructs are usually called Linguistic variables.
Definition 2.8. [4] The function assigning fuzzy truth degrees between 0 and 1 to elements of universal set, $\mu_{A}: X \rightarrow[0,1]$, A fuzzy rule base contains fuzzy rules $\mathrm{R}_{i}$

$$
R_{i}=\text { If }\left(x_{1} \text { is } A_{i 1}\right) \text { And }\left(x_{2} \text { is } A_{i 2}\right) \text { And } \ldots . .\left(x_{n} \text { is } A_{i n}\right) \text { then }\left(Y \text { is } B_{i}\right)
$$

where $A_{i j}$ and $B_{i}$ are linguistic values defined by fuzzy set, $x_{j}$ and $y$ are fuzzy inputs and outputs. The structure of the fuzzy rule is the following: If Premise Then Conclusion.

Definition 2.9. [9] The result obtained from fuzzy inference technique is then processed to produce a quantifiable result that is the level of stress. Defuzzification process of producing a quantifiable result in crisp logic, given fuzzy set to a crisp set. No systematic procedure for choosing a good defuzzification strategy.

## 3 Determination of Linguistic Variables andMembership Function

The major factors responsible for Polycystic Kidney Diseases are High blood pressure, Urea level in the blood and Potassium level in the blood. In this paper, these three major causes for kidney damage are considered. Taking these factors into account, Mamdani model is used to diagnose the level of creatinine in the kidney for each people. These rules can be used to find out the level of creatinine for each patient by just entering the values of risk in the rule viewer. Finally the level of creatinine are found out.

In this section, the required linguistic variables and their membership values are given. The linguistic variables associated with each of these factors are given in the following tables. Hence there are four input variables and one output variable.

Table 3.1: Linguistic Variable for Input Variables

| S.No | Input Variables | Linguistic Variables |
| :---: | :---: | :---: |
| 1. | Very Low | VL |
| 2. | Low | L |
| 3. | Medium | M |
| 4. | High | H |

Table 3.2: Linguistic Variable for Output Variable

| S.No | Output Variable | Linguistic Variable |
| :---: | :---: | :---: |
| 1. | Creatinine | Level of Creatinine |

## Membership function for all Input variables

The input variables are defined based on their level of risk and their corresponding membership function is computed. The range for all the Input Variables and their corresponding Membership functions are given.

Fuzzified Values for Blood Pressure:
All human beings are facing the problem of Blood pressure. The main reason for poly- cystic kidney diseases is the high blood pressure. So blood pressure is taken into con- sideration and the membership functions for blood pressure are given below.

Table 3.3: Fuzzified Values for Blood Pressure

| Input Variable | Range | Risk Level |
| :---: | :---: | :---: |
| Blood Pressure | $<130 \mathrm{~mm} \mathrm{Hg}$ | Very Low(VL) |
|  | $130 \mathrm{~mm} \mathrm{Hg}-149 \mathrm{~mm} \mathrm{Hg}$ | $\operatorname{Low}(\mathrm{L})$ |
|  | $150 \mathrm{~mm} \mathrm{Hg}-189 \mathrm{~mm} \mathrm{Hg}$ | Medium(M) |
|  | $>190 \mathrm{~mm} \mathrm{Hg}$ | $\operatorname{High}(\mathrm{H})$ |

$$
\begin{aligned}
& \mu_{V L}(x)=\left\{\begin{array}{ccc}
0 & x \leq 110 \\
\frac{\mathrm{x}-110}{10} & 110<x<120 \\
1 & x=120 \\
\frac{130-\mathrm{x}}{10} & 120<x<130 \\
0 & x \geq 130
\end{array} \quad \mu_{L}(x)=\left\{\begin{array}{cc}
0 & x \leq 130 \\
\frac{\mathrm{x}-130}{10} & 130<x<140 \\
1 & 140 \leq x \leq 141 \\
\frac{149-\mathrm{x}}{8} & 141<x<149 \\
0 & x \geq 149
\end{array}\right.\right. \\
& \mu_{M}(x)=\left\{\begin{array}{cc}
0 & x \leq 150 \\
\frac{\mathrm{x}-150}{20} & 150<x<170 \\
1 & 170 \leq x \leq 171 \\
\frac{189-\mathrm{x}}{18} & 171<x<189 \\
0 & x \geq 189
\end{array} \quad \mu_{H}(x)=\left\{\begin{array}{cc}
0 & x \leq 190 \\
\frac{\mathrm{x}-190}{10} & 190 \leq x \leq 200 \\
1 & x=200 \\
\frac{210-\mathrm{x}}{10} & 200<x \leq 210 \\
0 & x \geq 210 \\
&
\end{array}\right.\right.
\end{aligned}
$$



Figure 1: Fuzzified Values for Blood Pressure

## Fuzzified Values for Urea level in blood:

A common blood test, the Blood Urea Nitrogen (BUN) test reveals important information about how well kidneys are working. A BUN test measures the amount of urea nitrogen that's in blood. If kidney problems are the main concern, the Urea levels in the blood will likely also be measured. The membership functions for Urea level are given below.

Table 3.4: Fuzzified Values for Urea Level

| Input Variable | Range | Risk Level |
| :--- | :--- | :--- |
| Urea Level | $<10$ | Very Low(VL) |
|  | $10-20$ | Low(L) |
|  | $21-45$ | Medium(M) |
|  | $>46$ | $\operatorname{High}(\mathrm{H})$ |

$$
\left.\begin{array}{c}
\mu_{V L}(x)=\left\{\begin{array}{ccc}
0 & x \leq 5 \\
\frac{\mathrm{x}-5}{2} & 5<x<7 \\
1 & x=7 \\
\frac{10-\mathrm{x}}{3} & 7<x<10
\end{array} \quad \mu_{L}(x)=\left\{\begin{array}{cc}
0 & x \leq 10 \\
\frac{\mathrm{x}-10}{5} & 10<x<15 \\
1 & x=15 \\
\frac{20-\mathrm{x}}{5} & 15<x<20
\end{array}\right.\right. \\
0
\end{array} \quad x \geq 10 . \quad \begin{array}{ll}
0 & x \geq 20
\end{array}\right\}
$$



Figure 2: Fuzzified Values for Urea Level

## Fuzzified Values for Potassium Level

The major factor for Polycystic Kidney Diseases is Potassium Level. High lev-els of potassium in the blood (called hyperkalemia) is unpredictable and can be life-threatening. It can cause serious kidney and heart problems and cause sudden kidney failure. The membership functions for Potassium Level are given below.

Table 3.5: Fuzzified Values for Potassium Level

| Input <br> Variable | Range | Risk Level |
| :--- | :---: | :---: |
| Potassium <br> Level | $<3.5$ | Very <br> Low(VL) |
|  | $3.5-4.5$ | Low(L) |
|  | $4.6-6.5$ | Medium(M) |
|  | $>6.6$ | High(H) |

$$
\begin{gathered}
\mu_{V L}(x)=\left\{\begin{array}{cc}
0 & x \leq 2 \\
\frac{\mathrm{x}-2}{0.8} & 2 \leq x \leq 2.8 \\
1 & x=2.8 \\
\frac{3.5-\mathrm{x}}{0.7} & 2.8<x \leq 3.5 \\
0 & x \geq 3.5
\end{array} \quad \mu_{L}(x)=\left\{\begin{array}{cc}
0 & x \leq 3.5 \\
\frac{\mathrm{x}-3.5}{0.5} & 3.5<x<4 \\
1 & x=4 \\
\frac{4.5-\mathrm{x}}{0.5} & 4<x<4.5 \\
0 & x \geq 4.5
\end{array}\right.\right. \\
\mu_{M}(x)=\left\{\begin{array}{cc}
0 & x \leq 4.6 \\
\frac{\mathrm{x}-4.6}{0.9} & 4.6<x<5.5 \\
1 \\
\frac{6.5-\mathrm{x}}{1} & 5.5<x<6.5 \\
0 & x \geq 6.5
\end{array} \quad \mu_{H}(x)=\left\{\begin{array}{cc}
\frac{\mathrm{x}-6.6}{0.9} & 6.6<x<7.5 \\
1 \\
\frac{10-\mathrm{x}}{2.5} & x=7.5 \\
0 & 7.5<x<10 \\
0
\end{array}\right.\right.
\end{gathered}
$$



Figure 3: Fuzzified Values for Potassium Level

## Fuzzy Rules

In this section fuzzy rules are generated. Also rule viewer for linguistic variables are given below :
(1) If (BP is Very Low) and (UL is High) and (PL is Very Low)then Creatinine is Very Low.
(2) If (BP is Very Low) and (UL is High) and (PL is Medium) then Creatinine is Very Low.
(3) If (BP is Very Low) and (UL is High) and (PL is Low) then Creatinine is Very Low.
(4) If (BP is Very Low) and (UL is High) and (PL is High) then Creatinine is High
(5) If (BP is Very Low) and (UL is Medium) and (PL is Very Low) then Creatinine is Very Low.
(6) If (BP is Very Low) and (UL is Medium) and (PL is Medium) then Creatinine is Medium.
(7) If (BP is Very Low) and (UL is Medium) and (PL is Low) then Creatinine is Very Low.
(8) If (BP is Very Low) and (UL is Medium) and (PL is High) then Creatinine is Very Low.
(9) If (BP is Very Low) and (UL is Low) and (PL is Very low) then Creatinine is Very Low.
(10) If (BP is Very Low) and (UL is Low) and (PL is Medium) then Creatinine is Very Low.
(11) If (BP is Very Low) and (UL is Low) and (PL is low) then Creatinine is Very Low.
(12) If (BP is Very Low) and (UL is Low) and (PL is High) then Creatinine is Very Low.
(13) If (BP is Very Low) and (UL is Very Low) and (PL is Very Low) then Creatinineis Very Low.
(14) If (BP is Very Low) and (UL is Very Low) and (PL is Medium) then Creatinine is Very Low.
(15) If (BP is Very Low) and (UL is Very Low) and (PL is Low) then Creatinine is Very Low.
(16) If (BP is Very Low) and (UL is Very Low) and (PL is High) then Creatinine is Very Low.
(17) If (BP is Low) and (UL is High) and (PL is Very Low) then Creatinine is Very Low.
(18) If ( BP is Low) and (UL is High) and (PL is Medium) then Creatinine is High.
(19) If ( BP is Low) and ( UL is High) and ( PL is low) then Creatinine is low.
(20) If ( BP is Low) and (UL is High) and (PL is High) then Creatinine is High.
(21) If ( BP is Low) and (UL is Medium) and (PL is Very low) then Creatinine is Verylow.
(22) If ( BP is Low) and ( UL is Medium) and ( PL is Medium) then Creatinine is Medium.
(23) If (BP is Low) and (UL is Medium) and (PL is Medium) then Creatinine is Medium.
(24) If (BP is Low) and (UL is Medium) and (PL is High) then Creatinine is Low .
(25) If (BP is Low) and (UL is Low) and (PL is Very low) then Creatinine is low.
(26) If ( BP is Low) and (UL is Low) and (PL is Medium) then Creatinine is low.
(27) If (BP is Low) and (UL is Low) and (PL is low) then Creatinine is low.
(28) If (BP is Low) and (UL is Low) and (PL is High) then Creatinine is low.
(29) If (BP is Low) and (UL is Very Low) and (PL is Very Low) then Creatinine is Very Low.
(30) If (BP is Low) and (UL is Very Low) and (PL is Medium) then Creatinine is Very Low.
(31) If (BP is Low) and (UL is Very Low) and (PL is Low) then Creatinine is Very Low.
(32) If (BP is Low) and (UL is Very Low) and (PL is High) then Creatinine is Very Low.
(33) If (BP is Medium) and (UL is High) and (PL is Very Low) then Creatinine is Very Low.
(34) If ( BP is Medium) and (UL is High) and (PL is Medium) then Creatinine is Medium.
(35) If (BP is Medium) and (UL is High) and (PL is Low) then Creatinine is Very Low.
(36) If ( BP is Medium) and (UL is High) and (PL is High) then Creatinine is High.
(37) If (BP is Medium) and (UL is Medium) and (PL is Very Low) then Creatinine is Very Low.
(38) If (BP is Medium) and (UL is Medium) and (PL is Medium) then Creatinine is Medium.
(39) If ( BP is Medium) and (UL is Medium) and (PL is low) then Creatinine is Medium.
(40) If ( BP is Medium) and (UL is Medium) and (PL is High) then Creatinine is High.
(41) If ( BP is Medium) and (UL is Low) and (PL is Very Low) then Creatinine is Very Low.
(42) If (BP is Medium) and (UL is Low) and (PL is Medium) then Creatinine is Very Low.
(43) If (BP is Medium) and (UL is Low) and (PL is Low) then Creatinine is Very Low.
(44) If ( BP is Medium) and (UL is Low) and (PL is High) then Creatinine is High.
(45) If (BP is Medium) and (UL is Very Low) and (PL is Very Low) then Creatinine is Very Low.
(46) If (BP is Medium) and (UL is Very Low) and (PL is Medium) then Creatinine is Very Low.
(47) If (BP is Medium) and (UL is Very Low) and (PL is Low) then Creatinine is Very Low.
(48) If (BP is Medium) and (UL is Very Low) and (PL is High) then Creatinine is Medium.
(49) If (BP is High) and (UL is High) and (PL is Very Low) then Creatinine is High.
(50) If (BP is High) and (UL is High) and (PL is Medium) then Creatinine is High.
(51) If ( BP is High) and (UL is High) and (PL is Low) then Creatinine is High.
(52) If (BP is High) and (UL is High) and (PL is High) then Creatinine is High.
(53) If (BP is High) and (UL is Medium) and (PL is Very Low) then Creatinine is Medium.
(54) If (BP is High) and (UL is Medium) and (PL is Medium) then Creatinine is Low.
(55) If (BP is High) and (UL is Medium) and (PL is Low) then Creatinine is High.
(56) If (BP is High) and (UL is Medium) and (PL is High) then Creatinine is High.
(57) If (BP is High) and (UL is Low) and (PL is Very Low) then Creatinine is Very Low.
(58) If ( BP is High) and ( UL is Low) and (PL is Medium) then Creatinine is High.
(59) If ( BP is High) and (UL is Low) and (PL is Low) then Creatinine is Very Low.
(60) If (BP is High) and (UL is Low) and (PL is High) then Creatinine is High.
(61) If (BP is High) and (UL is Very Low) and (PL is Very Low) then Creatinine is Very Low.
(62) If (BP is High) and (UL is Very Low) and (PL is Medium) then Creatinine is Very Low.
(63) If (BP is High) and (UL is Very Low) and (PL is Low) then Creatinine is Very Low.
(64) If (BP is High) and (UL is Very Low) and (PL is High) then Creatinine is High.

## Rule Viewer for Linguistic Variables: Surface Viewer

From the Rule viewer, the level of Creatinine can be identified by giving the respec-tive data in the respective positions. Based on the rules generated, if the risk level of the factors mentioned in the Section 3 which are specified in the surface viewer are given,then the level of Creatinine of an individual can be found out.


Figure 4: Rule Viewer


Figure 5: Surface Viewer

## Conclusion

In this paper, fuzzy logic based fuzzy MAMDANI model has applied to diagnose the level of creatinine for Polycystic Kidney patients. The results are produced graphically using MATLAB. Using this MAMDANI model, the level of creatinine for people affected by Polycystic Kidney diseases can be found out. Increase in the creatinine level leads to kidney failure. Finally kidney failure leads to dialysis. Eventhough PKD is an inherited disorder,
some preventions can reduce the kidney failure. So taking the blood pressure medications prescribed by the doctor and eating a low-salt diet containing plenty of fruits, vegetables and whole grains helps to reduce kidney damage.

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# A Novel Fuzzy Time Series Forecasting Model based on Stochastic Process and Hydro Power Generation 

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#### Abstract

: Forecasting accuracy is one of the most favorable critical issues in fuzzy time series models. The study compares the application of two forecasting methods on the amount of hydropower production, the Fuzzy time series model and ARIMA model. Model discussed for the ARIMA model and Fuzzy time series model include the Sturges rule. A forecasting model is, therefore, a promising tool to predict the generation, consumption, and reservation of energy. In this paper, a long term forecasting model for hydropower production using the autoregressive integrated moving average (ARIMA) time series method is proposed. The collected data was obtain from the Secondary data in India. The electricity generation in this plant demonstrates an upward trend in the future. Although the power capacity of the hydropower plant is significantly affected by environmental variability, having a forecasting model and a long-term plan will greatly benefit renewable energy production to keep up with economic growth. Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) were compared. The results were display numerically and graphically.


Keywords:
ARIMA, Furzy Time Series, Hydro Power Generation, Mean Absolute Error, Root Mean Square Error, Forecasting.
AMS Classifications: $62 \mathrm{M} 10.62 \mathrm{M} 20,62 \mathrm{G} 05$

## INTRODUCTION

In the last decade, fuzzy time series have received more attention to deal with the vagueness and incompleteness inherent in time series data. Different types of models have been developed moreover to improve forecasting accuracy or decrease computation overhead. However, the issues
of controlling uncertainty in forecasting, effectively partitioning intervals, and consistently achieving forecasting accuracy with different interval lengths have been rarely investigated. In the literature survey most of the model is of first order fuzzy time series model. In the past decade many forecasting models based on the concepts of fuzzy time series have been proposed. It has been applied to predict roll number, temperature, crop production and stock index, etc.

Time series forecasting studied the relationships on the chronological set of past data measured over time to forecast the future values.

Forecasting performance are frequently conducted by using statistical tools like regression analysis, moving averages, integrated moving average and autoregressive integrated moving average. Fuzzy set theory and fuzzy logic was first introduced by Zadeh (1965) which provides a general method for handling uncertainty and vagueness in information available in linguistic terms. Song and Chissom (1993) used the fuzzy set theory given by Zadeh to develop models for fuzzy time series forecasting.

Song and Chissom (1993) presented the concept of fuzzy time series based on the historical enrollments of the University of Alabama. Fuzzy time series are used to handle forecasting problems. They presented the time-invariant fuzzy time series model and the time-variant fuzzy time series model based on the fuzzy set theory for forecasting the enrollments of the University of Alabama.

Wong et al.,(2010) proposed Traditional Time Series Method (ARIMA model and Vector ARMA model) and Fuzzy Time Series Method (Two-factor model, Heuristic model, and Markov model) for the forecasting problem. Reuter et al., (2010) presented an artificial neural network for modeling and forecasting of fuzzy time series. Analysis and forecasting of time series with fuzzy data may be carried out with the aid of artificial neural networks.

This paper applies Autoregressive Integrated Moving Average (ARIMA) forecasting model, the most popular and widely used forecasting models for univariate time series data.

Although it is applied across various functional areas, it's application is very limited in agriculture, mainly due to unavailability of required data and also due to the fact that agricultural product depends typically on monsoon and other factors, which the model failed to incorporate. In this context, it is worth mentioning, few applications of ARIMA model for forecasting agriculture product.

Applying ARIMA model Padhan(2012) forecasted annual productivity of selected 34 agricultural product in India with annual data from 1950 to 2010; Hossian et al. (2006) forecasted
three different varieties of pulse prices namely motor, mash and mung in Bangladesh with monthly data from Jan1998 to Dec 2000; Wankhade et al. (2010) forecasted pigeon pea production in India with annual data from 1950-1951 to 2007-2008; Saeed(2000) forecasted wheat production in Pakistan with annual data form 1947-48 to 1988-89;

Shukla and Jharkharia(2011) forecasted Ahmedabad wholesale vegetable market in India; Khin et al. (2008) forecasted natural rubber price in world market; Shukla and Jharkharia (2011) forecasted wholesale vegetable market in Ahmedabad; Assis et al. (2010) forecasted cocoa bean prices in Malaysia along Nochai and Nochai (2006) forecasted palm oi in Thailand; Masuda and Goldsmith (2009) forecasted world Soybean productions;

Cooray (2006) forecasted Sri Lanka's monthly total production of tea and paddy beyond Sept 1988 using monthly data from January 1988 to September 2004. With these exceptions, there is paucity of studies regarding applications of ARIMA model for forecasting agricultural products. Our main findings are as follows.

## BASIC DEFINITION OF FUZZY TIME SERIES

Song and Chissom (1993) presented the concept of fuzzy time series based on the historical enrollments of the University of Alabama. Fuzzy time series are used to handle forecasting problems. They presented the time-invariant fuzzy time series model and the time-variant fuzzy time series model based on the fuzzy set theory for forecasting the enrollments of the University of Alabama. Let U be the universe of discourse, where $\mathrm{U}=\{\mathrm{u} 1, \mathrm{u} 2, \ldots, \mathrm{un}\}$. A fuzzy set Ai of U is defined by

$$
\widetilde{A}_{l}=\mu_{\bar{A}_{l}}\left(\mu_{1}\right) / u_{1}+\mu_{\widetilde{A_{l}}}\left(\mu_{2}\right) / u_{2}+\ldots \ldots \ldots+\mu_{A_{l}}\left(\mu_{n}\right) / u_{n}
$$

Where

$$
\mu_{\widetilde{A_{l}}} \text { is the membership function of } \widetilde{A_{l}}, \mu_{\widetilde{A_{l}}}: \mathrm{U} \rightarrow[0,1]
$$

$\mu_{\bar{A}_{l}}$ ( ui) denotes the membership value of ui in $\widetilde{A}_{l}, \mu_{\bar{A}_{l}}$ ( ui)

## Fuzzy time series:

$Y(t,(t=\ldots, 0,1,2, \ldots)$ let $Y(t)$ be the universe of discourse defined by the fuzzy set $\mu i(t)$. If $\mathrm{F}(\mathrm{t})$ consists of $\mu \mathrm{i}(\mathrm{t})(\mathrm{i}=1,2,3, \ldots), \mathrm{F}(\mathrm{t})$ is called a fuzzy time series on $\mathrm{Y}(\mathrm{t})$.

## Fuzzy Relationship:

If there exists a fuzzy relationship $\mathrm{R}(\mathrm{t}-1, \mathrm{t})$, such that $\mathrm{F}(\mathrm{t})=\mathrm{F}(\mathrm{t}-1)^{\circ} \mathrm{R}(\mathrm{t}-1, \mathrm{t})$, where ${ }^{\circ}$ is an arithmetic operator, then $\mathrm{F}(\mathrm{t})$ is said to be caused by $\mathrm{F}(\mathrm{t}-1)$. The relationship between $\mathrm{F}(\mathrm{t})$ and $\mathrm{F}(\mathrm{t}-1)$ can be denoted by $\mathrm{F}(\mathrm{t}-1) \rightarrow \mathrm{F}(\mathrm{t})$.

Time invariant fuzzy time series: Suppose $\mathrm{F}(\mathrm{t})$ is calculated by $\mathrm{F}(\mathrm{t}-1)$ only, and $\mathrm{F}(\mathrm{t})=\mathrm{F}(\mathrm{t}-1)^{\circ}$ $\mathrm{R}(\mathrm{t}-1, \mathrm{t})$. For any t , if $\mathrm{R}(\mathrm{t}-1, \mathrm{t})$ is independent of t , then $\mathrm{F}(\mathrm{t})$ is considered a time invariant fuzzy time series. Otherwise, $\mathrm{F}(\mathrm{t})$ is time - variant.

## Fuzzy logical relationship:

Suppose $\mathrm{F}(\mathrm{t}-1)=\mathrm{Ai}$ and $\mathrm{F}(\mathrm{t})=\mathrm{Aj}$, a fuzzy logical relationship can be defined as $\mathrm{Ai} \rightarrow \mathrm{Aj}$. Where Ai and Aj are called the left-hand side and the right hand side of the fuzzy logical relationship, respectively.

## ARIMA MODELS

It is popularly known as Box - Jenkins (BJ) Methodology. Autoregressive Integrated Moving Average Models were used in many studies. Pal, et. al., (2007) use double exponential smoothing method and ARIMA for forecasting milk production. Sankar and Prabakaran, (2012) forecasted milk production in Tamil Nadu using Autoregressive (AR), moving average (MA) and Autoregressive Integrated Moving Average (ARIMA) methods. Chaudhari and Tingre (2013) used ARIMA for forecasting milk production.

Hossain and Hassan, (2013) forecasted milk, meat and egg production in Bangladesh using Cubic and Linear models. Time series when differentiated follows both AR and MA models and thus is known as autoregressive integrated moving average. In ARIMA ( $p, d, q$ ) time series, $p$ denotes the number of autoregressive terms (AR), d the number of times the series has to be differenced before it becomes stationary ( I ), and q the number of moving average terms (MA). Auto Regressive Process of order (p) is,

$$
Y_{t}=\mu+\phi_{1} Y_{t-1}+\phi_{2} Y_{t-2}+\ldots \ldots . . \Phi_{p} Y_{t-p}+\varepsilon_{t}:
$$

Moving Average Process of order (q) is,

$$
Y_{t}=\mu-\theta_{1} \varepsilon_{t-1}-\theta_{2} \varepsilon_{t-2}-\ldots-\theta_{p} \varepsilon_{t-p}+\varepsilon_{t}:
$$

And the general form of ARIMA model of order ( $p, d, q$ ) is
$Y_{t}=\phi_{1} Y_{t-1}+\phi_{2} Y_{t-2}+\ldots \ldots . . \phi_{p} Y_{t-p}+\mu-\theta_{1} \varepsilon_{t-1}-\theta_{2} \varepsilon_{t-2}-\ldots-\theta_{p} \varepsilon_{t-p}+\varepsilon_{t}$ :
ARIMA model includes following steps-

## Model identification:

At first, the data is checked for stationarity with the help of the autocorrelation function (ACF) and partial autocorrelation function (PACF). The next step in the identification process is to find the initial values for the orders of non-seasonal parameters p and q , which are obtained by looking for significant correlations in the ACF and PACF plots.

## Estimation:

Generally, this calculation is done by simple least squares but sometimes we have to resort to nonlinear (in parameter) estimation methods. Since software packages are available for easy and convenient usage, software package SPSS were used for the study.

## Diagnostic checking:

For the adequacy of the model, the residuals are examined from the fitted model and Alternative models are considered, if necessary. If the first identified model appears to be inadequate then other ARIMA models are tried until a satisfactory model fit to the data.

Different models are obtained for various combinations of AR and MA individually and collectively (Makridakis el al. 1998), the best model is obtained based on minimum value of Schwarz's Bayesian Information Criterion (BIC) is a model selection tool. BIC is given by the formula:

$$
B I C=-2 \log L+d * \log (N)
$$

where N is the sample size of the data and d is the total number of parameters. The lower BIC score signals a better model. The performances of different approaches have been evaluated on the basis of Mean Absolute Error (MAE) and Root Mean Square Error (RMSE), which are given by

$$
\begin{gathered}
\text { MAE }=\frac{\sum_{t=1 \mid}^{n} Y_{t}-F_{t} \mid}{n} \\
\text { RMSE }=\sqrt{\frac{1}{n}} \sum_{t=1}^{n}\left(Y_{t}-F_{t}\right)^{2}
\end{gathered}
$$

## Forecasting:

Nine-year forecast, from 2022 to 2030 is done because forecasting errors increase rapidly if we go too far out in the future.

Table 1: Forecasting the method of ARIMA $(1,1,1)$ Models

| Sl.N <br> o | Year | Hydro <br> Power | Predicted | Sl.No | Year | Hydro <br> Power | Predicted |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1995 | 3963.77 | 4000.658 | 15 | 2009 | 5307.227 | 5152.017 |
| 2 | 1996 | 3957.082 | 4006.118 | 16 | 2010 | 5307.294 | 5180.255 |
| 3 | 1997 | 4870.091 | 4614.000 | 17 | 2011 | 5090.157 | 5100.258 |
| 4 | 1998 | 5181.323 | 4872.235 | 18 | 2012 | 2125.82 | 3678.496 |
| 5 | 1999 | 4502.29 | 4540.855 | 19 | 2013 | 3915.12 | 4486.952 |
| 6 | 2000 | 5367.981 | 5047.227 | 20 | 2014 | 4111.131 | 4573.690 |
| 7 | 2001 | 4603.738 | 4683.253 | 21 | 2015 | 5121.272 | 5056.005 |


| 8 | 2002 | 2737.409 | 3688.009 | 22 | 2016 | 3077.125 | 4102.065 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 9 | 2003 | 2334.426 | 3404.393 | 23 | 2017 | 6469.425 | 5691.970 |
| 10 | 2004 | 3916.55 | 4156.504 | 24 | 2018 | 5489.891 | 5276.819 |
| 11 | 2005 | 5807.139 | 5129.880 | 25 | 2019 | 5489.891 | 5303.348 |
| 12 | 2006 | 6469.919 | 5536.728 | 26 | 2020 | 5307.294 | 5242.517 |
| 13 | 2007 | 6340.202 | 5555.242 | 27 | 2021 | 5489.891 | 5276.819 |
| 14 | 2008 | 5625.227 | 5268.469 |  |  |  |  |

Table 2: BIC Value for various ARIMA Models

| ARIMA (p,d,q) | BIC value |
| :---: | :---: |
| ARIMA (0,1,0) | 14.720 |
| ARIMA (1,1,0) | 14.746 |
| ARIMA (0,1,1) | 14.729 |
| ARIMA (1,1,1) | $\mathbf{1 4 . 6 9 7}$ |
| ARIMA (2,1,1) | 15.049 |
| ARIMA (1,1,2) | 15.042 |
| ARIMA (0,1,2) | 14.728 |

Table 3: Fitted a Model Statistics

| Fit Statistic | Mean |
| :---: | :---: |
| Stationary R-squared | .256 |
| R-squared | .101 |
| RMSE | 1209.533 |
| MAE | 21.251 |
| Normalized BIC | 14.697 |

## Result and Discussion:

The stationarity of data is checked with the help of ACF and PACF. In SPSS software all ARIMA models were tested for accurate fit based on the criteria of minimum BIC values. Table-1 shows that the predicted value of hydro power generation. Table -2 shows that BIC values of ARIMA (p,d,q). ARIMA $(1,1,1)$ has least BIC (14.697) value. Table-3 shows that the fitted model statistic value of hydro power generation from 2022-2030.

In Figure. 1 we can see that the values of ACF and PACF lies between -0.5 to 0.5 which indicates that the data is stationary. In Figure. 2 shows that the actual and forecasted values of hydro power generation.


Figure1: The plot of Hydro Power Generation in residuals of ACF and PACF by ARIMA (1,1,1) Model


Figure 2: Time series plot of Actual and Forecasted values
Table 4: A Comparison of the Error of Proposed Models

| Models | Fuzzy | ARIMA |
| :--- | :--- | :--- |
| RMSE | 1320.786 | 1209.533 |
| MAE | 22.210 | 21.251 |

## CONCLUSION

In this paper, we have presented efficient techniques to accurately predict time series data of Hydro Power Generation production. We have presented efficient techniques to accurately predict time series data of Hydro Power Generation production. The time series forecast based on
comparison of two models across the nine periods ahead in the forecast horizon. The accuracy of forecast of production of Hydro Power Generation as obtained by various methods is shown in Table 4 by presenting the RMSE and MAE error month wise for the period from 1995-2021. The as obtained by various methods is also presented. The Root mean square errors are two models from Table 4. Moreover, the ARIMA $(1,1,1)$ model developed for this study could be modified in term of learning rule, different training techniques, different of equation of ARIMA Model. Forecasting the future production of Hydro Power Generation through the most accurate univariate time series model can help the Indian government as well as the production is Hydro Power Generation industry to perform better strategic planning and also to help them in maximizing revenue and minimizing the Hydro Power Generation production.

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# Genome Comparisons of SARS-CoV and SARS-CoV-2 using nucleotide positions in codon as a fuzzy sets model 

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#### Abstract

The current availability of genome data stored in biological databases helps to compare the viral genomes. The genome comparison helps to define virus heterogeneity and allows researchers to better understand about viral evolution and pathogenesis and its host interactions. Thus in this study, we used fraction of nucleotides at each of a codon three base positions as fuzzy sets to compare the genomes of the major coronavirus such as SARS-CoV and SARS-CoV-2.

\section*{Introduction}

Severe acute respiratory syndrome coronavirus (SARS-CoV) and the 2019 novel coronavirus (SARS-CoV-2) are the primary pathogens that predominantly target the human respiratory system [1]. Their infections can cause mild respiratory illness to acute pneumonia and even respiratory failure. SARS-CoV was an extremely fatal virus that went away following extensive public health interventions [2]. The SARS-CoV epidemic ended in June 2003, with a total of 8098 cases recorded worldwide, 774 fatalities, and a case fatality rate of $97 \%$, with the majority of cases acquired nosocomially [3]. In contrast, the new SARS-CoV-2 virus that began in Wuhan, China, and spread over the globe caused a worldwide health emergency. As of February 2022, COVID-19 has been associated to roughly 280 million illnesses and over 5.3 million deaths, with instances continuing to rise due to the lack of a viable therapy [4]. This on-going pandemic impacted negatively on healthcare systems and the global economy, necessitating the development of efficient medicines to prevent the disease's spread and severity. For the coronavirus that causes COVID19, there are currently no target-specific drugs available; however, novel therapeutic options that target the viral replication cycle are being researched [5].

The first step in discovering a novel medicine is to look for genomes that are closely similar. Recent genome sequencing projects have generated a huge amount of data on the function and structure of biological molecules and sequences. We have access to a vast number of genomes


including SARS-CoV and SARS-CoV-2, protein structures, and genes, all of which have their expression levels tested in tests. Genomics has opened up new avenues for drug development, notably high-throughput sequencing and characterisation of expressed genes [6]. Knowing all of the genes and their activities might lead to more effective preventative measures, as well as changes in medication research strategy and development methods. Handling such a large volume of data, which is sometimes inaccurate and ambiguous, necessitates the use of strong integrated bioinformatics systems and novel technologies [7]. In genomic comparisons, fuzzy logic and fuzzy technologies are now commonly employed. A fuzzy set is a collection of items that has a range of membership grades. A membership (characteristic) function that assigns a grade of membership to each object ranging from zero to one characterises such a set. As a result, in this investigation, we used fuzzy sets to estimate the distance between two genomes, SARS-CoV and SARS-CoV2 , which can indicate considerable similarities between the genomes in this study.

The genomes of SARS-CoV (NC_004718.3) and SARS-CoV-2 (NC_045512.2) were obtained from the NCBI genome database [8].

The complete genome of SARS-CoV comprises of 29751 bp holding 13 genes and having $40.8 \%$ of $\mathrm{G}+\mathrm{C}$ content. The number of nucleotides at the three base sites of a codon in the coding sequences is shown in Table. 1 and we have the corresponding fractions (Table 2) as fuzzy set. This set can be considered as a point in the hypercube $\mathrm{I}^{12}$.

Table 1: The number of nucleotides at each of a codon's three base positions in the coding sequence of SARS-CoV.

|  | A | T | G | C |
| :---: | :---: | :---: | :---: | :---: |
| First base | 8282 | 8913 | 6015 | 5797 |
| Second base | 8272 | 8901 | 6035 | 5799 |
| Third base | 8297 | 8271 | 6038 | 5790 |

Table 2: The fractions of nucleotides at each of a codon's three base positions in the coding sequence of SARS-CoV.

|  | A | T | G | C |
| :--- | :---: | :---: | :---: | :---: |
| First base | 0.2855 | 0.3072 | 0.2073 | 0.1998 |
| Second base | 0.2851 | 0.3068 | 0.2080 | 0.1999 |
| Third base | 0.2921 | 0.2912 | 0.2126 | 0.2039 |

The fuzzy set of genome frequencies of SARS-CoV is
( $0.2855,0.3073,0.2074,0.1998,0.2852,0.3069$,
$0.2081,0.1999,0.2922,0.2913,0.2126,0.2039) \in I^{12}$
The complete genome of SARS-CoV-2 comprises of 29903 bp holding 11 genes and having $38 \%$ of G+C content. The number of nucleotides at the three base sites of a codon in the coding sequences is shown in Table. 3 and we have the corresponding fractions (Table 4) as fuzzy set.
Table 3: The number of nucleotides at each of a codon's three base positions in the coding sequence of SARS-CoV-2.

|  | A | T | G | C |
| :---: | :---: | :---: | :---: | :---: |
| First base | 8732 | 9360 | 5722 | 5341 |
| Second base | 8716 | 9386 | 5709 | 5344 |
| Third base | 8714 | 9367 | 5388 | 5364 |

Table 4: The fractions of nucleotides at each of a codon's three base positions in the coding sequence of SARS-CoV-2.

|  | A | T | G | C |
| :--- | :---: | :---: | :---: | :---: |
| First base | 0.3010 | 0.3227 | 0.1973 | 0.1841 |
| Second base | 0.3005 | 0.3236 | 0.1968 | 0.1842 |
| Third base | 0.3069 | 0.3299 | 0.1897 | 0.1889 |

The fuzzy set of genome frequencies of SARS-CoV-2 is
(0.3010, 0.3227, 0.1973, 0.1841, 0.3005, 0.3236,
$0.1968,0.1842,0.3069,0.3299,0.1897,0.1889) \in I^{12}$
The distance between the two sets is estimated using $\mathrm{d}(\mathrm{x}, \mathrm{y})=\frac{\sum_{i=1}^{n}\left|x_{i}-y_{i}\right|}{\sum_{i=1}^{n} \max \left(x_{i}, y_{i}\right)}$, where $\mathrm{x}=\left(x_{1}, x_{2}, \ldots, x_{n}\right)$ and $\mathrm{y}=\left(y_{1}, y_{2}, \ldots, y_{n}\right) \in I^{n}$

Using the distance given in the above equation, it is possible to compute the distance between these two fuzzy sets representing the frequencies of the nucleotides of SARS-CoV and SARS-CoV-2

$$
\mathrm{d}(\text { SARS-CoV and SARS-CoV-2 })=\frac{0.2069}{3.1162} \approx 0.0663
$$

In general, the similarities between the organisms are compared based on the sequencealigned dissimilarity to illustrate the homology of organisms in relation to genotype and phenotype. Where as in this study, we used fraction of nucleotides at each of a codon three base positions as fuzzy sets to compare the genomic characterization of the major coronavirus such as SARS-CoV and SARS-CoV-2. Thus we show evidence with fuzzy sets that using features from a large number of viral genomes, the genomes can be compared to define virus heterogeneity, which will allow researchers to learn more about viral genome evolution, control, and pathogenesis, as well as the basic mechanism of virus-host interaction.

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# PREDICTING THE CHANCE OF DEATH RATE FOR COVID 19 USING BIOMATHEMATICAL MODELLING VIA FUZZY INFERNCE SYSTEM 

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#### Abstract

The Indian Covid-19 pandemic is part of a global Corona pandemic, outbreak that started in Wuhan, China, in 2019. As a result of Covid-19, many people died all over the world. As a necessary consequence, vaccination has been discovered as a method of protecting people from communicable diseases Covid19. Vaccines significantly boost the amount of antibiotics in the human body. Despite having been vaccinated, some people were affected and even died. The main goal of this paper is to predict the chance of death in India after being vaccinated. The primary goal of this article is to show how a mathematical model can help to solve a biological problem in this society by implementing the fuzzy inference system method with the MATLAB Tool Set. This study provides a set of tools for predicting the possibility of death rate after vaccination in India.


Keywords: Fuzzy Inference System, Covid 19 dataset, Bio-mathematical Model, Optimal Solution

## 1 Introduction

A variety of biological issues arise in everyday life in this world, posing problems for this society. Biomathematical models are useful in resolving societal issues. Covid 19 is a terrible disease in biological issues that causes problems all over the world. The Covid 19 pandemic is a massive global outbreak that began in Wuhan, China in December 2019. On January 30, 2020, the first case of Covid 19 was filed in Kerala, India [22][26][27]. Covid 19 has been classified into several variants as it continues to spread from person to person. A few types of covid 19 variants are listed below, along with an explanation of how they are used to obtain treatment: Covid-19 pneumonia was primarily analyzed by CT scan, and research was conducted to assist radiologists in determining the pattern of analysis for atypical pneumonias that are similar to covid 19[1]; Using FIS and Deep Neural Network covid 19 patients were identified using the Hybrid Diagnose Strategy, which produced accurate results when compared to other recent methodologies in terms of accuracy, error, precision, macro-average, macro-average [2]; Implementing a deep transfer learning network to automatically classify the covid 19 , pneumonia, and tuberculosis from X-rays [3]; Covid 19 was
detected from the lungs using a Fuzzy integral based CNN ensemble[4] ; A fuzzy graph approach was used to analyses the Covid 19 outbreak [5]; Routine COVID-19 Pneumonia Severity Biomarkers May Show Different Results in Kidney Transplant Recipients [6]; Radiotherapy could be a promising treatment option for COVID-19 pneumonia management [7]. Doctors have a difficult time treating people because the symptoms of diseases differ from person to person, and as a result, many people have died [28]. So, after extensive research, scientists discovered a vaccine to help combat the Covid 19 pandemic. In November 2020, the Indian government allocated Rs. 900 crores to the department of biotechnology to aid in the development of a covid 19 vaccine [24]. India's budget for 2021 includes $\$ 35,000$ crore for vaccine procurement [29].

In ancient times, Boolean logic, such as yes or no, 0 or 1, true or false, was used[8]. Professor Lotfi A. Zadeh proposed fuzzy logic for the first time in 1965[9] [10] [11]. Fuzzy logic is involved with the membership function in linguistic values [12]. Fuzzy logic has a wide range of applications that help humans make decisions in a similar way [13] [14][15]. The fuzzy inference system is a type of fuzzy logic that has a wide range of practical applications[16]. Essentially, a fuzzy inference system deals with the terms "If" and "Then," with "If" defining the antecedent and "Then" defining the consequence[17]. The Mamdani method, which was introduced in 1976, the Takagi, Sugeno, and Kang method, which was introduced in 1985, and the Tsukamoto method are the three types of fuzzy inference systems[18]. In general, membership functions with linguistic values and parameters are used in fuzzy inference systems [19]. In the FIS model, a crisp value was converted to a fuzzified value. Then, with knowledge based, rules will be generated using a Fuzzy Inference System[20]. As a result, fuzzified values were converted to crip values, as shown in Figure 1,


Figure 1

During this pandemic, the government is requiring people to get vaccinated in order to protect them from becoming infected with Covid 19. The first vaccination campaign in India began on January 16, 2021[23][25]. It should take two doses to complete the vaccination. Vaccination greatly helps in the increase of antibiotics in the human body, but it cannot guarantee that covid will not affect those who have been vaccinated. Even after vaccination, some people are still infected with Covid 19, and some people die as a result. It is difficult to predict whether the risk of death cases increases or decreases after vaccination. To solve this biological problem, the Fuzzy Inference System and the MATLAB toolset, which provides a toolset for prediction, were used in the Biomathematical model to predict the chance of death rate after being vaccinated for Covid 19.

This article is structured as follows: Section 2, will discussed the data collection. Section 3 explains about the algorithm. Section 4, discusses the fuzzy Hungarian method algorithm that was used in our problem, and Section 5, discusses the steps involved in finding the optimal solution for the fuzzified value. Section 6, introduces a new fuzzy inference system model that can be used to generate a tool set for predicting the change in death rate after being vaccinated for covid 19 in India. Section 7, discusses the implementation of the Fuzzy Inference System (FIS). Section 8, discusses the rules indicating the change in death rate after covid-19 vaccination, and Section 9 discusses the visualization results.

## 2 Algorithm

Step 1: Data Collection
Step 2: Determine the Fuzzy Hungarian Method Algorithm
Step 3: Find Optimal Solution using Fuzzy Hungarian Algorithm
Step 4: Introduce the new Fuzzy Inference System Model
Step 5: The methodology used in the MATLAB tool set in the Fuzzy Inference system model is described.

Step 6: Explains the rules that indicate the chance of death after being vaccinated for covid 19.
Step 7: The result of the fuzzy inference system model, as well as its visualisation, are described in detail.

## 3 Implementation

### 3.1 Data collection

A dataset of vaccinated people in India was discovered using the Kaggle website. In the dataset, India is divided into four regions: north, south, west, and east. The total number of vaccines
allocated to the specific region, as well as the number of people who received dose 1 and dose 2, are all included in the dataset.

| India | Total Vaccination <br> Doses | Dose 1 | Dose 2 | Population |
| :--- | :--- | :--- | :--- | :--- |
| North | 517708793 | 287780407 | 229928386 | 425358882 |
| South | 376246485 | 200830580 | 175415905 | 319502247 |
| West | 389139955 | 207190527 | 181949428 | 314802352 |
| East | 385153598 | 210818117 | 174335481 | 369741397 |

Table 1: Dataset

Figure 2 depicts a graphical representation of the given data,


Figure 2

### 3.2 Fuzzy Hungarian Method Algorithm

Find the Optimal solution for the fuzzified dataset using the Fuzzy Hungarian Method, as detailed below [21].
Case 1: Verify whether the problem is balanced or unbalanced. If it is balanced, proceed to step 3; otherwise, proceed to step 2.
Case 2: To make the unbalanced problem balanced, set a dummy source or dummy destination to zero.

Case 3:

- Subtract the column minimum from each column after subtracting the row minimum.
- Cover all zeros in the resulting matrix with the fewest horizontal and vertical lines possible.


## Case 4:

- If the result is feasible by confirming that the number of rows is equal to the single fuzzified number of zero, proceed to step 5.
- If not, take the minimum value of all uncovered elements and subtract it from all uncovered values.
- Add it to the elements at the intersection of the vertical and horizontal lines.
- If step 3 and 4 are completed successfully, proceed to step 5 .

Case 5: Find the Optimal solution by comparing the feasible solution to the original dataset.

### 3.3 Steps Involves to Find Optimal Solution

Case 1: India's vaccinated details data set. The $4 * 4$ matrix is Fuzzified using the normalized method, and the result is shown below,

| INDIA | TOTAL <br> VACCINATION <br> DOSES | DOSE 1 | DOSE 2 | POPULATION |
| :--- | :--- | :--- | :--- | :--- |
| North | 0.310330679 | 0.317421328 | 0.301890193 | 0.297577606 |
| South | 0.225533792 | 0.221515808 | 0.230316675 | 0.223521167 |
| West | 0.233262537 | 0.228530819 | 0.238895027 | 0.220233159 |
| East | 0.230872991 | 0.232532045 | 0.228898106 | 0.258668067 |

Table 2: Normalized Dataset
Case 2: In the Hungarian Assignment method, a normalized dataset was used.
Case 3: The Hungarian Assignment method was used to find the row and column minimums, which were then subtracted from the respective entire row and column, as shown in table 3 .

| India | TotalVaccination Doses | Dose 1 | Dose 2 | Population |
| :--- | :--- | :--- | :--- | :--- |
| North | 0.010778187 | 0.019843721 | 0.004312586 | 0 |
| South | 0.002043098 | 0 | 0.008800867 | 0.00200536 |
| West | 0.011054492 | 0.00829766 | 0.018661867 | 0 |
| East | 0 | 0.00363394 | 0 | 0.029769962 |

Table 3: Subtract Row Minimum and Column Minimum Values
Case 4: The diagonal method yielded a feasible solution, with the number of rows equal to a single fuzzified number of zero.

| India | Total Vaccination Doses | Dose 1 | Dose 2 | Population |
| :---: | :---: | :---: | :---: | :---: |
| North | 0.006465601 | 0.017574233 | $\phi$ | $\phi$ |
| South | $母$ | 0 | 006757769.004274848 |  |
| West | 0.006741906 | 0.0060281720 | 014349281 | $\phi$ |
| East | $\phi$ | 0.005677038 | $\phi$ | 0.034082548 |

Table 4: Feasible Solution
The optimal solution for the fuzzy Hungarian assignment problem is shown in table 5,

| Inida | Total <br> Vaccination <br> Doses | Dose 1 | Dose 2 | Population |
| :--- | :--- | :--- | :--- | :--- |
| North | 0.310330679 | 0.317421 | 0.30189 | 0.2975776 |
| South | 0.225533792 | 0.221516 | 0.230317 | 0.2235212 |
| West | 0.233262537 | 0.228531 | 0.238895 | 0.2202332 |
| East | 0.230872991 | 0.232532 | 0.228898 | 0.2586681 |

Table 5: Optimal Solution from original dataset
Case 5: The diagonal method yields a feasible and optimal solution for a normalized dataset, as shown in table 3 ,

| INDIA | OPTIMAL SOLUTION |
| :--- | :--- |
| North | 0.301890193 |
| South | 0.221515808 |
| West | 0.220233159 |
| East | 0.230872991 |

Table 6: Optimal Solution

### 3.4 New model of Fuzzy Inference System

Covid 19 causes a massive pandemic all over the world. Due to covid-19, a large number of people are experiencing a variety of problems. The majority of people died as a result of a lack of treatment and hospital facilities. Since it is so easy to spread from one person to another, the number of covid cases is rapidly increasing. After extensive research, a researcher discovered vaccination. Vaccinations that help to increase the amount of antibiotics in the human body. Vaccination is usually given in two doses for covid 19. In some cases, people died even after receiving the vaccination. As a result, predicting the possibility of death rate after vaccination is difficult. This paper discussed the possibility of death rate after taking vaccination for the covid 19
in India. The data set was obtained from the Kaggle website, and it categorises Indian regions as north, south, east, and west, describing the total number of vaccines allocated to each region, as well as the number of people who received dose 1 and dose 2. The Fuzzy Hungarian Assignment Method was used to find the optimal solution for the normalised dataset, which was then used to fix the parameters in the Fuzzy inference system model. A diagram of a fuzzy inference system can provide a detailed description of the expert system's entire process. The introduction of a new model of a fuzzy inference system is depicted in Figure 3.


Figure 2

### 3.5 Methodology

This model employs the normalised method to convert positive real values into a fuzzified dataset. By implementing the fuzzified dataset, the "Hungarian Assignment Method" is used to find the optimal solution. Now, using the Sugeno method, a Fuzzy Inference System model is built with three input values: the number of people who have already received the entire vaccine, the number of people who have received dose 1 and the number of people who have received dose 2 , and the output value as the chance of death rate after vaccination for covid 19, as shown in Figure 4.


Figure 4

Then, with the parameter shown in table 7, set the membership function to very low, low, normal, and high.

| Linguistic Value | Parameter |
| :--- | :--- |
| Very Low | $\left[\begin{array}{lll}0 & 0.2202 & 0.23\end{array}\right]$ |
| Low | $\left[\begin{array}{lll}0.15 & 0.2215 & 0.25\end{array}\right]$ |
| Normal | $\left[\begin{array}{lll}0.2 & 0.2308 & 0.35\end{array}\right]$ |
| High | $\left[\begin{array}{lll}0.3 & 0.3019 & 0.5\end{array}\right]$ |

Table 7: Linguistic Value and Parameter
Parameter was fixed by using the optimal solution which was found with the help of fuzzy Hungarian assignment method as shown in figure 5


Figure 5

### 3.6 Rules Indicate the Chance of Death Rate of Covid-19

The rules establish the framework for a given situation. Given a model of a fuzzy inference system for predicting the chance of death due to covid 19. Three inputs were used: Totally vaccinated count, Dose 1 count, Dose 2 count, and membership with linguistic functions of Very Low, Low, Normal High, and the output was chance of death rate with membership functions of minimum, moderate, and maximum. The rules indicating as
IF (Totally Vaccinated count is low) and (Dose 1 count is low) and (Dose 2 count is low) then (Chance of Death rate is low)
As indicating in the table below, there are $4^{\wedge} 3$ (Four linguistic to the power of Three inputs) $=64$

| No. of Rules | Totally <br> Vaccinated <br> count | Dose 1 Count | Dose 2 Count | Chance of <br> Death Rate |
| :--- | :--- | :--- | :--- | :--- |
| 1 | Very Low | Very Low | Very Low | Maximum <br> Possibility |
| 2 | Very Low | Very Low | Low | Maximum <br> possibility |
| 3 | $\ldots$ | Very Low | Normal | Maximum <br> Possibility |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\cdots$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $\ldots$ | High | High | High | Minimum <br> 64 |

As shown in Figure 6, the MATLAB toolset was used to generate 64 rules.


Figure 6

## 4 Visualization Results

The normalised dataset obtained from Kaggle was used to estimate the chance of death after being vaccinated for covid -19 prediction implemented in the MATLAB toolset. Figure 7 depicts


Figure 7
As a result of this discovery, a MATLAB toolset that was provided in this paper helps to predict the chance of death rate with the appropriate input dataset of totally vaccinated count, dose 1 count, and dose 2 count of covid 19 vaccination, Figure 8 shows a visualisation of the possible death rate for a dataset collected from the Kaggle website.


Figure 8

## Conclusion

There are numerous issues in the field of biology that exist in this society. Mathematical concepts and models aid in the solution of biological problems. Covid 19 is one of the deadliest diseases that has caused a pandemic throughout the world in the field of biology. Vaccination is the only way to protect people from the pandemic. So, after a long and complex research process, the covid 19 scientific research community discovered the vaccination that can be taken as the two do. In this paper, the Kaggle website was used to obtain a dataset of covid 19 vaccinated count, which
was fuzzified using normalisation. The optimal solution for the normalised dataset was found using the Fuzzy Hungarian Assignment Method. The probability of death after vaccination due to covid 19 tool set with the parameter by implementing optimal solution was estimated using the MATLAB tool box of fuzzy inference system, and the MATLAB toolset was provided to predict the probability of death rate after vaccination for covid 19 based on the inputs Totally vaccinated Count, Dose 1 Count, and Dose 2 Count. The death rate can be predicted using this tool set.

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# The Detour Cototal Domination Number of a Graph 

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#### Abstract

Let $G=(V, E)$ be a connected graph with at least two vertices. A detour dominating set $S \subseteq V$ is called a detour cototal dominating set of $G$, if $\langle V-S\rangle$ has no isolated vertices. The detour cototal domination number $\gamma_{d c t}(G)$ of $G$ is the minimum order of its detour cototal dominating sets of $G$. Some of its general properties are studied. Connected graphs of order $n \geq 2$ with detour cototal domination number $n$ or $n-2$ are characterized. It is shown that for every pair $a$ and $b$ of positive integers with $2 \leq a \leq b$, there exists a connected graph $G$ such that $d n(G)=a$ and $\gamma_{d c t}(G)=b$.


Keywords: detour cototal domination number, cototal domination number, domination number, detour number, detour set.

AMS Subject Classification: 05C12, 05C69.

## 1. Introduction

By a graph $G$ we mean a finite, undirected connected graph without loops or multiple edges. Unless and otherwise stated, the graph $G=(V, E)$ has $n=|V|$ vertices and $m=|E|$ edges. For basic definitions and terminologies, we refer [1]. For vertices $u$ and $v$ in a graph $G$, the detour distance $D(u, v)$ is the length of a detour distance $D(u, v)$ is the length of a longest $u-v$ path in $G$. A $u-v$ path of length $D(u, v)$ is called a $u-v$ detour. These concepts were studied by Chartrand et al. [4]. The closed detour interval $I_{D}[u, v]$ consists of $u, v$ and all vertices in some $u-v$ detour of $G$. For $S \subseteq V(G), I_{D}[S]=\cup_{u, v \in S} I_{D}[u, v]=V(G)$. A
subset $S$ of $V$ of a graph $G$ is called a detour set if $I_{D}[S]=V(G)$. The detour number $d n(G)$ of $G$ is the minimum cardinality taken over all detour sets in $G$. These concepts were studied by Chartrand [3,7,10].

A set $D \subseteq V(G)$ is a dominating set of $G$ if every vertex in $\langle V-D\rangle$ is adjacent to some vertex in $D$. The domination number $\gamma(G)$ is the minimum order of its dominating sets and any dominating set of order $\gamma(G)$ is called $\gamma$-set of $G$. The domination number of a graph was studied in [6]. A set $D \subseteq V$ is called a detour dominating set of $G$ if $D$ is a detour set of $G$ and a dominating set of $G$. The detour domination number of a graph was studied in $[5,8,9]$. A dominating set $S$ of $G$ is a cototal dominating set if every vertex $v \in V-S$ is not an isolated vertex in $\langle V-S\rangle$. The co-total domination number $\gamma_{c t}(G)$ of $G$ is the minimum cardinality of a cototal dominating set. The cototal domination number of a graph was studied in $[2,11,12,13]$. The following theorem are used in the sequel.

Theorem 1.1. [3] Each end vertex of a non-trivial connected graph $G$ belongs to every detour set of $G$. Moreover if the set $S$ of all end-vertices of $G$ is a detour set, then $S$ is the unique minimum detour set of $G$.

Theorem 1.2. [3] For the star graph $G=K_{1, n-1}(n \geq 3), d n(G)=n-1$.
Theorem 1.3. [3] Let $G$ be a double star of order $(n \geq 4), d n(G)=n-2$.

## 2.The detour cototal domination number of a graph

Definition 2.1. Let $G=(V, E)$ be a connected graph with at least two vertices. A detour dominating set $S \subseteq V$ is called a detour cototal dominating set of $G$, if $\langle V-S\rangle$ has no isolated vertices. The detour cototal domination number $\gamma_{d c t}(G)$ of $G$ is the minimum order of its detour cototal dominating sets of $G$ and any detour cototal dominating set of order $\gamma_{d c t}(G)$ is called the $\gamma_{d c t}$-set of $G$.

Example 2.2. For the graph $G$ given in Figure 2.1, $S=\left\{v_{1}, v_{2}, v_{3}\right\}$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G)=3$.


Theorem 2.3. Each end vertex of a graph $G$ belongs to every detour cototal dominating set of $G$.

Proof: Each detour cototal dominating set is a detour set of $G$, the results follows form Theorem 1.1.

Theorem 2.4. For the complete graph $G=K_{n}(n \geq 3), \gamma_{d c t}(G)=2$.
Proof: Let $x, y$ be two adjacent vertices of $G$. Then $S=\{x, y\}$ is a $\gamma_{d c t}$-set of $G$ so that $\gamma_{d c t}(G)=2$.

Theorem 2.5. For the star graph $G=K_{1, n-1}, \gamma_{d c t}(G)=n$.
Proof: Let $S$ be the set of end vertices of $G$. By Theorem 2.3, $S$ is a subset of every detour cototal dominating set of $G$. Since $\langle V-S\rangle$ contains an isolated vertex, $S=V$ is the unique detour cototal dominating set of $G$ so that $\gamma_{d c t}(G)=n$.
Theorem 2.6. For the path $G=P_{n},(n \geq 3), \gamma_{d c t}(G)=\left\{\begin{array}{l}\frac{n+2}{3} \text { when } n \text { is odd } \\ \frac{n+5}{3} \text { when } n \text { is even }\end{array}\right.$
Proof: Let $P_{n}$ be $v_{1}, v_{2}, \ldots, v_{n}$. We consider the following cases.
Case (i): $n \equiv 0(\bmod 4)$.
Let $n=4 k$. Let $S=\left\{v_{1}, v_{4}, v_{7}, \ldots, v_{4 k-1}, v_{4 k}\right\}$ is a detour cototal dominating set of $G$ and so $\gamma_{d c t}(G) \leq \frac{4 k+2}{3}$. We prove that $\gamma_{d c t}(G)=\frac{4 k+2}{3}$. On the contrary, suppose that $\gamma_{d c t}(G)<$ $\frac{4 k+2}{3}-1$. Then there exists a detour cototal dominating set $S^{\prime}$ such that $\left|S^{\prime}\right| \leq \frac{4 k+2}{3}-1$. Then there exists $x \in S$ such that $x \notin S^{\prime}$. Hence $x$ is either not dominated by an element of $S^{\prime}$ or $\left\langle V-S^{\prime}\right\rangle$ contains an isolated vertex. Hence it follows
that $S^{\prime}$ is not a detour cototal dominating set of $G$ which is a contradiction. Therefore $\gamma_{d c t}(G)=\frac{4 k+2}{3}=\frac{n+2}{3}$.

Case $(\mathbf{i i}): n \equiv 1(\bmod 4)$.
Let $n=4 k-1$. Let $S=\left\{v_{1}, v_{4}, v_{7}, \ldots, v_{4 k-1}\right\}$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G) \leq \frac{4 k+1}{3}$. We prove that $\gamma_{d c t}(G)=\frac{4 k+1}{3}$. On the contrary, suppose that $\gamma_{d c t}(G)<\frac{4 k+1}{3}-1$. Then there exists a detour cototal dominating set $S^{\prime}$ such that $\left|S^{\prime}\right| \leq$ $\frac{4 k+1}{3}-1$. Then there exists $x \in S$ such that $x \notin S^{\prime}$. Hence $x$ is either not dominated by an
element of $S^{\prime}$ or $\left\langle V-S^{\prime}\right\rangle$ contains an isolated vertex. Hence it follows that $S^{\prime}$ is not a detour cototal dominating set of $G$ which is a contradiction. Therefore $\gamma_{d c t}(G)=\frac{4 k+1}{3}=\frac{n+1}{3}$.
Case (iii): $n \equiv 2(\bmod 4)$.
Let $n=4 k-2$. Let $S=\left\{v_{1}, v_{4}, v_{7}, \ldots, v_{4 k-2}\right\}$ is a detour cototal dominating set of $G$ and so $\gamma_{d c t}(G) \leq \frac{4 k}{3}$. We prove that $\gamma_{d c t}(G)=\frac{4 k}{3}$. On the contrary, suppose that $\gamma_{d c t}(G) \leq$ $\frac{4 k}{3}-1$. Then there exists a detour cototal dominating set $S^{\prime}$ such that $\left|S^{\prime}\right| \leq \frac{4 k}{3}-1$. Then there exists $x \in S$ such that $x \notin S^{\prime}$. Hence $x$ is either not dominated by an element of $S^{\prime}$ or $\left\langle V-S^{\prime}\right\rangle$ contains an isolated vertices. Hence it follows that $S^{\prime}$ is not a detour cototal dominating set of $G$ which is a contradiction. Therefore $\gamma_{d c t}(G)=\frac{4 k}{3}=\frac{n}{3}$.
Case (iv): $n \equiv 3(\bmod 4)$.
Let $n=4 k-3$. Let $S=\left\{v_{1}, v_{4}, v_{7}, \ldots, v_{4 k-3}\right\}$ is a detour cototal dominating set of $G$ and so $\gamma_{d c t}(G) \leq \frac{4 k+4}{3}$. We prove that $\gamma_{d c t}(G)=\frac{4 k+4}{3}$. On the contrary, suppose that $\gamma_{d c t}(G)<$ $\frac{4 k+4}{3}-1$. Then there exists a detour cototal dominating set $S^{\prime}$ such that $\left|S^{\prime}\right| \leq \frac{4 k+4}{3}-1$. Then there exists $x \in S$ such that $x \notin S$. Hence $x$ is either not dominated by an element of $S^{\prime}$ or $\left\langle V-S^{\prime}\right\rangle$ contains an isolated vertex. Hence it follows that $S^{\prime}$ is not a detour cototal dominating set of $G$ which is a contradiction. Therefore $\gamma_{d c t}(G)=\frac{4 k+4}{3}=$ $\frac{n+4}{3}$.
Theorem 2.7. For the wheel graph $W_{n}=K_{1}+C_{n-1},(n \geq 5), \gamma_{d c t}(G)=2$.
Proof: Let $x$ be a vertex of $K_{1}$ and $C_{n-1}$ be $v_{1}, v_{2}, \ldots, v_{n-1}, v_{1}$. Then $S=\left\{x, v_{1}\right\}$ is a $\gamma_{d c t^{-}}$ set of $G$ so that $\gamma_{d c t}(G)=2$.
Theorem 2.8. Let $G$ be a double star of order $(n \geq 4)$. Then $\gamma_{d c t}(G)=n-2$.
Proof: Let $x$ and $y$ be the central vertices of $G$ and $S=\left\{x_{1}, x_{2}, \ldots, x_{n-2}\right\}$ be the set of end vertices of $G$. By Theorem 2.3, $S$ is a subset of every detour cototal dominating set of $G$ and so $\gamma_{d c t}(G) \geq n-2$. Since $S$ is a detour dominating set of $G$ and $\langle V-S\rangle$ has no isolated vertex, $S$ is not a detour cototal dominating set of $G$ and so $\gamma_{d c t}(G)=n-$ 2.

Theorem 2.9. For the complete bipartite graph $G=K_{m, n},(1 \leq m \leq n)$,
$\gamma_{d c t}(G)=\left\{\begin{array}{cl}n+1 & m=1, n \geq 1 \\ 2 & \text { otherwise }\end{array}\right.$
Proof: If $m=1$ and $n \geq 1$, then the result follows from Theorem 2.4 and 2.5 . So let $m \geq 2$. Let $U=\left\{u_{1}, u_{2}, \ldots, u_{m}\right\}$ and $V=\left\{v_{1}, v_{2}, \ldots, v_{n}\right\}$ be the bipartite sets of $G$. Let $S=\left\{u_{1}, v_{1}\right\}$. Then $S$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G)=2$.

Theorem 2.10. For the helm graph $G=H_{r}, \gamma_{d c t}(G)=r+1$.
Proof: Let $x$ be the central vertex of $G$ and $S$ be the set of $r$ end vertices of $G$. By Theorem $2.3, S$ is a subset of every detour cototal dominating set of $G$. Since $x$ is not dominated by any vertex of $S, S$ is not a detour cototal dominating set of $G$ and so $\gamma_{d c t}(G) \geq r+1$. Let $S^{\prime}=$ $S \cup\{x\}$. Then $S^{\prime}$ is a detour dominating set of $G$. Since $\left\langle V-S^{\prime}\right\rangle$ has no isolated vertices, $S^{\prime}$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G)=r+1$.

Theorem 2.11. Let $G$ be a connected graph of order $n \geq 5$ with $C(G) \geq 4$. Then $\gamma_{d c t}(G) \leq n-3$, where $C(G)$ is the length of a longest cycle in $G$.

Proof: Let $C: v_{1}, v_{2}, v_{3}, \ldots, v_{p}, v_{1}$ be a longest cycle in $G$.
Case (i): No vertex of $C$ is a cut vertex of $G$. Then $S=V(G)-V(C)$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G) \leq n-|C(G)| \leq n-3$.

Case (ii): One vertex of $C$ is a cut vertex of $G$. Without loss of generality, let us assume that $v_{1}$ is a cut vertex of $G$. Then $S=V(G)-\left\{v_{1}, v_{2}, v_{p}\right\}$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G) \leq n-3$.

Case (iii): At least two vertices of $C$ are cut vertices of $G$. Then in Case (ii), we can prove that $\gamma_{d c t}(G) \leq n-3$.

Remark 2.12. The bound in Theorem 2.11, can be sharp. For the graph $G$ in Figure 2.2. $S=$ $\left\{v_{3}, v_{5}\right\}$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G)=2=n-3$.


Theorem 2.13. Let $G$ be a connected graph of order $n \geq 3$. Then $\gamma_{d c t}(G)=n$ if and only if $G=K_{1, n-1}$.

If $G=K_{1, n-1}$, then by Theorem 2.5, $\gamma_{d c t}(G)=n$. Conversely let $\gamma_{d c t}(G)=n$.
If $G$ is a tree, then $G=K_{1, n-1}$, which satisfies the requirements of this theorem. So, let us assume that $G$ is not a tree. Therefore $G$ contains a cycle. If $G=C_{n}, n \geq 4$, then by Theorem 2.8, $\gamma_{d c t}(G) \leq n-2$, which is a contradiction. Therefore $G \neq C_{n},(n \geq 4)$. If $C(G) \geq 4$, then by Theorem 2.12, $\gamma_{d c t}(G) \leq n-3$, which is a contradiction. Therefore $C(G)=3$. Let $C$ : $v_{1}, v_{2}, v_{3}, v_{1}$ be a longest cycle in $G$.

Case (i): No vertex of $C$ is a cut vertex of $G$. Then $S=V(G)-V(C)$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G) \leq n-C(G) \leq n-3$.

Case (ii): One vertex of $C$ is a cut vertex of $G$. Without loss of generality, let us assume that $v_{1}$ is a cut vertex of $G$. Then $S=V(G)-\left\{v_{1}, v_{2}, v_{p}\right\}$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G) \leq n-3$.

Case (iii): At least two vertices of $C$ are cut vertices of $G$. Let $v_{1}$ and $v_{2}$ are cut vertices of $G$. Then $S=V(G)-\left\{v_{1}, v_{2}\right\}$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G) \leq n-$ 2 , which is a contradiction.

Case (iv): All the vertices of $C$ are cut vertices of $G$. Then $S=V(G)-\left\{v_{1}, v_{2}, v_{3}\right\}$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G) \leq n-3$, which is a contradiction.

Remark 2.14. There is no connected graph of order $n \geq 3$ with $\gamma_{d c t}(G)=n-1$.
Theorem 2.15. Let $G$ be a connected graph of order $n \geq 4$. Then $\gamma_{d c t}(G)=n-2$
if and only if $G$ is either double star or $K_{1, n-1}+\mathrm{e}$ or $C_{4}$ or $K_{4}$ or $K_{4}-\{e\}$.
Proof: Let $\gamma_{d c t}(G)=n-2$. If $n=3$, then $G$ is either $C_{3}$ or $K_{1,2}$. If $G=K 3$, then $\gamma_{d c t}(G)=2=n-1$, which is a contradiction. If $G=K_{1,2}$, then $\gamma_{d c t}(G)=3=n$, which is a contradiction.

If $n=4$, then $G$ is either $P_{4}$ or $K_{1,3}$ or $K_{1,3}+e$ or $K_{4}$ or $K_{4}-\{e\}$. If $G=P_{4}$, then $\gamma_{d c t}(G)=2=n-2$, which satisfies the requirement of this theorem. If $G=K_{1,3}$ then $\gamma_{d c t}(G)$ $=\mathrm{n}$, which is a contradiction. If $G=C_{4}$, then $\gamma_{d c t}(G)=2=n-2$, which satisfies the requirement of this theorem. If $G=K_{4}$, then $\gamma_{d c t}(G)=n-2=2$, which satisfies the requirement of this theorem. If $G=K_{4}-\{e\}$, then $\gamma_{d c t}(G)=n-2$, which satisfies the requirement of this theorem. So let $n \geq 5$. If $G$ is a tree, then $G$ is a double star which satisfies the
requirement of this theorem. So assume that $G$ is not a tree. Then $G$ contains a cycle. If $G=$ $C_{n}(n \geq 5)$ then by Theorem 2.11, $\gamma_{d c t}(G) \leq n-3$, which is a contradiction. Therefore $G \neq$ $C_{n}(n \geq 5)$. By Theorem 2.11, $C(G) \leq 3$. Let $C: v_{1}, v_{2}, v_{3}, v_{1}$ be a longest cycle in $G$. If only one vertex of $C$, say $v_{1}$ has degree more than three and $\operatorname{deg}\left(v_{2}\right)=\operatorname{deg}\left(v_{3}\right)=2$, then $G=$ $K_{1, n-1}+e$, which satisfies the requirement of this theorem. If at least two vertices having degree more than three, then $\gamma_{d c t}(G) \leq n-3$, which is a contradiction. The converse is clear.

Theorem 2.16. For every pair $a$ and $n$ of positive integers with $2 \leq a \leq n$ and $a \neq n-1$, there exists a connected graph $G$ of order $n$ such that $\gamma_{d c t}(G)=a$.

Proof: Let $a=n$. Let $G=K_{1, n-1}$, then by Theorem 2.13, $\gamma_{d c t}(G)=n$. So, let $2 \leq a \leq n-$ 2. Let $H$ be a complete graph with $a-2$ vertices with $V(H)=\left\{v_{1}, v_{2}, \ldots\right.$,
$\left.v_{n-2}\right\}$. Let $G$ be a graph obtained from $H$ by adding new vertices $x, z_{1}, z_{2}, \ldots, z_{a-1}$ by joining $x$ with $v_{1}$ and $v_{n-2}$ and joining $v_{n-2}$ with each $z_{i}(1 \leq i \leq a-1)$. The graph $G$ is shown in Figure 2.3.

We prove that $\gamma_{d c t}(G)=a$. Let $Z=\left\{z_{1}, z_{2}, \ldots, z_{a-1}\right\}$ be the set of end vertices of $G$. By Theorem 2.3, $Z$ is a subset of every detour cototal dominating set of $G$. Since $x$, $v_{1}, v_{2}, \ldots, v_{n-2} \notin I_{D}[Z], \mathrm{Z}$ is not a detour cototal dominating set of $G$ and so $\gamma_{d c t}(G) \geq a$. Let $Z_{1}=Z \cup\{x\}$. Then $Z_{1}$ is a detour set of $G$ as well as a cototal dominating set of $G$ so that $\gamma_{d c t}(G)=a$.


Figure 2.3
Theorem 2.17. For every pair $a$ and $b$ of positive integers with $2 \leq a \leq b$, there
exists a connected graph $G$ such that $d n(G)=a$ and $\gamma_{d c t}(G)=b$.
Proof: For $a=b$, let $G$ is a double star with a end vertices. Then by Theorem 1.3 and 2.6, $d n(G)$ $=\gamma_{d c t}(G)=a$. For $b=a+1$, Let $G=K_{1, a}$. Then by Theorem 1.2 and 2.13, $d n(G)=$ $\gamma_{d c t}(G)=a+1=b$. So let $2<a<b$ and $b \neq a+1$.

Let $P_{i}: x_{i}, y_{i}, z_{i}(1 \leq i \leq b-a+1)$ be a path on three vertices. Let $G$ be the graph obtained from $P_{i}(1 \leq i \leq b-a+1)$ by adding new vertices $x, y, h_{1}, h_{2}, \ldots, h_{a-1}$ and introducing the edges $x x_{i}(1 \leq i \leq b-a+1)$ and $y z_{i}(1 \leq i \leq b-a+$ 1). The graph $G$ is shown in Figure 2.4.

First we prove that $d n(G)=a$. Let $Z=\left\{h_{1}, h_{2}, \ldots, h_{a-1}\right\}$ be the set of detour vertices of $G$. By Theorem $1.1, Z$ is a subset of every detour set of $G$ and so $d n(G) \geq a-1$. Since $x_{i}, y_{i}, z_{i} \notin I_{D}[Z]$ for all $i(1 \leq i \leq a-1)$. $Z$ is not a detour set of $G$ and so $d n(G) \geq$ $a$. Let $Z_{1}=Z \cup\{x\}$. Then $Z_{1}$ is a detour set of $G$ so that $d n(G)=a$.

Next we prove that $\gamma_{d c t}(G)=b . H_{i}=\left\{x_{i}, y_{i}, z_{i}\right\}(1 \leq i \leq b-a+1)$. By Theorem 2.3, $Z$ is a subset of every detour cototal dominating set of $G$ and also every detour cototal dominating of $G$ contains at least one vertex from each $H_{i}(1 \leq i \leq b-a+1)$ and so $\gamma_{d c t}(G) \geq b-a+1+a-1=b . \operatorname{Let} S=Z \cup\left\{y_{1}, y_{2}, \ldots\right.$,
$\left.y_{b-a+1}\right\}$. Then $S$ is a detour cototal dominating set of $G$ so that $\gamma_{d c t}(G)=b$.


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# ISOLATE DOUBLE GEODETIC NUMBER OF A GRAPH 

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#### Abstract

Let $G$ be a connected graph with atleast two vertices. A set $S$ of vertices of $G$ is called a double geodetic set of $G$ if for each pair of vertices $x, y$ in $G$ there exist vertices $u, v$ in $S$ such that $x, y \in I[u, v]$. The double geodetic number $d g(G)$ of $G$ is the minimum cardinality of a double geodetic set. A double geodetic set $S$ of $G$ such that the subgraph $<\mathrm{S}>$ induced by $S$ has atleast one isolated vertex is called an isolate double geodetic set of $G$. An isolate double geodetic number $d g_{0}(G)$ is the minimum cardinality of an isolate double geodetic set in $G$.In this paper we obtain the isolate double geodetic number of some graphs.


Key words : double geodetic set, double geodetic number, isolate double geodetic number

## Introduction

By a graph $G=(V, E)$, we mean a finite undirected connected graph without loops or multiple edges. As usual $n=|V|$ and $m=|E|$ denote the number of vertices and edges of a graph $G$ respectively. The distance $d(x, y)$ is the length of a shortest $x-y$ path in $G$. It is known that the distance is a metric on the vertex set of $G$. An $x-y$ path of length $d(x, y)$ is called an $x-$ $y$ geodesic. For any vertex $u$ of $G$, the eccentricity of $u$ is $e(u)=\max \{d(u, v): v \in V\}$. A vertex $v$ is an eccentric vertex of $u$ if $e(u)=d(u, v)$. The neighborhood of a vertex $v$ is the set $N(v)$ consisting of all vertices $u$ which are adjacent with $v$. A vertex $v$ is an extreme vertex of $G$ if the subgraph induced by its neighbors is complete. The set of all extreme vertices is denoted by $\operatorname{Ext}(G)$.

The diameter is defined by $\operatorname{diam}(G)=\max \{e(v): v \in V\}$. Two vertices $u$ and $v$ are said to be antipodal vertices if $d(u, v)=\operatorname{diam}(G)$. If $e=\{u, v\}$ is an edge of a graph $G$ with
$\operatorname{deg}(u)=1$ and $\operatorname{deg}(v)>1$, then we call $e$ a pendant edge, $u$ a pendant vertex and $v$ a support vertex.

The closed interval $I[x, y]$ consists of all vertices lying on some $x-y$ geodesic of $G$, while for $S \subseteq V,[S]=\bigcup_{x, y \in S} I[x, y]$. A set $S$ of vertices is a geodetic set if $I[S]=V$ and the minimum cardinality of a geodetic set is the geodetic number $G$. A geodetic set $S$ of $G$ such that the subgraph $<\mathrm{S}>$ induced by $S$ has atleast one isolated vertex is called an isolate geodetic set of $G$. An isolate geodetic number is the minimum cardinality of an isolate geodetic set in $G$. Let $G$ be a connected graph with atleast two vertices. A set $S$ of vertices of $G$ is called a double geodetic set of $G$ if for each pair of vertices $x, y$ in $G$ there exist vertices $u, v$ in $S$ such that $\quad x, y \in I[u, v]$. The double geodetic number $d g(G)$ of $G$ is the minimum cardinality of a double geodetic set. .

## 1. Preliminary Results

Theorem 1.1 Each extreme vertex of a connected graph $G$ belongs to every double geodetic set of $G$. In particular, if the set of all end vertices of $G$ is a double geodetic set, then it is the unique double geodetic set of $G$.

Proposition 1.2 For a non trivial connected graph $G, g(G)=2$ if and only if $d g(G)=2$.
Definition 1.3 The helm graph is the graph obtained from an wheel graph by adjoining a pendent edge at each node of the cycle.

Definition 1.4 The graph corresponding to the skeleton of an $n$ - antiprism is called the $n$-antiprism graph and it is denoted by $Q_{n}$.

Definition 1.5 A prism graph is a graph that has one of the prisms as its skeleton.
Definition 1.6 The triangular snake $T_{n}$ is obtained from the path $P_{n}$ by replacing each edge of the path by a triangle $C_{3}$

## 2. Isolate Double Geodetic Number of a graph

## Definition 2.1

Let $G$ be a connected graph with atleast three vertices. An isolate double geodetic set of a graph $G$ is a double geodetic set $S$ such that the subgraph $<S>$ induced by $S$ has atleast one isolate vertex. The minimum cardinality of an isolate double geodetic set of $G$ is an isolate double geodetic number and is denoted by $d g_{0}(G)$.

## Example 2.2



For the graph $G$ in figure 2.1 , it is easily seen that $S=\left\{v_{1}, v_{2}, v_{7}, v_{8}\right\}$ is the unique minimum double geodetic set of $G$. In this set does not have any isolate vertices .So it is not the isolate double geodetic set of $G$.Then we include one vertex $v_{4}$ or $v_{5}$ in $S$.Now, the sets $S_{1}=$ $\left\{v_{1}, v_{2}, v_{4}, v_{7}, v_{8}\right\}$ and $S_{2}=\left\{v_{1}, v_{2}, v_{5}, v_{7}, v_{8}\right\}$ are the minimum isolate double geodetic set of $G$. Therefore, $d g_{0}(G)=5$. Also, the double geodetic number and isolate double geodetic number can be different.

## Theorem 2.3

For the non trivial connected graph $G$, then $2 \leq d g(G) \leq d g_{0}(G) \leq n-1$.

## Proof

Any double geodetic set needs atleast two vertices, $d g(G) \geq 2$ and every isolate double geodetic set of $G$ is double geodetic set of $G$. ie) $d g(G) \leq d g_{0}(G)$. We know that for the connected graph , $2 \leq d g_{0}(G) \leq n-1$. Hence $d g_{0}(G) \leq n-1$.

## Proposition 2.4

Let $G$ be a connected graph of $n \geq 3$, then $d g(G)=2$ iff $d g_{0}(G)=2$..

## Proof

First we assume $\operatorname{dg}(G)=2$. Let $S=\{u, v\}$ be the double geodetic set of $G$ and $n \geq 3$. Clearly it is non adjacent vertices. Then the set $S$ has two isolated vertices. Hence $d g_{0}(G)=2$. Conversely, assume $d g(G)=2$, clearly $d g_{0}(G)=2$ then $d g(G)=2$.
Theorem 2.5
For the helm graph $H_{n}, n \geq 3$ then $d g_{0}\left(H_{n}\right)= \begin{cases}4 & \text { if } n=3 \\ n & \text { if } n \geq 4\end{cases}$
Proof
Case (i) For $n=3$

For the helm graph $H_{3}$ contains three end vertices is denoted by $\left\{v_{1}, v_{4}, v_{6}\right\}$. By a theorem 1.1, $S=\left\{v_{1}, v_{4}, v_{6}\right\}$. It is not a double geodetic set. Since any pair of vertices $\left\{v_{i}, v_{7}\right\}$ where $1 \leq$ $i \leq 6$ does not belongs to any geodesic of $S$. So we choose a vertex $v_{7}$ such that $S=$ $\left\{v_{1}, v_{4}, v_{6}, v_{7}\right\}$ is a minimum double geodetic set and $\langle S\rangle$ has isolated vertices. Hence $S$ is a minimum isolate double geodetic set and $d g_{0}\left(H_{3}\right)=4$.


Case (ii) For $n \geq 4$
Let $n \geq 4$ the helm graph $H_{n}, \operatorname{diam}\left(H_{n}\right)=n$ and $H_{n}$ contains $n$ end vertices. By a theorem 1.1, the double geodetic set $S$ contains the $n$ end vertices.and every pair of vertices in $H_{n}$ lies between the geodesic in $S$.Also, it is not adjacent to each other. Therefore $S$ is a minimum isolate double geodetic set and hence $d g_{0}\left(H_{n}\right)=n$.

Theorem 2.6
For the $n$ - antiprism graph $Q_{n}=2 n, n$ is odd, $n \geq 3$, then $d g_{0}\left(Q_{n}\right)=2$.

## Proof

Let $n$-antiprism graph of order $2 n, n$ is odd. Let $u, v$ be two vertices of $Q_{n}$. Clearly $\operatorname{diam}\left(Q_{n}\right)=d(u, v)$. Clearly $u$ and $v$ are the antipodal vertices and it is easily seen that $S=\{u, v\}$ be the geodetic set of $Q_{n}$. ie) $g\left(Q_{n}\right)=2$. By a Proposition 1.2, $d g\left(Q_{n}\right)=2$. By a Proposition 2.4, $d g_{0}\left(Q_{n}\right)=2$.

## Remark

Some of the graphs does not have the isolate double geodetic set. For example, $n$ antiprism graph , $n$ is even, then it is does not have the isolate double geodetic set.

## Theorem 2.7

For the $n$-prism graph $Y_{n}, n \geq 3, n$ is even then $d g_{0}\left(Y_{n}\right)=2$.
Proof
This is follows from the theorem 2.6.

## Theorem 2.8

For the triangular snake graph $T_{n}, n \geq 4$, then $d g_{0}\left(T_{n}\right)=n+1$.

## Proof

Let $\left\{u_{1}, u_{2}, \ldots . . u_{n}, v_{1}, v_{2}, v_{n-1}\right\}$ be the vertices of $T_{n}$. Here, $\left\{u_{1}, u_{2}, \ldots . u_{n}\right\} \in P_{n}$. And $v_{1}, v_{2}, \ldots \ldots . v_{n-1}$ be the vertices joined to $u_{i}$ and $u_{i+1}$ for $1 \leq i \leq n-1 . u_{1}$ and $u_{n}$ be the end vertices of $P_{n}$. By a theorem 1.1, the double geodetic set $S$ contains $u_{1}, u_{n}$ also any pair of vertices $\left(v_{i}, v_{i+1}\right), 1 \leq i \leq n-1$ does not exist in the $u_{1}-u_{n}$ geodesic. Therefore , $\quad S^{\prime}=$ $\left\{u_{1}, u_{n}, v_{1}, v_{2}, \ldots ., v_{n-1}\right\}$ is the double geodetic set and $\left\langle S^{\prime}>\right.$ has isolated vertices. Now, $| S^{\prime} \mid=$ $n+1$.

## Theorem

For any integers $a, b$ with $2 \leq a \leq b$. there is a connected graph $G$ with $g_{0}(G)=a$ and $g_{0}(G)=b$.

## Proof

Case (i) $a=2$
This is follows from Theorem 2.6.
Case (ii) $a>2$
This is follows from Theorem 2.5.
Case (iii) $a=b$
Consider the graph $C_{4}$, the isolate geodetic number of $C_{4}$ is 2 . ie) $g_{0}\left(C_{4}\right)=2$.and the isolate double geodetic number is also 2. Therefore, $g_{0}\left(C_{4}\right)=d g_{0}\left(C_{4}\right)$
Case (iv) $a<b$

Let $G$ be the graph in figure 2.3 formed from the path $P_{5}: p_{1}, p_{2}, p_{3}, p_{4}, p_{5}$ of order 5 . By adding ' $b-2$ ' new vertices $u_{1}, u_{2}, x_{1}, x_{2}, \ldots \ldots, x_{a-3}, y_{1}, y_{2}, \ldots ., y_{b-a-1}$ to $P_{5}$ and joining the vertex $u_{1}$ to $p_{1}$ and $u_{2}$ to $p_{2}$ and joining each vertex $y_{i}(1 \leq i \leq b-a-1)$ to $u_{1}$ and $u_{2}$. Also joining each vertex $x_{i}(1 \leq i \leq a-3)$ to $p_{4}$ and $p_{5}$. Let $S=\left\{u_{1}, u_{2}, p_{5}, x_{1}, x_{2}, \ldots, x_{a-3}\right\}$ be the set of vertices, all the vertices of $G$ lies between the geodesic of $S$. Thus $S$ is the geodetic set of $G$.Also the set s has
isolated vertices. Therefore, $g_{0}(G)=a-3+3=a$. But the set $S$ is not a double geodetic set. So we add the vertices $p_{1}$ and $y_{1}, y_{2}, \ldots . . y_{b-a-1}$ in the double geodetic set. Now, $S^{\prime}=S \cup$ $\left\{p_{1}, y_{1}, y_{2}, \ldots ., y_{b-a-1}\right\}$ is the minimum double geodetic set of $G$. In this set does not have isolated vertices. So it is not a isolate double geodetic set. Now, choose the vertex $p 3$ in $S^{\prime}$. Hence

$$
d g_{0}(G)=b-a-1+4+a-3=b
$$


figure 2.3|

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# STANDARDIZING THE INFLUX OF VECHICLES BY USING THE METHOD OF FUZZY TRANSPORTATION PROBLEMS 

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#### Abstract

This article gives a brief study on "STANDARDIZING THE INFLUXOFVECHICLES BY USING THE METHOD OF FUZZY TRANSPORTATION PROBLEMS using the method of Fuzzy Assignment problem to find out which districts in Tamil Nadu faces the majority of accidents because of not wearing helmets along with the period of occurrence.


## Introduction

The pace of society has necessitated the increased use of motor vehicles. The regulation of movement of motor vehicles is necessary for an orderly conduct of business in day to day life. Road accident is most unwanted thing to happen to a road user, though they happen quite often. Most of the road users are quite well aware of the general rules and safety measures while using roads but it is only the laxity on part of road users, which cause accidents and crashes. Main cause of accidents and crashes are due to human errors. The Motor Vehicles (Amendment) Bill, 2016 has been listed for consideration and passage in the current Budget Session of Parliament. It seeks to address issues related to road accidents, third party insurance and road safety

The assignment problem is to resolve the problem of assigning a number of origins to the equal number of destinations at a minimum cost or maximum profit. To find solutions to assignment problems, various algorithms such as linear programming [1-4], Hungarian algorithm [5], neural network [6], genetic algorithm [7] have been developed. In recent years, fuzzy transportation and fuzzy assignment problems have received much concentration.
. Majority of the accidents that happens all over the country is mainly due to this recklessness. Though many factors can be attributed for accidents, the major reason is not wearing helmet while driving. Based on this, in this chapter it is aimed to find out which district in Tamil Nadu faces the most number of accidents because of not wearing helmets. This is achieved with the aid of Fuzzy assignment problem in fuzzy logic.

### 1.2 PRELIMINARIES

In this section some basic definitions and fuzzy arithmetic operations are defined:

## Definition 1.2.1 [1]

The characteristic function $\mu_{\mathrm{A}}$ of a crisp set $\mathrm{A} \subseteq \mathrm{X}$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the Universal set X fall within a specified range i.e. $\mu_{\tilde{A}}: \mathrm{X} \rightarrow[0,1]$. The assigned value indicates the membership grade of the element in the set A.

The function $\mu_{\tilde{A}}$ is called membership function and the set $\tilde{A}=\left\{\left(\mathrm{x}, \mu_{\tilde{A}}(\mathrm{x})\right) ; \mathrm{x} \in \mathrm{X}\right\}$ defined by $\mu_{\tilde{A}}$ for each $\mathrm{x} \in \mathrm{X}$ is called a fuzzy set.

Definition 1.2.2 [11]
A generalized fuzzy number $\tilde{\mathrm{A}}=(\mathrm{a}, \mathrm{b}, \mathrm{d}$; w) is said to be generalized triangular fuzzy number if its membership function is given by

$$
\mu_{\tilde{A}}(\mathrm{x})=\left\{\begin{array}{c}
\frac{\mathrm{w}(\mathrm{x}-\mathrm{a})}{(\mathrm{b}-\mathrm{a})}, \mathrm{a} \leq \mathrm{x} \leq \mathrm{b} \\
\frac{\mathrm{w}(\mathrm{~d}-\mathrm{x})}{(\mathrm{d}-\mathrm{b})}, \mathrm{b} \leq \mathrm{x} \leq \mathrm{d} \\
0, \text { elsewhere }
\end{array}\right.
$$

## IMPLEMENTATION OF FUZZY ASSIGNMENT PROBLEM TO DETERMINE THE NUMBER OF ACCIDENTS IN DISTRICTS OF TAMIL NADU

In this section, the algorithm for solving a fuzzy assignment problems and the method of solving an assignment problem using generalized fuzzy numbers is given. The ranking of generalized fuzzy numbers using the incenter of the centroids of the generalized trapezoidal fuzzy number is given by $\tilde{A}$ $=(a, b, c, d ; w)$
where $\alpha=\frac{\sqrt{(c-3 b+2 d)^{2}+w^{2}}}{6}, \beta=\frac{\sqrt{(2 c+d-a-2 b)^{2}}}{3}$ and $\quad \gamma=\frac{\sqrt{(3 c-2 a-b)^{2}+w^{2}}}{6}$.
The ranking function of the generalized trapezoidal fuzzy number is given by

$$
\begin{align*}
& R(\tilde{A})=\left(\tilde{x}_{0} \times \tilde{y}_{0}\right)= \\
& \left(\left(\frac{\alpha\left(\frac{a+2 b}{3}\right)+\beta\left(\frac{b+c}{2}\right)+\gamma\left(\frac{2 c+d}{3}\right)}{\alpha+\beta+\gamma}, \frac{\alpha\left(\frac{w}{3}\right)+\beta\left(\frac{w}{2}\right)+\gamma\left(\frac{w}{3}\right)}{\alpha+\beta+\gamma}\right) X\left(\frac{\alpha\left(\frac{w}{3}\right)+\beta\left(\frac{w}{2}\right)+\gamma\left(\frac{w}{3}\right)}{\alpha+\beta+\gamma}\right)\right) \tag{1}
\end{align*}
$$

## FUZZY ASSIGNMENT PROBLEM

Suppose there are n works to be performed and n persons are available for doing the works. Assume that each person can do each work at a time, though with unreliable grade of efficiency. Let $\tilde{c}_{i j}$ be the fuzzy cost if the $\mathrm{i}^{\text {th }}$ person is assigned the $\mathrm{j}^{\text {th }}$ work, the problem is to find a minimum fuzzy cost with fuzzy assignment.

## ALGORITHMS ON FUZZY ASSIGNMENT PROBLEMS []

The algorithm for the decision making problem is as follows:

## Algorithm :

Step 1: First test whether the given fuzzy cost matrix of an fuzzy assignment problem is a balanced one or not. If it is a balanced then go to step 3. If not go to step 2 .

Step 2: Introduce dummy rows or columns with zero fuzzy costs so as to form a balanced one.
Step 3: Find the rank of each cell $\tilde{c}_{i j}$ of the chosen fuzzy cost matrix by using the ranking procedure as mentioned in section 3 and determine the minimum element in each row and its corresponding fuzzy element.

Step 4: For each row in the fuzzy cost matrix of table II, subtract the minimum fuzzy element obtained in step 3 in the row from each fuzzy element in that row to get the reduced fuzzy cost matrix.

Step 5: Find the rank of each cell $\tilde{c}_{i j}$ of the reduced fuzzy cost matrix obtained in step 4 by the minimum element in each column and its corresponding fuzzy element.

Step 6: Similarly for each column, subtract the minimum fuzzy element obtained in step 5 in the column from each fuzzy element of that column to get the first modified fuzzy cost matrix.

Step 7: Find the rank of each cell $\tilde{c}_{i j}$ of the first modified fuzzy cost matrix obtained in step 6 by using the ranking procedure. Thus, the first modified matrix is obtained.

Step 8: If the rank of any cell of the first modified matrix obtained in step 7,
i.e., $\mathrm{R}\left(\tilde{c}_{i j}\right)=0$, then draw the minimum number of horizontal and vertical lines to cover all such type of cells in the resulting matrix. Let the minimum number of lines be N . Now there may arise two cases:

Case (i) if $\mathrm{N}=\mathrm{n}$, the number of rows (columns) of given matrix, then an optimal fuzzy assignment can be made. So make the $\mathrm{R}\left(\tilde{c}_{i j}\right)=0$ assignment to get the required optimal solution.

Case (ii) if $\mathrm{N}<\mathrm{n}$, then determine the minimum element in the matrix and its corresponding fuzzy element which is not covered by the N lines.

Step 9: If it falls under step 8 of case (ii) then, subtract this minimum fuzzy element from all uncovered fuzzy elements and add the same fuzzy element at the intersection of horizontal and vertical lines. Thus, the second modified fuzzy cost matrix is obtained.

Step 10: Find the rank of each cell $\tilde{c}_{i j}$ of the second modified fuzzy cost matrix obtained in step 9 by using the ranking procedure as mentioned in section 3. Thus, the second modified matrix is obtained.

Step 11: Again repeat step 8, step 9 and step 10 until minimum number of lines become equal to the number of rows (columns) of the given matrix i.e., $\mathrm{N}=\mathrm{n}$. Step 12: (to make $\mathrm{R}\left(\tilde{c}_{i j}\right)=0$ assignment). Examine the rows successively until a row-wise exactly single $\mathrm{R}\left(\tilde{c}_{i j}\right)=0$ is found, mark this $\mathrm{R}\left(\tilde{c}_{i j}\right)=0$ by ' $\mathbf{O}$ ' to make the assignment, Then, mark a cross ( ${ }^{*}$ ) over all $\mathrm{R}\left(\tilde{c}_{i j}\right)=0$ if lying in the column of the marked ' $\mathbf{O}$ ' $\left(\mathrm{R}\left(\tilde{c}_{i j}\right)=0\right)$, showing that they cannot be considered for future assignment. Continue in this manner until all the rows have been examined. Repeat the same procedure for columns also.

Step 13: Repeat step 12 successively until one of the following situations arises:
(i) If no unmarked $\left(\mathrm{R}\left(\tilde{c}_{i j}\right)=0\right)$ is left, then the process ends; or
(ii) If there lie more than one of the unmarked $\left(\mathrm{R}\left(\tilde{c}_{i j}\right)=0\right)$ in any column or row, then mark ' $\mathbf{O}$ ' one of the unmarked $\left(\mathrm{R}\left(\tilde{c}_{i j}\right)=0\right)$ arbitrarily and mark a asterisk $\left({ }^{*}\right)$ in the cells of remaining zeros in its rows and columns. Repeat the process until no unmarked $\left(\mathrm{R}\left(\tilde{c}_{i j}\right)=0\right)$ is left in the matrix.

Step 14: Thus exactly one marked ' O ' $\left(\mathrm{R}\left(\tilde{c}_{i j}\right)=0\right)$ in each row and each column of the matrix is obtained. The assignment corresponding to these marked ' $\mathbf{O}$ ' $\left(\mathrm{R}\left(\tilde{c}_{i j}\right)=0\right)$ will give the optimal assignment.

Step 15: Substitute the optimal assignment obtained in step 14 in the original fuzzy cost matrix to get the optimal fuzzy assignment.

## ANALYSIS OF ACCIDENTS IN DISTRICTS OF TAMIL NADU

The web portal "ROAD ACCIDENT ANALYSIS IN TAMIL NADU" launched by the Government of Tamil Nadu, Home (Transport) Department, provides monthly analysis on the number of accidents in Tamil Nadu. The portal gives exclusive data on each and every factor associated with the accidents. Based on the data collected from this website [] about the accidents on the districts of Tamil Nadu from January to August 2019, from the top 5 districts that have faced the major accidents apart from Kancheepuram, Using fuzzy assignment model, the districts that have faced the most accidents in a bimonthly period is found out. Since Kancheepuram stands first in all months, the remaining four districts are here taken into consideration

|  | January \& February | March \& April | May \& June July \& August |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Kancheepuram | 26 | 35 | 30 | 33 |
| Coimbatore | 26 | 20 | 18 | 24 |
| Vellore | 20 | 12 | 29 | 23 |
| Thiruvallur | 20 | 23 | 22 | 18 |
| Salem | 13 | 22 | 18 | 18 |

## COMPUTATION OF THE PROBLEM

To illustrate the proposed algorithms, consider a fuzzy assignment problem with four districts and numbers of accidents. The fuzzy assignment costs in Table II and Table III are costs without restrictions and with restrictions respectively.

Table II: Fuzzy assignment cost

|  | I | II | III | IV |
| ---: | :--- | :--- | :--- | :---: |
| A | $23,24,26,27 ; 0.1)$ | $8,19,20,21 ; 0.1)$ | $16,17,18,20 ; 0.1)$ | $(20,21,24,26 ; 0.1)$ |
| B | $16,18,20,21 ; 0.1)$ | $;, 9,12,14 ; 0.1)$ | $23,25,29,30 ; 0.1)$ | $(20,22,23,24 ; 0.1)$ |
| C | $18,19,20,22 ; 0.2)$ | $20,21,23,24 ; 0.2)$ | $19,20,22,24 ; 0.2)$ | $(15,17,18,19 ; 0.2)$ |
| D | $8,9,13,15 ; 0.2)$ | $9,21,22,23 ; 0.2)$ | $16,17,18,22 ; 0.2)$ | $(15,16,18,20 ; 0.2)$ |

Table III: Fuzzy assignment costs when restrictions are made

|  | I | II | III | IV |
| :---: | :---: | :---: | :---: | :---: |
| A | - | $(18,19,20,21 ; 0.1)$ | $16,17,18,20 ; 0.1)$ | $(20,21,24,26 ; 0.1)$ |
| B | $16,18,20,21 ; 0.1)$ | - | $23,25,29,30 ; 0.1)$ | $(20,22,23,24 ; 0.1)$ |
| C | $18,19,20,22 ; 0.2)$ | $(20,21,23,24 ; 0.2)$ | - | $(15,17,18,19 ; 0.2)$ |
| D | $8,9,13,15 ; 0.2)$ | $(19,21,22,23 ; 0.2)$ | $16,17,18,22 ; 0.2)$ | - |

Step 1: The fuzzy assignment problem shown in table II is a balanced one.
Step 2: Using Step 3 of the proposed algorithm, the rank of each cell $R\left(\widetilde{c_{1 J}}\right)$ is now computed.

$$
\text { Thus, } \quad \alpha=\frac{\sqrt{(26-3(24)+2(27))^{2}+0.1^{2}}}{6}=\frac{8}{6}=1.33, \boldsymbol{\beta}=2.67 \text { and } \quad \gamma=1.33
$$

With these values, we now compute the rank of each cell as follows. Thus.

$$
\begin{aligned}
& R(11)=\left(\left(\frac{1.33\left(\frac{23+2(24)}{3}\right)+2.67\left(\frac{24+26}{2}\right)+1.33\left(\frac{2(26)+27}{3}\right)}{1.33+2.67+1.33}\right) \times\right. \\
& \left.\left(\frac{1.33\left(\frac{0.1}{3}\right)+2.67\left(\frac{0.1}{2}\right)+1.33\left(\frac{0.1}{3}\right)}{1.32 .67+1.33}\right)\right)=1
\end{aligned}
$$

In the same way the ranks of the remaining cells are computed and they are tabulated below:

TABLE IV: Ranks of table II

|  | I | II | III | IV |
| :---: | :---: | :---: | :---: | :---: |
| A | 1 | 0.96 | 0.70 | 0.90 |
| B | 4.75 | 0.42 | 1.08 | 0.90 |
| C | 0.78 | 2.58 | 3.57 | 2.28 |
| D | 1.39 | 5.38 | 4.37 | 1.39 |

Step 3: The minimum fuzzy element obtained in Step 2 in each row and the reduced fuzzy cost matrix is presented in table V .

TABLE V: Reduced Fuzzy Cost Matrix

|  | I | II | III | IV |
| :--- | :--- | :--- | :--- | :--- |
| A | $(3,6,9,7 ; 0.1)$ | $(-2,-1,3,5 ; 0.1)$ | $(-4,-1,2,4 ; 0.1)$ | $(0,1,3,5 ; 0.1)$ |
| B | $(2,6,11,14 ; 0.1)$ | $(-7,-2,2,7 ; 0.1)$ | $(9,13,20,23 ; 0.1)$ | $(6,10,14,17 ; 0.1)$ |
| C | $(-4,-1,2,4 ; 0.2)$ | $(-2,1,4,6 ; 0.2)$ | $(-3,0,3,6 ; 0.2)$ | $(-7,-3,-1,1 ; 0.2)$ |
| D | $(-7,-1,2,7 ; 0.2)$ | $(4,8,13,15 ; 0.2)$ | $(1,4,9,14 ; 0.2)$ | $(0,3,9,12 ; 0.2)$ |

Step 4: Using Step 5 of the proposed algorithm the rank of each cell $\mathrm{R}\left(\tilde{c}_{i j}\right)$ is calculated.
Step 5: Subtract the minimum fuzzy element obtained in Step 2 in each row from every fuzzy element in that row of table II and the reduced fuzzy cost matrix is presented in table V (a).

TABLE V(a): Reduced fuzzy cost matrix

|  | I | II | III | IV |
| :--- | :--- | :--- | :--- | :--- |
| A | $(-1,4,10,11 ; 0.1)$ | $(-6,-3,4,9 ; 0.1)$ | $(-8,-3,2,8 ; 0.1)$ | $(-4,-1,4,9 ; 0.1)$ |
| B | $(-5,4,13,21 ; 0.1)$ | $(-14,-5,5,14 ; 0.1)$ | $(2,11,22,30 ; 0.1)$ | $(-1,8,16,24 ; 0.1)$ |
| C | $(-5,0,5,21 ; 0.2)$ | $(-3,2,7,13 ; 0.2)$ | $(-4,1,6,13 ; 0.2)$ | $(-6,-4,2,8 ; 0.2)$ |
| D | $(-14,-6,5,14 ; 0.2)$ | $(-3,6,14,22 ; 0.2)$ | $(-6,2,10,21 ; 0.2)$ | $(-7,1,10,19 ; 0.2)$ |

Step 5: Subtract the minimum fuzzy element obtained in Step 4 in each column from every fuzzy element in that column of table $V(a)$ and the first modified fuzzy cost matrix is presented in table VI

TABLE VI: First modified fuzzy cost matrix

|  | I | II | III | IV |
| :--- | :--- | :--- | :--- | :--- |
| A | $(-16,-5,6,16 ; 0.1)$ | $(7,2,13,19 ; 0.1)$ | $(-14,-5,7,17 ; 0.1)$ | $(-12,-3,7,17 ; 0.1)$ |
| B | $(0,-9,9,28 ; 0.1)$ | $(-19,-1,18,35 ; 0.1)$ | $(-12,6,27,35 ; 0.1)$ | $(13,3,21,38 ; 0.1)$ |


| C | $(-14,-2,6,14 ; 0.2)$ | $(-13,-2,9,27 ; 0.2)$ | $(-11,0,11,19 ; 0.2)$ | $(-12,-1,10,19 ; 0.2)$ |
| :--- | :--- | :--- | :--- | :--- |
| D | $(-28,-8,9,28 ; 0.2)$ | $(-17,1,20,38 ; 0.2)$ | $(-20,-3,-16,35 ; 0.2)$ | $(-21,-4,16,33 ; 0.2)$ |

Step 6: Using Step 7 of the proposed algorithm in the rank of each cell $\mathrm{R}\left(\tilde{c}_{i j}\right)$ is computed
Step 7: Using step 8 of the proposed algorithm for table VIII, it falls under case (i) i.e., $N=n$, the resulting matrix is given in table IX.

TABLE IX: Resulting Matrix

|  | I | II | III | IV |
| :--- | :--- | :--- | :--- | :--- |
| A | 0.18 | 0.02 | 0 | 0.05 |
| B | 0.36 | 0 | 0.46 | 0.40 |
| C | 0 | 0.38 | 0.04 | 0.07 |
| D | 0.05 | 0.02 | 0.36 | 0 |

Step 8: Using 12 and 14 of the proposed Algorithm. Tables X show the optimal assignment.

|  | I | II | III | IV |
| :--- | :--- | :--- | :---: | :---: |
| A | 0.18 | 0.02 | $\mathbf{0}$ | 0.05 |
| B | 0.36 | $\mathbf{0}$ | 0.46 | 0.40 |
| C | $\mathbf{0}$ | 0.38 | 0.04 | 0.07 |
| D | 0.05 | 0.02 | 0.36 | $\mathbf{0}$ |

TABLE X: optimal assignment
Step 9: The optimal assignment from table X is A-III, B-II, C-I, D-IV.
Step 10: From the original fuzzy assignment cost matrix presented in table II, the optimal fuzzy cost assignment is calculated and presented in table XI.

TABLE XI: Optimal Fuzzy Cost Assignment

| Optimal <br> Assignment | A-III | B-II | C-I | D-IV |
| :---: | :---: | :---: | :---: | :---: |
| Fuzzy cost | $16,17,18,20 ; 0.1$ | $7,9,12,14: 0.1$ | $18,19,20,22 ; 0.2$ | $15,16,18,20 ; 0.2$ |

Step 11: Using step 16 of the proposed algorithm, the minimum total fuzzy cost is (56, $61,68,76: 0.1$ ). Thus from the computation it is clear that Coimbatore has faced major accidents during the months of May and June, Vellore during March and April , Thiruvallur during January and February and finally Salem has faced the majority of accidents during July and August.

## FACTORS ATTRIBUTING TO THESE ACCIDENTS

From the accident details collected for Salem city, 15 major accident road corridors have been identified to have highest rate of occurrence of accidents and severity. Also Traffic has become a predominantly challenging issue of Salem city, due to the construction of fly-over bridges across the city. Apart from this, the accidents in Thiruvallur are associated with reckless driving in the highways as inferred by the police officials. Though several steps are being taken by them to reduce these accidents, it is necessary each individual realizes the need to drive safely. Finally, Coimbatore houses the entry point to the familiar hill station Kodaikanal on one side and also the enroute to Kerala on the other. During summer, these two places houses a lots and lots of visitors, hence the mishaps occurring in the hill stations are pretty much high.

## CONCLUSION

Mathematics has always been a integral part of our lives aiding in finding solutions to problems that are considered to be unpredictable at the start. Implementing fuzzy logic into assignment problem, associates a membership to each factor based upon its frequency, thereby helping us to obtain solution much faster. In this project an attempt has been made to determine the number of accidents in certain districts of Tamil Nadu, based upon their frequencies and the reason behind the occurrence has been analysed and studied.

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# Application of Fuzzy TOPSIS Method to identify the Prominent District in Tamil Nadu in the Cultivation of Sorghum bicolor 

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## 1. Introduction

Sorghum bicolor is one of the five top cereal grains produced worldwid, along with maize, wheat, rice and barely. It is the main cereal food for over 750 million people living in semi-arid tropical regions of Africa, Asia and Latin America. Cholam grains are used by people, who often do not have the means to feed themselves with food sources of energy, rich in protein, vitamins, minerals [3].

In India, cholam ranks third both in area and in production after rice and wheat. Cholam is grown over an area of 7.92 million hectares with a total production of 7.92 million tones with an average productivity of 1020 kg / hectare. Cholam is important source of animal feed[4]. The term cholam includes four groups they are grai cholam (as food and feed), sweet stalk cholam (as forage and for animal feed), Sudan grass (for forage and pasture) and broom corn (for making brooms).

Cholam can also be useful in the production of ethanol and other bio-industrial products such as bio-plastic, especially, in dry areas where other crops cannot be easily grown. Cholam is grown for a variety of uses. It is the staple food crop for millions of peoples and its grain is used as an animal feed in several regions of the country.

The notion of a fuzzy set was initially prposed by Zadeh [6] during 1965 , which is an extension of the basic set theory, that associates each member to a certain degree of membership value in the range $[0,1]$. This study employs Fuzzy Topsis method because of its simplicity and its approach to consider unlimited alternatives and criterion in the decision making process [2]. The prime aim of this article is to identify the city which surpasses the cultivation of Sorghum bicolor in Tamil Nadu amongst the 20 cities taken for consideration in terms of its Area, Yield and Production. The aim is to utilise Fuzzy TOPSIS method has been used to identify which city among the 20 selected cities of Tamil Nadu stands first in the cultivation of Sorghum bicolor.

### 1.1. Preliminaries

In this section, the basic definitions required for the article are studied.

Definition 1.1. [5:339] A fuzzy set $A$ in $X$ is characterized by a membership(Characteristic) function $f_{A}(x)$ which associates each point in $X$ a real number in the interval $[0,1]$, with the value of $f_{A}(x)$ at $x$ representing the "grade of membership" of $x$ in $A$.

## Definition 1.2. [5:81]

A triangular fuzzy number $\tilde{a}$ is defined by a triplet $\tilde{a}=\left(a_{1}, a_{2}, a_{3}\right)$ whose membership function is defined by

$$
\mu_{\tilde{a}}(x)= \begin{cases}0 ; & \text { if } \mathrm{x}<\mathrm{a}_{1} \\ \frac{x-a_{1}}{a_{2}-a_{1}} ; & \text { if } \mathrm{a}_{2} \geq \mathrm{x} \geq \mathrm{a}_{1} \\ \frac{x-a_{2}}{a_{3}-a_{2}} ; & \text { if } \mathrm{a}_{3} \geq \mathrm{x} \geq \mathrm{a}_{2} \\ 0 ; & \text { if } \mathrm{x}>\mathrm{a}_{3}\end{cases}
$$

where $a_{2}$ represents the value for which $\mu_{\tilde{a}_{1}}$ and $a_{1}$ and $a_{3}$ are the most extreme values on the left and on the right of the fuzzy number $\tilde{a}$, respectively with membership $\mu_{\tilde{a}}\left(a_{1}\right)=\mu_{\tilde{a}}\left(a_{3}\right)=0$.

## Definition 1.3. [5:4192]

Triangular fuzzy number $\tilde{a}=\left(a_{1}, a_{2}, a_{3}\right)$ and $\tilde{b}=\left(b_{1}, b_{2}, b_{3}\right)$, then the operation with these fuzzy numbers are defined as follows

$$
\begin{gathered}
\tilde{a}(+) \tilde{b}=\left(a_{1}, a_{2}, a_{3}\right)(+)\left(b_{1}, b_{2}, b_{3}\right)=\left(a_{1}+b_{1}, a_{2}+b_{2}, a_{3}+b_{3}\right) \\
\tilde{a}(-) \tilde{b}=\left(a_{1}, a_{2}, a_{3}\right)(-)\left(b_{1}, b_{2}, b_{3}\right)=\left(a_{1}-b_{1}, a_{2}-b_{2}, a_{3}-b_{3}\right) \\
\tilde{a}(\times) \tilde{b}=\left(a_{1}, a_{2}, a_{3}\right)(\times)\left(b_{1}, b_{2}, b_{3}\right)=\left(a_{1} \cdot b_{1}, a_{2} \cdot b_{2}, a_{3} \cdot b_{3}\right) \\
\tilde{a}(/) \tilde{b}=\left(a_{1}, a_{2}, a_{3}\right)(/)\left(b_{1}, b_{2}, b_{3}\right)=\left(a_{1} / b_{3}, a_{2} / b_{2}, a_{3} / b_{1}\right) \\
k \tilde{a}=k\left(a_{1}, a_{2}, a_{3}\right)\left(k a_{1}, k a_{2}, k a_{3}\right)
\end{gathered}
$$

## Definition 1.4. [11:4192]

Let $\tilde{a}=\left(a_{1}, a_{2}, a_{3}\right)$ and $\tilde{b}=\left(b_{1}, b_{2}, b_{3}\right)$ be two triangular fuzzy numbers. The distance between them is calculated by

$$
d(\tilde{a}, \tilde{b})=\sqrt{\frac{1}{3}\left[\left(a_{1}-b_{1}\right)^{2}+\left(a_{2}-b_{2}\right)^{2}+\left(a_{3}-b_{3}\right)^{2}\right]}
$$

## 2. Multicriteria decision making

Decision Making plays a vital role when there is a large amount of data presented to a user, out of which deriving a result by viewing the data is practically impossible. In this section, Fuzzy TOPSIS method is implemented to find out which city in Tamil Nadu stands first in the cultivation of Sorghum bicolor. The final output of this method gives the ranking of each cities based on the cultivation pattern.

### 2.1. Some Basic Definitions

The basic definitions required are as follows.

Definition 2.1[7:4191] The weight vector $W=\left(w_{1}, w_{2}, \ldots ., w_{n}\right)$ composed of the individual weights $w_{j}(j=1,2 \ldots, n)$ for each criterion $C_{j}$ satisfies $\sum_{j=1}^{n} w_{j}=1$.

Definition 2.2[7:4191] The normalized decision matrix $R=\left[r_{i j}\right]_{m \times n}$ with $i=1,2, \ldots, m$, and $j=$ $1,2, \ldots, n$. The normalized value $r_{i j}$ is calculated by $r_{i j}=\frac{x_{i j}}{\sqrt{\sum_{i=1}^{m} x_{i j}^{2}}}$ with $i=1,2 \ldots, m ; j=1,2, \ldots, n$

Definition 2.3[7:4191] The weighted normalized decision matrix $Q=\left[q_{i j}\right]_{m \times n}$ with $i=$ $1,2 \ldots m, j=1,2, \ldots, n$ is calculated by multiplying the normalized decision matrix by its associated weights. The weighted normalized value $q_{i j}$ is calculated by using the formula $q_{i j}=w_{i} x r_{i j}$ with $i=$ $1,2 \ldots, m, j=1,2, \ldots, n$. Definition $2.4[7: 4191]$ The Euclidean distance from the positive ideal solution $M^{+}$for each alternative $M_{i}$, is calculated by using the formula $d i^{+}=\sqrt{\sum_{i=1}^{n}\left(d i j^{+}\right)^{2}}$ where $d_{i j}^{+}=q_{j}^{+}-q_{i j}$, with $i=1,2, \ldots m$

Definition 2.5[7:4191] The Euclidean distances from the negative ideal solution $M^{-}$for each alternative $M_{i}$, is calculated by using the formula $d_{i}^{-}=\sqrt{\sum_{i=1}^{n}\left(d i j^{-}\right)^{2}}$ where $d_{i j}^{-}=q_{j}^{-}-q_{i j}$, with $i=1,2, \ldots m$

Definition 2.5[7:4192] The relative closeness $\eta_{i}$ for each alternative $M_{i}$, with respect to positive ideal solution is computed using the following formula: $\eta_{i}=\frac{d_{i}^{-}}{d_{i}^{+}+d_{i}^{-}}$.

## 3. Algorithm for the Fuzzy TOPSIS Method Used in Decision Making [7]

In this section, the algorithm applied in fuzzy TOPSIS method is given. The significance of using these fuzzy numbers is that they are represented as triangular fuzzy numbers, which proves very effective when it comes to compute data that are not precise[1]. This method involves a group of decision-makers who participate in the decision making criteria. They are represented by,$G=$ $S_{1}, S_{2}, \ldots, S_{L}$. The weight vector for each member of the group consisting of L members is represented as $W^{l}=w_{1}^{l}, w_{l}^{l}, \ldots, w_{n}^{l}$ with $l=1,2, \ldots, \mathrm{~L}$, where $w_{j}^{l}$ is the weight assigned by the member of the group $S_{l}$ to criteria $C_{j}$ and this weight satisfies teh condition, $0 \leq w_{j}^{l} \leq 1$, and $\sum_{i=1}^{n} w_{j}^{l}=1$.

Initially, a normalized decision matrix $\tilde{Q}=\left[\tilde{q}_{i j}\right]_{m \times n}$ with $i=1,2 \ldots m$, and $j=1,2, \ldots, n$ is constructed for every group member whose respective weight vector is $W^{l}$ for $l=1,2, \ldots, L$. The weighted fuzzy normalized decision matrix for individual members is now given by ${ }^{l} \tilde{q}=\left[{ }^{[ } \tilde{q}_{i j}\right]_{m \times n}$ which is computed as

$$
l \tilde{q}=\left[\begin{array}{ccc}
w_{1}^{l} \tilde{q}_{11} & \cdots & w_{n}^{l} \tilde{q}_{1 n} \\
\vdots & \ddots & \vdots \\
w_{1}^{l} \tilde{q}_{m 1} & \cdots & w_{1}^{l} \tilde{q}_{m n}
\end{array}\right]
$$

with $i=1,2, . . m ; j=1,2 \ldots, n$; and $l=1,2, \ldots, L$.
With this brief description we now present the algorithm for Fuzzy TOPSIS method [7] as follows:

Step 1 Suppose that $J_{1}$ and $J_{2}$ represent the criteria benefit and criteria cost, respectively, then for every group member, the positive ideal solutions, $M^{+}$and the negative ideal solutions $M^{-}$which are the benefits and costs respectively $l=1,2, \ldots, L$ are computed with the aid of: ${ }^{l} M^{+}=$ $\left({ }^{l} \tilde{q}_{1}^{+}, \tilde{q}_{2}^{+}, \ldots,{ }^{l} \tilde{q}_{n}^{+}\right),{ }^{l} M^{-}=\left({ }^{l} \tilde{q}_{1}^{-}, \tilde{q}_{2}^{-}, \ldots,{ }^{l} \tilde{q}_{n}^{-}\right) \quad$ where $\quad{ }^{l} \tilde{q}_{j}^{+}=\left(\max _{i}^{l} \tilde{q}_{i j}, j \in J_{1} ; \min _{i}^{l} \tilde{q}_{i j}, j \in\right.$ $\left.J_{2}\right),{ }^{1} \tilde{\mathrm{q}}_{\mathrm{j}}^{-}=\left(\min _{i}^{l} \tilde{q}_{i j}, j \in J_{1} ; \max _{i}^{l} \tilde{q}_{i j}, j \in J_{2}\right)$

Step 2 The consequent step is to compute the distance of each alternative from the positive and negative ideal solution. Thus, the distance of the alternative $M_{i}$ from the positive ideal solution
represented by ${ }^{l} d_{i}^{+}$for the group member $S_{l}$, is given by ${ }^{l} d_{i}^{+}=\sum_{j=1}^{n} d\left({ }^{l} \tilde{q}_{i j}{ }^{l} \tilde{q}_{j}^{+}\right)$, with $i=$ $1,2, \ldots, m ; l=1,2, \ldots, L$

In a similar way, the distance of alternative $M_{i}$ from the negative ideal solution ${ }^{l} d_{i}^{-}$of the group member $S_{l}$, is given by ${ }^{l} d_{i}^{-}=\sum_{j=1}^{n} d\left({ }^{l} \tilde{q}_{i j}, \tilde{q}_{j}^{-}\right)$, with $i=1,2, \ldots, m ; l=1,2, \ldots, L$. where the distance $d\left({ }^{l} \tilde{q}_{i j},{ }^{l} \tilde{q}_{j}^{+}\right)$and $d\left({ }^{l} \tilde{q}_{i j},{ }^{l} \tilde{q}_{j}^{-}\right)$between two fuzzy numbers is given by the equation $d(\tilde{a}, \tilde{b})=\sqrt{\frac{1}{3}\left[\left(a_{1}-b_{1}\right)^{2}+\left(a_{2}-b_{2}\right)^{2}+\left(a_{3}-b_{3}\right)^{2}\right]}$

Step 3 The relative closeness ${ }^{l} \eta\left(M_{i}\right)$ for each alternative $M_{i}$ of each member in the group is now computed. Thus, the relative closeness with respect to its positive ideal solution is given by ${ }^{l} \eta\left(M_{i}\right)=\frac{d_{i}^{-}}{l d_{i}^{+}+{ }^{l} d_{i}^{-}}$

After calculating the relative closeness value ${ }^{l} \eta_{i}\left(M_{i}\right)$ for each member $l$ the relative-closeness matrix is then framed and it is given by

$$
C=\left[\begin{array}{ccc}
l_{\eta}\left(M_{1}\right) & \ldots & { }^{L} \eta\left(M_{1}\right) \\
\vdots & \ddots & \vdots \\
{ }^{l} \eta\left(M_{m}\right) & \ldots & { }^{l} \eta\left(M_{m}\right)
\end{array}\right]
$$

If the importance weights if each members of the group is introduced into its relative closeness, the resulting matrix is the weighted relative-closeness matrix denoted by $C \beta$ and it is given by,

$$
C \beta=\left[\begin{array}{ccc}
\beta_{1}^{1} \eta\left(M_{1}\right) & \cdots & \beta_{1}^{L} \eta\left(M_{1}\right) \\
\vdots & \ddots & \vdots \\
\beta_{1}^{1} \eta\left(M_{m}\right) & \cdots & \beta_{1}^{L} \eta\left(M_{m}\right)
\end{array}\right]
$$

Step 4 The group positive $M_{G}^{+}$and group negative ideal solution $M_{G}^{-}$, respectively are now identified with the aid of $M_{G}^{+}=\left(Q_{G 1}^{-}, Q_{G 2}^{-}, \ldots, Q_{G L}^{-}\right)=,\left(\max _{i} \beta_{1}^{1} \eta\left(M_{i}\right), \max _{i} \beta_{2}^{2} \eta\left(M_{i}\right), \ldots, \max _{i} \beta_{L}^{L} \eta\left(M_{i}\right)\right)$

$$
M_{G}^{-}=\left(Q_{\bar{G} 1}^{-}, Q_{\bar{G} 2}^{-}, \ldots, Q_{\bar{G} L}^{-}\right)=\left(\min _{i} \beta_{1}^{1} \eta\left(M_{i}\right), \min _{i} \beta_{2}^{2} \eta\left(M_{i}\right), \ldots, \min _{i} \beta_{L}^{L} \eta\left(M_{i}\right)\right)
$$

Step 5 The distances from the group positive $M_{G}^{+}$and the group negative ideal solution $M_{G}^{-}$for each alternative $M_{i}$, is computed using the following formula, where the distance of the group positive
ideal solution $M_{G}^{+}$is $d_{G i}^{+}=\sqrt{\sum_{l=1}^{L}\left(\beta_{l}^{l}\left(\eta\left(M_{i}\right)\right)-q_{G i}^{+}\right)^{2}}$ with $i=1,2, \ldots, m$. and The distances of the group negative ideal solution $M_{G}^{-}$is $d_{G i}^{-}=\sqrt{\sum_{l=1}^{L}\left(\beta_{l}^{l}\left(\eta\left(M_{i}\right)\right)-q_{G i}^{-}\right)^{2}}$ with $i=1,2, \ldots, m$.
Step 6 The penultimate step is the computation of the group relative closeness $\eta_{i}$ with respect to the group psotive ideal solution $M_{G}^{+}$of each alternative $M_{i}$ which is given by $\eta_{G}\left(M_{i}\right)=\frac{d_{G i}^{-}}{d_{G i}^{+}+d_{G i}^{-}}$

Step 7 Now the best alternatives are the ones with the highest value of $\eta G\left(M_{i}\right)$ as they are nearer to the positive ideal solution and they are ranked accordingly.

### 3.1. Determining the Surpassing cities in Sorghum bicolor Cultivation using Fuzzy TOPSIS method

In this section, initially collected data about various parameters such as Total Area, Production and Yield of the crop at each district in Tamilnadu is tabulated and then the Fuzzy TOPSIS method is used to find the best city in Sorghum bicolor cultivation.

Sorghum bicolor is the crop for grain for human and animal consumption. Sorghum bicolor is produced in areas that are too hot, a minimum average temperature of $25 \cdot C$ is necessary to grain production. Sorghum bicolor is the fifth most cereal crop in the world after rice, wheat, corn and barley. There are 20 cities which cultivates Sorghum bicolor. The aim of this study is to compare various parameters such as the total area on which the crop is cultivated, the production of the crop at each cities and the yield of the crop at each cities during the last five years and to find out which city is most suitable for the cultivation of Sorghum bicolor.

The following table gives a detailed account of cultivation of Sorghum bicolor in various cities [8] of Tamilnadu in terms of its Area, Production and Yield.

| Area production and yield in Tamiladu State |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2015-2016 |  |  | 2016-2017 |  |  | 2017-2018 |  |  | 2018-2019 |  |  | 2019-2020 |  |  |
|  | Area | Production | Yield | Area | Production | Yield | Area | Production | Yield | Area | Production | Yield | Area | Production | Yield |
| Coimbatore | 30562 | 20146 | 659 | 23114 | 18714 | 810 | 22088 | 12134 | 549 | 25001 | 23105 | 924 | 28.157 | 37454 | 0 |
| Thiruppur | 19972 | 4791 | 240 | 11971 | 8389 | 701 | 3666 | 311 | 221 | 332203 | 13301 | 401 | 36974 | 11147 | 0 |
| Dindugul | 28679 | 46017 | 1605 | 29156 | 50907 | 1746 | 226003 | 27509 | 1217 | 33498 | 85437 | 2550 | 57504 | 166362 | 0 |
| Theni | 10092 | 31617 | 3133 | 9677 | 32659 | 3375 | 6401 | 18849 | 2945 | 7744 | 19468 | 2514 | 9875 | 36899 | 0 |
| Madurai | 9846 | 12052 | 1224 | 9554 | 15651 | 1638 | 6305 | 6707 | 1064 | 10490 | 20040 | 1910 | 15523 | 39497 | 0 |
| Ramauadhapuram | 1771 | 3142 | 194 | 1734 | 1634 | 942 | 2385 | 1104 | 463 | 3237 | 6842 | 2114 | 4019 | 7939 | 0 |
| Thirmelveli | 1860) | 1604 | 862 | 2295 | 2610 | 1137 | 8557 | 1932 | 755 | 1715 | 428.4 | 2498 | 2156 | 10098 | 0 |
| Virudhunagar | 10465 | 13959 | 133.1 | 96562 | 11315 | 1172 | 10735 | 10239 | 954 | 14798 | 36842 | 260 | 16446 | 63313 | 0 |
| Thuthukudi | 8959 | 18871 | 2106 | 8231 | 17833 | 2167 | 8831 | 5370 | 608 | 15886 | 21144 | 1331 | 18107 | 62685 | 0 |
| Karur | 20581 | 7246 | 352 | 16059 | 14482 | 927 | 7773 | 7672 | 987 | 21225 | 26595 | 1253 | 25985 | 47497 | 0 |
| Vellore. | 6888. | 4028 | 585 | 6107 | 9685 | 1586 | 5545 | 4569 | 824 | 5907 | 5451 | 923 | 7047 | 19612 | 2783 |
| Thinusuamalai | 6.39 | 565 | 88.1 | 590 | 689 | 1168 | 6.18 | 496 | 765 | 789 | 1122 | 1421 | 773 | 1622 | 0 |
| Villupuram | 1743 | 1553 | 891 | 71 | 75 | 1053 | 540 | 428 | 793 | 318 | 513 | 1474 | 824 | 1711 | 2076 |
| Dharmapuri | 15696 | 12545 | 799 | 9200 | 12461 | 1354 | 10187 | 9563 | 939 | 17776 | 19025 | 1070 | 17365 | 45461 | 2617 |
| Krishagiri | 5116 | 14703 | 2874 | 4205 | 10715 | 2548 | 3651 | 2241 | 614 | 4980 | 7136 | 1433 | 415 | 9194 | 2082 |
| Salem | 20702 | 19459 | 9410 | 17397 | 17105 | 983 | 19108 | 16405 | 859 | 31653 | 31948 | 1009 | 47452 | 85010 | 1791 |
| Namakkal | 19800 | 13817 | 698 | 9917 | 8476 | 855 | 5438 I | 35587 | 654 | 88750 | 16421 | 1853 | 85992 | 160096 | 186: |
| Perambalur | 3195 | 4296 | 1344 | 1363 | 1628 | 1194 | 1230 | 917 | 745 | 3637 | 6391 | 1757 | 2165 | 3666 | 0 |
| Ariyalur | 35211 | 2514 | 714 | 1674 | 2136 | 1278 | 1028 | 1168 | 1136 | 872 | 1140 | 1306 | 775 | 2003 | 0 |
| Thiruchirapalli | 22795 | 13258 | 582 | 25056 | 14121 | 564 | 20761 | 10910 | 525 | 24608 | 17616 | 416 | 32295 | 55740 | 0 |
| 3 | ' 1 | - 1 |  | 1 | 1. $\cdot$ | ${ }^{\circ}$ | * | $\cdots \cdots$ |  | - | 1. 1 | 1 | - | 1 1 | . |

Now, the total area under Cultivation for the last five years is added and it will be denoted as the cost criteria, where as the criterion benefit is obtained by adding production value and Yield value of the past five years. This is given in the following table.

| Alternatives $\left(M_{i}\right)$ | Area | Production + yield $\left(C_{k}\right)$ |
| :---: | :---: | :---: |
| Coimbatore | 12922 | 14495 |
| Thiruppur | 105786 | 39502 |
| Dindugul | 17144 | 383350 |
| Theni | 43789 | 151459 |
| Madurai | 51718 | 99783 |
| Ramanathapuram | 13146 | 26424 |
| Thirunelveli | 16583 | 25780 |
| Virudhunagar | 62096 | 136263 |
| Pudukottai | 1987 | 6462 |
| Karur | 91623 | 107411 |


| Vellore | 31494 | 50046 |
| :---: | :---: | :---: |
| Thiruvanamalai | 3439 | 8732 |
| Villupuram | 3526 | 6715 |
| Dharmapuri | 70224 | 105834 |
| Krishnagiri | 22367 | 53540 |
| Salem | 136312 | 175509 |
| Namakkal | 258843 | 386643 |
| Perambalur | 1159 | 17382 |
| Ariyalur | 7869 | 13395 |
| Thiruchirapalli | 125515 | 13252 |
| $\boldsymbol{x}_{\boldsymbol{i j}}$ | 13679201338 | 1544922715 |
| $\sqrt{\sum_{\boldsymbol{i}=\mathbf{1}}^{m} \boldsymbol{x}_{\boldsymbol{i j}} \mathbf{2}}$ | 369854.0433 | 399305.50489 |

where $x_{i j}$ is indicates the rating of the alternative $M_{i j}$.

With the aid of this table the Normalized fuzzy decision matrix is computed by using

$$
r_{i j}=\frac{x_{i j}}{\sqrt{\sum_{i=1}^{m} x_{i j}^{2}}}
$$

where $r_{i j}$ is the $(i, j)^{t h}$ entry in Normalized fuzzy decision matrix with $i=1,2 \ldots m ; j=1,2, \ldots, n$. The Normalized values for each Alternative with respect to its $\operatorname{cost}\left(C_{1}\right)$ and benefit $\left(C_{2}\right)$ are computed as follows. Thus, The normalized value for the Alternative $M_{1}$, whose cost $\left(x_{11}\right)$ is 12922 and benefit $\left(x_{12}\right)$ is 14495 , is obtained by

$$
r_{11}=\frac{x_{11}}{\sqrt{\sum_{i=1}^{m} x_{i j}^{2}}}=\frac{12922}{369854.0433}=0.3439
$$

In the similar way other entries are calculated and they are tabulated in the following table.

| Alternatives | Value of $r_{i j}$ with respect to $\left(C_{1}\right)$ | Value of $r_{i j}$ with respect to $\left(C_{2}\right)$ |
| :---: | :---: | :---: |
| $M_{1}$ | 0.3439 | 0.0363 |
| $M_{2}$ | 0.2860 | 0.0989 |
| $M_{3}$ | 0.0463 | 0.9600 |
| $M_{4}$ | 0.1189 | 0.3793 |
| $M_{5}$ | 0.1398 | 0.2499 |
| $M_{6}$ | 0.0355 | 0.0662 |
| $M_{7}$ | 0.0448 | 0.0646 |
| $M_{8}$ | 0.1679 | 0.3412 |
| $M_{9}$ | 0.0054 | 0.0162 |
| $M_{10}$ | 0.2477 | 0.2690 |
| $M_{11}$ | 0.0852 | 0.1253 |
| $M_{12}$ | 0.0093 | 0.0219 |
| $M_{13}$ | 0.0095 | 0.0168 |
| $M_{14}$ | 0.1899 | 0.2650 |
| $M_{15}$ | 0.0605 | 0.1341 |
| $M_{16}$ | 0.3686 | 0.4395 |
| $M_{17}$ | 0.6999 | 0.9683 |
| $M_{18}$ | 0.031 | 0.0435 |
| $M_{19}$ | 0.0213 | 0.0335 |
| $M_{20}$ | 0.3313 | 0.0332 |
|  |  |  |

The values in Table 3 are now converted as Fuzzy Triangular Numbers and the values are listed below

Table 3: Normalized values as fuzzy triangular numbers

| Alternatives | Value of $r_{i j}$ with respect to $\left(C_{1}\right)$ | Value of $r_{i j}$ with respect to $\left(C_{2}\right)$ |
| :---: | :---: | :---: |
| $M_{1}$ | $(0.3430,0.3439,0.3447)$ | $(0.0350,0.0363,0.0378)$ |
| $M_{2}$ | $(0.2849,0.2860,0.2872)$ | $(0.0976,0.0989,0.1011)$ |
| $M_{3}$ | $(0.0454,0.0463,0.0476)$ | $(0.9652,0.9600,0.9664)$ |


| $M_{4}$ | (0.1180,0.1189,0.1196) | (0.3781,0.3793,0.3806) |
| :---: | :---: | :---: |
| $M_{5}$ | (0.1386,0.1398,0.1408) | (0.2438,0.2499,0.2512) |
| $M_{6}$ | (0.0343,0.0355,0.0364) | (0.0654,0.0662,0.0679) |
| $M_{7}$ | (0.0436,0.0448,0.0457) | (0.0637,0.0646,0.0659) |
| $M_{8}$ | (0.1660,0.1679,0.1688) | (0.3404, $0.3412,0.34225)$ |
| $M_{9}$ | (0.0048,0.0054,0.0065) | (0.0151,0.0162,0.0173) |
| $M_{10}$ | (0.2466,0.2477,0.2484) | (0.2682,0.2690,0.2703) |
| $M_{11}$ | (0.0846,0.0852,0.0863) | (0.1246,0.1253,0.1264) |
| $M_{12}$ | (0.0084,0.0093,0.0106) | (0.0208,0.0219,0.0228) |
| $M_{13}$ | (0.0083,0.0095,0.0108) | (0.0156,0.0168,0.0179) |
| $M_{14}$ | (0.1891,0.1899,0.1996) | (0.2641, $0.2650,0.2663)$ |
| $M_{15}$ | (0.0596,0.0605,0.0615) | (0.1332,0.1341,0.1353) |
| $M_{16}$ | (0.3678,0.3686,0.3695) | (0.4382, $0.4395,0.4408)$ |
| $M_{17}$ | (0.6991,0.6999,0.7010) | (0.9672,0.9683,0.9697) |
| $M_{18}$ | (0.0302,0.031,0.0322) | (0.0424,0.0435,0.0444) |
| $M_{19}$ | (0.0203,0.0213,0.0221) | (0.0327,0.0335,0.0348) |
| $M_{20}$ | (0.3304,0.3313,0.3321) | (0.0324,0.0332,0.347) |

In the process of decision making for cultivation of Sorghum bicolor, the criteria of decision makers are not given the same importance. Therefore, a weighted vector $W$ is introduced to denote the importance of weight for that criterion regarding the opinion of the decision maker. For this three levels of importance are assigned for each criterion: They are Very important, moderate and unimportant. They express their preferences according to Table 4.

Table 4: Opinions of the decision makers in form of weights

| Decision makers(DM) | Criteria | $\left(C_{2}\right)$ |
| :--- | :--- | :--- |
| $D M 1$ (Rainfall) | Moderate | Moderate |
| $D M 2$ ( Soil ) | Important | Very Important |
| $D M 3$ (Temperature) | Very important | Important |

The weights assigned for the tables very important, moderate and unimportant are 0.95 , $0.5,0.05$, respectively. So the weight vector for DM1 (Rainfall) is $W^{1}=(0.5,0.5)$ for DM2 (Soil) is $W^{2}=(0.95,0.5)$ for DM3 (Temperature) is $W^{3}=(0.05,0.95)$. Similarly, an importance weight vector is assigned to each decision maker $\beta=\left(\beta_{1}, \beta_{2}, \beta_{3}\right)$.

Now the Normalized weight with respect to its cost $\left(C_{1}\right)$ and benefit $\left(C_{2}\right)$ is obtained by using the weight $W^{1}=(0.5,0.5)$. Thus, the normalized weighted value $q_{i j}$ for the alternatives $M_{i}$, is calculated as $q_{i j}=w_{i} x r_{i j}$ with $i=1,2, \ldots m$, and $j=1,2, \ldots n$. This is computed as follows The normalized weight for cost $\left(C_{1}\right)$, is given by

$$
W^{1} C_{1}=(0.3439 \times 0.5,0.2860 \times 0.5,0.0463 \times 0.5=0.1720,0.1430,0.0232)
$$

In the similar way other entries are calculated.. Next the positive ideal solution $M^{+}$and negative ideal solutions $M^{-}$are identified as given in the algorithm and the positive ideal solution for cost $\left(C_{1}\right)$, and benefit $\left(C_{2}\right)$ are $(0.0027,0.4842)$, the negative ideal solution for cost $\left(C_{1}\right)$, and benefit $\left(C_{2}\right)$ are $(0.3500,0.0081)$. Thus, the Euclidean distance for positive ideal solution ${ }^{1} d_{i}^{+}$of the alternative $M_{1}$ is,

$$
\sqrt{(0.1720-0.0027)^{2}+(0.0182-0.4842)^{2}}=0.2459
$$

The Euclidean distance for negative ideal solution ${ }^{1} d_{i}^{-}$of the alternative $M_{1}$ is given by,

$$
\sqrt{(0.1720-0.3500)^{2}+(0.0182-0.0081)^{2}}=0.0317
$$

The relative closeness ${ }^{1} \eta_{i}$ for the alternative $M_{1}$ is given by

$$
\frac{0.0317}{0.2459+0.0317}=\frac{0.0317}{0.2776}=0.1142
$$

Similarly the other values are calculated and they are tabulated below:
Table 5 Normalized weight of each alternative for $W^{1}=(0.5,0.5)$

| Alternatives | $W^{1} C_{1}$ | $W^{1} C_{2}$ | ${ }^{1} d_{i}^{+}$ | ${ }^{1} d_{i}^{-}$ | ${ }^{1} \eta_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{1}$ | $(0.1711,0.1720,0.1730)$ | $(0.0175,0.0182,0.0196)$ | 0.2459 | 0.0317 | 0.1142 |
| $M_{2}$ | $(0.1422,0.1430,0.1441)$ | $(0.0487,0.0495,0.0507)$ | 0.2087 | 0.0445 | 0.1758 |
| $M_{3}$ | $(0.0225,0.0232,0.0242)$ | $(0.4791,0.4800,0.4811)$ | 0.0004 | 0.3295 | 0.9988 |
| $M_{4}$ | $(0.0587,0.0595,0.0615)$ | $(0.1891,0.1897,0.1920)$ | 0.0899 | 0.1173 | 0.5661 |
| $M_{5}$ | $(0.0687,0.0699,0.0712)$ | $(0.1240,0.1250,0.1263)$ | 0.1335 | 0.0921 | 0.4082 |
| $M_{6}$ | $(0.0169,0.0178,0.0186)$ | $(0.0319,0.0331,0.0342)$ | 0.2037 | 0.1109 | 0.3525 |
| $M_{7}$ | $(0.0215,0.0224,0.2238)$ | $(0.0311,0.0323,0.0334)$ | 0.2081 | 0.1079 | 0.3415 |
| $M_{8}$ | $(0.0838,0.0840,0.0855)$ | $(0.1695,0.1706,0.1718)$ | 0.1049 | 0.0971 | 0.4806 |
| $M_{9}$ | $(0.0018,0.0027,0.0038)$ | $(0.0075,0.0081,0.0092)$ | 0.2267 | 0.1206 | 0.3473 |
| $M_{10}$ | $(0.1221,0.1239,0.1244)$ | $(0.1337,0.1345,0.1353)$ | 0.1370 | 0.0671 | 0.3288 |
| $M_{11}$ | $(0.0411,0.0426,0.0440)$ | $(0.0620,0.0627,0.0638)$ | 0.1793 | 0.0974 | 0.3520 |
| $M_{12}$ | $(0.0038,0.0047,0.0053)$ | $(0.0100,0.0109,0.0117)$ | 0.2240 | 0.1192 | 0.3473 |
| $M_{13}$ | $(0.0039,0.0048,0.0056)$ | $(0.0076,0.0084,0.0095)$ | 0.2264 | 0.1192 | 0.3449 |
| $M_{14}$ | $(0.0942,0.0950,0.0964)$ | $(0.1318,0.1325,0.1332)$ | 0.1322 | 0.1123 | 0.4593 |
| $M_{15}$ | $(0.0301,0.0303,0.0311)$ | $(0.0664,0.0671,0.0683)$ | 0.1816 | 0.1056 | 0.3677 |
| $M_{16}$ | $(0.1830,0.1843,0.1850)$ | $(0.2191,0.2198,0.2210)$ | 0.1028 | 0.0723 | 0.4129 |
| $M_{17}$ | $(0.3490,0.3500,0.3509)$ | $(0.4836,0.4842,0.4853)$ | 0.1206 | 0.2267 | 0.6527 |
| $M_{18}$ | $(0.0148,0.0155,0.0167)$ | $(0.0210,0.0218,0.0227)$ | 0.2154 | 0.1121 | 0.3423 |
| $M_{19}$ | $(0.0101,0.0107,0.0112)$ | $(0.0156,0.0168,0.0179)$ | 0.2185 | 0.1152 | 0.3452 |
| $M_{20}$ | $(0.1650,0.1657,0.1668)$ | $(0.0154,0.0166,0.0178)$ | 0.2452 | 0.0340 | 0.4126 |
| $M^{+}$ | $(0.0027$ | 0.4842 |  |  |  |
| $M^{-}$ | 0.3500 | 0.0081 |  |  |  |

In the similar way, the nomalized weights with respect to the cost and benefit using the weight $W^{2}=(0.95,0.05)$ are now computed

Table 7 Normalized weight of each alternative $W^{2}=(0.95,0.05)$

| Alternatives | $W^{2} C_{1}$ | $W^{2} C_{2}$ | ${ }^{2} d_{i}^{+}$ | ${ }^{2} d_{i}^{-}$ | ${ }^{2} \eta_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{1}$ | $(0.3254,0.3267,0.3278)$ | $(0.0011,0.0018,0.0031)$ | 0.1055 | 0.1143 | 0.5200 |
| $M_{2}$ | $(0.2704,0.2717,0.2726)$ | $(0.0038,0.0050,0.0064)$ | 0.0730 | 0.1546 | 0.6793 |
| $M_{3}$ | $(0.0429,0.0440,0.0452)$ | $(0.0469,0.0480,0.0493)$ | 0.0015 | 0.3877 | 0.9961 |
| $M_{4}$ | $(0.1121,0.1130,0.1143)$ | $(0.0179,0.0190,0.0204)$ | 0.0125 | 0.3049 | 0.9603 |
| $M_{5}$ | $(0.1319,0.1328,0.1340)$ | $(0.0112,0.0125,0.0136)$ | 0.0176 | 0.2833 | 0.9415 |
| $M_{6}$ | $(0.0324,0.0337,0.0349)$ | $(0.0022,0.0033,0.0046)$ | 0.0028 | 0.3984 | 0.9937 |
| $M_{7}$ | $(0.0415,0.0426,0.0437)$ | $(0.0026,0.0032,0.0043)$ | 0.0034 | 0.3873 | 0.9912 |
| $M_{8}$ | $(0.1586,0.1595,0.1609)$ | $(0.0162,0.0171,0.0186)$ | 0.0247 | 0.2557 | 0.9119 |
| $M_{9}$ | $(0.0039,0.0051,0.0062)$ | $(0.0002,0.0008,0.0016)$ | 0.0023 | 0.4353 | 0.9947 |
| $M_{10}$ | $(0.2341,0.2353,0.2368)$ | $(0.0124,0.0135,0.0144)$ | 0.0542 | 0.1847 | 0.7731 |
| $M_{11}$ | $(0.0800,0.0809,0.0822)$ | $(0.0057,0.0063,0.0075)$ | 0.0075 | 0.3411 | 0.9785 |
| $M_{12}$ | $(0.0074,0.0088,0.0099)$ | $(0.0009,0.0011,0.0023)$ | 0.0022 | 0.4305 | 0.9949 |
| $M_{13}$ | $(0.0079,0.0090,0.0102)$ | $(0.0001,0.0008,0.0021)$ | 0.0023 | 0.4302 | 0.9946 |
| $M_{14}$ | $(0.1792,0.1804,0.1817)$ | $(0.0123,0.0133,0.0146)$ | 0.0319 | 0.2349 | 0.8804 |
| $M_{15}$ | $(0.0564,0.0575,0.0587)$ | $(0.0055,0.0067,0.0076)$ | 0.0044 | 0.3690 | 0.9882 |
| $M_{16}$ | $(0.3491,0.3502,0.3516)$ | $(0.0212,0.0220,0.0234)$ | 0.1199 | 0.0994 | 0.4533 |
| $M_{17}$ | $(0.6638,0.6649,0.6660)$ | $(0.0472,0.0484,0.0502)$ | 0.4354 | 0.0023 | 0.0053 |
| $M_{18}$ | $(0.0283,0.0295,0.0307)$ | $(0.0011,0.0022,0.0035)$ | 0.0027 | 0.4037 | 0.9934 |
| $M_{19}$ | $(0.0191,0.0202,0.0215)$ | $(0.0010,0.0017,0.0029)$ | 0.0024 | 0.4156 | 0.9943 |
| $M_{20}$ | $(0.1330,0.1347,0.1358)$ | $(0.0011,0.0017,0.0029)$ | 0.0190 | 0.2811 | 0.9367 |
| $M^{+}$ | 0.0051 | 0.0484 |  |  |  |
| $M^{-}$ | 0.6649 | 0.0008 |  |  |  |

Table 8 Normalized weight of each alternative for $W^{3}=(0.05,0.95)$

| Alternatives | $W^{3} C_{1}$ | $W^{3} C_{2}$ | ${ }^{3} d_{i}^{+}$ | ${ }^{3} d_{i}^{-}$ | ${ }^{3} \eta_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{1}$ | $(0.0160,0.0172,0.0188)$ | $(0.0332,0.0345,0.0357)$ | 0.7842 | 0.0000 | 0.0000 |
| $M_{2}$ | $(0.0133,0.0143,0.0152)$ | $(0.0932,0.0940,0.0954)$ | 0.6823 | 0.0004 | 0.0006 |
| $M_{3}$ | $(0.0016,0.0023,0.0034)$ | $(0.9111,0.9120,0.9134)$ | 0.0001 | 0.7754 | 0.9999 |
| $M_{4}$ | $(0.0047,0.0059,0.0068)$ | $(0.3593,0.3603,0.3614)$ | 0.3132 | 0.1083 | 0.2569 |
| $M_{5}$ | $(0.0061,0.0070,0.0083)$ | $(0.2363,0.2374,0.2387)$ | 0.4658 | 0.0424 | 0.0834 |
| $M_{6}$ | $(0.0010,0.0018,0.0028)$ | $(0.0614,0.0629,0.0642)$ | 0.7344 | 0.0012 | 0.0016 |
| $M_{7}$ | $(0.0011,0.0022,0.0036)$ | $(0.0602,0.0614,0.0627)$ | 0.7370 | 0.0012 | 0.0016 |
| $M_{8}$ | $(0.0072,0.0084,0.0098)$ | $(0.3205,0.3214,0.3226)$ | 0.3583 | 0.0841 | 0.1901 |
| $M_{9}$ | $(0.0001,0.0003,0.0014)$ | $(0.0143,0.0154,0.0167)$ | 0.8181 | 0.0006 | 0.0007 |
| $M_{10}$ | $(0.0113,0.0124,0.0237)$ | $(0.2543,0.2556,0.2569)$ | 0.4414 | 0.0502 | 0.1021 |
| $M_{11}$ | $(0.0034,0.0043,0.0055)$ | $(0.1179,0.1190,0.1203)$ | 0.6414 | 0.0079 | 0.0122 |
| $M_{12}$ | $(0.0039,0.0047,0.0058)$ | $(0.0200,0.0208,0.0223)$ | 0.8084 | 0.0003 | 0.0004 |
| $M_{13}$ | $(0.0001,0.0005,0.0015)$ | $(0.0151,0.0160,0.0177)$ | 0.8170 | 0.0006 | 0.0007 |
| $M_{14}$ | $(0.0082,0.0095,0.0104)$ | $(0.2504,0.2518,0.2526)$ | 0.4465 | 0.0486 | 0.0982 |
| $M_{15}$ | $(0.0021,0.0030,0.0044)$ | $(0.1263,0.1274,0.1289)$ | 0.6281 | 0.0094 | 0.0147 |
| $M_{15}$ | $(0.0174,0.0184,0.0195)$ | $(0.4166,0.4175,0.4189)$ | 0.2527 | 0.1489 | 0.0000 |
| $M_{17}$ | $(0.0339,0.0350,0.0364)$ | $(0.9183,0.9199,0.9211)$ | 0.0012 | 0.7895 | 0.9984 |
| $M_{18}$ | $(0.0009,0.0016,0.0025)$ | $(0.0404,0.0413,0.0425)$ | 0.7719 | 0.0004 | 0.0005 |
| $M_{19}$ | $(0.0004,0.0011,0.0023)$ | $(0.0302,0.0318,0.0327)$ | 0.7887 | 0.0030 | 0.0037 |
| $M_{20}$ | $(0.0154,0.0166,0.0178)$ | $(0.0305,0.0315,0.0327)$ | 0.7896 | 0.0000 | 0.0000 |
| $M^{+}$ | 0.0003 | 0.9199 |  |  |  |
| $M^{-}$ | 0.0184 | 0.0315 |  |  |  |
|  |  |  |  |  |  |

In this table we combine the relative closeness values $\eta_{G i}$ from the above three tables.

Table 9 Relative closeness value for each alternative

| Alternatives | ${ }^{17} 7$ | $2_{7 i}$ | $3_{7 / i}$ |
| :---: | :---: | :---: | :---: |
| $M_{1}$ | 0.1142 | 0.5200 | 0 |
| $M 2$ | 0.1758 | 0.6793 | 0.0006 |
| $\Lambda_{3}$ | 0.9998 | 0.9961 | 0.9999 |
| $M_{4}$ | 0.5661 | 0.9603 | 0.2569 |
| $M_{5}$ | 0.4082 | 0.9415 | 0.0834 |
| $M_{6}$ | 0.3525 | 0.9937 | 0.0016 |
| $M_{7}$ | 0.3415 | 0.9912 | 0.0016 |
| $\mathrm{Mr}_{8}$ | 0.4806 | 0.9119 | 0.1901 |
| $M_{9}$ | 0.3473 | 0.9947 | 0.0007 |
| $M_{10}$ | 0.3288 | 0.7731 | 0.1021 |
| M11 | 0.3520 | 0.9785 | 0.0122 |
| $M_{12}$ | 0.3473 | 0.9949 | 0.0004 |
| $M_{13}$ | 0.3449 | 0.9946 | 0.0002 |
| $M_{11}$ | 0.4593 | 0.8804 | 0.0982 |
| M15 | 0.3677 | 0.9882 | 0.0147 |
| M16 | 0.4129 | 0.4552 | 0.0000 |
| M17 | 0.6527 | 0.0053 | 0.9984 |
| $M_{18}$ | 0.3423 | 0.9934 | 0.0005 |
| $\cdots{ }_{19}$ | 0.3452 | 0.9943 | 0.0037 |
| $M_{20}$ | 0.4126 | 0.9367 | 0.0000 |

In the following table the important weight vector with weight $\beta=(0.33,0.33,0.33)$ to the relative closeness matrix in the group positive ideal solutions and group negative ideal solutions are computed as follows.

The relative closeness value $\beta_{1}^{1} \eta\left(M_{i}\right)$ for positive ideal solution are given by $(0.3296,0.3287,0.3300)$ The relative closeness value $\beta_{1}^{1} \eta\left(M_{i}\right)$ for negative ideal solution are given by $(0.0377,0.0017,0)$.

The important weight vector $\beta$ for alternative $M_{1}$ is given by

$$
(0.1142 \times 0.33,0.5200 \times 0.33,0)=(0.0377,0.1716,0.0000)
$$

And also each alternative $M_{i}$ the distances from the group positive ideal solution $M_{G}^{+}$and from the group negative ideal solution $M_{G}^{-}$, respectively and they are tabulated

Table 10 Ranking of alternatives

| Alternatives | $\begin{gathered} \beta^{1} \eta\left(M_{i}\right) \\ 1 \end{gathered}$ | $\begin{gathered} \beta^{2} \eta\left(M_{i}\right) \\ 2 \end{gathered}$ | $\begin{gathered} \beta^{3} \eta\left(M_{i}\right) \\ 3 \end{gathered}$ | $\begin{aligned} & d^{+} \\ & G i \end{aligned}$ | ${ }^{d_{G i}}$ | $\eta G i$ | Ranking |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{1}$ | 0.0377 | 0.1716 | 0.0000 | 0.1099 | 0.0289 | 0.2082 | 18 |
| $M_{2}$ | 0.0580 | 0.2242 | 0.0002 | 0.1934 | 0.0499 | 0.2050 | 19 |
| $M_{3}$ | 0.3296 | 0.3287 | 0.3300 | 0.0000 | 0.5228 | 1.0000 | 1 |
| $M_{4}$ | 0.1868 | 0.3169 | 0.0848 | 0.0806 | 0.1287 | 0.6149 | 3 |
| $M_{5}$ | 0.1347 | 0.3107 | 0.0275 | 0.1298 | 0.1057 | 0.4488 | 7 |
| $M_{6}$ | 0.1163 | 0.3279 | 0.0005 | 0.1541 | 0.1126 | 0.4221 | 8 |
| $M_{7}$ | 0.1127 | 0.3271 | 0.0005 | 0.1555 | 0.1115 | 0.4176 | 15 |
| $M_{8}$ | 0.1586 | 0.3009 | 0.0627 | 0.1013 | 0.1080 | 0.5160 | 5 |
| M9 | 0.1146 | 0.3283 | 0.0002 | 0.1550 | 0.1126 | 0.4208 | 9 |
| M10 | 0.1085 | 0.2551 | 0.0336 | 0.1475 | 0.0743 | 0.3350 | 16 |
| M11 | 0.1162 | 0.3229 | 0.0040 | 0.1519 | 0.1093 | 0.4187 | 13 |
| M12 | 0.1146 | 0.3283 | 0.0001 | 0.1550 | 0.1126 | 0.4200 | 11 |
| M13 | 0.1138 | 0.3282 | 0.0002 | 0.1553 | 0.1124 | 0.4199 | 12 |
| $M_{14}$ | 0.1516 | 0.2905 | 0.0324 | 0.1258 | 0.2976 | 0.7091 | 2 |
| M15 | 0.1213 | 0.3261 | 0.0048 | 0.1058 | 0.1122 | 0.5147 | 6 |
| M16 | 0.1363 | 0.1502 | 0.0000 | 0.0693 | 0.0097 | 0.0138 | 20 |
| M17 | 0.2154 | 0.0017 | 0.3295 | 0.1199 | 0.1402 | 0.5390 | 4 |
| M18 | 0.1130 | 0.3278 | 0.0002 | 0.1556 | 0.1120 | 0.4185 | 14 |
| M19 | 0.1139 | 0.3281 | 0.0012 | 0.1546 | 0.1123 | 0.4208 | 9 |
| M20 | 0.1362 | 0.3091 | 0.0000 | 0.0378 | 0.1042 | 0.2161 | 17 |
| $\mathrm{M}^{+}$ | 0.3296 | 0.3287 | 0.3300 |  |  |  |  |
| $\mathrm{M}^{-}$ | 0.0377 | 0.0017 | 0 |  |  |  |  |

From the above table, it is clear that, the Alternative $\mathrm{M}_{3}$, which corresponds to Dindugal stands first in Sorghum bicolor cultivation.

## 4. Conclusion

Every choice put forth in front of a person comprises of both positive and negative parameters imbibed in it. In some cases the positive parameter may be higher, thus leading to the ignorance of the negativity in it and vice versa. Hence it is mandatory, that a proper decision is made in such circumstances so that the best parameter is always chosen. Decision making process plays a vital role on these cases, thus presenting the user with the best option.

## 5. References

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