A National Conference on Pure and Discrete Mathematics: Exploring the Technological era's Applications in Science and Engineering

PDMTSE-2k25

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Since 1878

From Darkness Lead Us unto Light

ABOUT ST.JOHN'S COLLEGE,PALAYAMKOTTAI

Founded in 1878 by the Church Missionary Society of England

St. John's College has been a beacon of academic excellence and value-based education for over 147 years. The institution is an Aided First Grade Postgraduate and Research Institution affiliated with Manonmaniam Sundaranar University, Tirunelveli. It is dedicated to providing quality education to students from all sections of society, fostering intellectual growth, ethical values, and holistic development.

St. John's College takes pride in its team of highly qualified and experienced faculty, who are committed to nurturing students academically and professionally. The college offers undergraduate, postgraduate, and research programs across various disciplines, ensuring a strong foundation in academics and research.

The campus is well-equipped to support students' learning experiences. It features spacious classrooms, well-equipped laboratories, a comprehensive library with an extensive collection of books and digital resources, and a computer center with high-speed internet access. Additionally, the vast and well-maintained campus provides an ideal environment for both academic and extracurricular activities.

The college actively promotes co-curricular and extracurricular activities

The college fosters leadership, teamwork, and social responsibility through NSS, NCC, YRC, and the Placement Cell. Guided by the motto "From Darkness Lead Us unto Light," it provides a disciplined yet empowering environment for holistic growth. With nearly 147 years of excellence, St. John's College remains committed to academic rigor, character building, and innovation.

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ABOUT THE DEPARTMENT OF MATHEMATICS

The Department of Mathematics at St. John's College has been an integral part of the institution since its inception, evolving from modest beginnings to meet diverse academic and intellectual needs. Initially, Mathematics was offered as part of English Language Papers, but due to rising student demand, it was introduced as a major subject in 1945. The department soon gained recognition for producing university rank holders under Madras University and later Madurai Kamaraj University.

In 1967, the Department of Mathematics at St. John's College introduced its M.Sc. program, becoming the first in the southern districts to do so under Madurai Kamaraj University. This milestone provided students with advanced knowledge and research opportunities in mathematics. Renowned professors like Dr. Burn and Dr. S. Arumugam played a key role in shaping the department's legacy, fostering a strong academic environment. Their mentorship led to the success of many distinguished alumni, including IAS officers, educators, and administrators, who have made significant contributions in their respective fields.

The M.Phil. program, introduced in 2007, has achieved remarkable success, with students earning gold medals and university ranks. Upgraded to a Research Department in 2017, it now specializes in combinatorics and optimization, including Graph Theory, Topology, Operations Research, and Fuzzy Set Theory. Several scholars continue its tradition of academic and research excellence. Today, the department has over 160 students and 10 faculty members, six of whom hold Ph.D. degrees.

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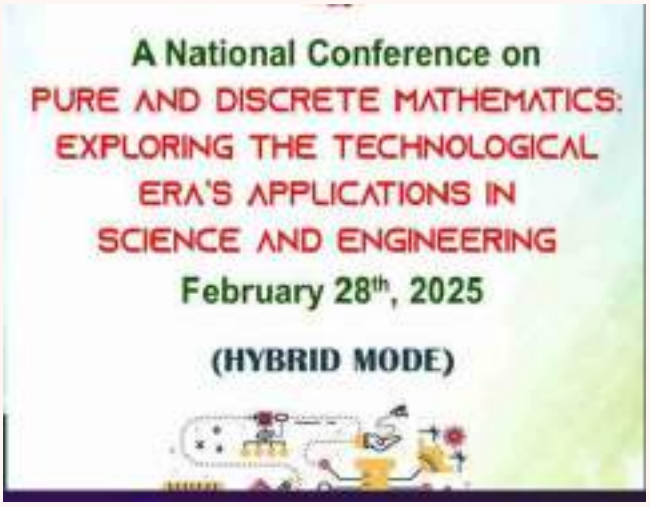
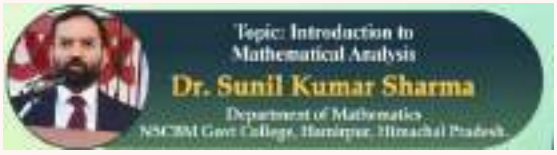
ABOUT THE PDMTSE-2K25 CONFERENCE



The National Conference on Pure and Discrete Mathematics brings together researchers, academicians, and industry experts to discuss advancements and applications in pure and discrete mathematics.The event bridges theory and practice, highlighting discrete mathematics' role in Computer Science, AI, Network Theory, Cryptography, and Optimization. Participants will share research, engage in discussions, and collaborate across disciplines.

We are grateful to all the resource persons Dr. R. Thangadurai, Dr. K. Thiyagu, and Dr. Sunil Kumar Sharma for delivering lectures at the conference. We also extend our gratitude to each and every author for their contributions in the form of articles in these proceedings and for their cooperation in successfully bringing out this book.



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MESSAGE FROM THE SECRETARY

I am delighted to know that the PG and Research Department of Mathematics

St. John's College, Palayamkottai, is organizing "A National Conference on Pure and Discrete Mathematics: Exploring the Technological Era's Applications in Science and Engineering (PDMTSE-2K25)" on 28th February 2025.

Mathematics, in its pure and discrete forms, continues to be the foundation of numerous scientific and technological innovations. Its impact extends across diverse fields, including cryptography, computational algorithms, artificial intelligence, and data science. As we navigate an era of rapid scientific and technological transformation, the synergy between pure and discrete mathematics is proving to be increasingly crucial in solving complex real-world problems. From securing digital communications to optimizing machine learning algorithms, the contributions of mathematics remain indispensable in shaping the future of science and engineering.

I extend my deepest gratitude to the esteemed speakers for sharing their expertise, the participants for their engagement and scholarly contributions, and the organizing team for their dedication in bringing this conference to fruition. I firmly believe that PDMTSE-2K25 will serve as a catalyst for meaningful discussions, collaborative research, and groundbreaking discoveries that will drive future advancements in mathematics and its applications in technology.

Best wishes for the grand success of PDMTSE-2K25.

Mr. G. JEYACHANDRAN
SECRETARY

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MESSAGE
FROM THE FORMER VICE-CHANCELLOR

A successful education is one which leads to a successful life. Mathematical education is a particularly important compound of such an education. This is for two reasons. On the one hand mathematics is one of the worthwhile human activities like art, literature, and music. On the other hand, mathematics is a key element in science and technology, vital to the understanding, control and development of resources of the world around. These two aspects of Mathematics, referred to as pure mathematics and applied Mathematics, should be present in a well- balanced Mathematics education.

Research in mathematics education has undergone a major transformation since the 1990s. Breaking the disciplinary boundaries and paradigmatic straightjackets, it has embarked on many new directions. The computer is enabling mathematicians to create new areas of studies in mathematics.Keeping all the above aspects in mind, the Department of Mathematics of St. John's College, my alma mater, is holding a one-day seminar inviting scholars across the country from diverse specialized areas- from Graph Theory to Statistics, from Data analysis to Cyber Security, from Functional Analysis to Artificial Intelligence. I wish the event a great success.

PRO.K.A. MANIKUMAR, PH.D.
FORMER VICE-CHANCELLOR
SWAMI VIVEKANAND UNIVERSITY,
SAGAR (MP)

Since 1878

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MESSAGE FROM THE PRINCIPAL

Warm greetings to one and all.
I am delighted that the PG and Research Department of Mathematics at St. John's College, Palayamkottai, is hosting the National Conference on Pure and Discrete Mathematics: Exploring the Technological Era's Applications in Science and Engineering. PDMTSE-2K25 This initiative is truly commendable, as mathematics forms the foundation of everything around us.

Mathematics is integral to every aspect of life—it is the language of the universe, the structure behind nature, and the driving force of technological progress. As Albert Einstein famously stated, "As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality." This profound statement highlights the intricate balance between mathematical certainty and real-world applicability.With its vast range of applications, mathematics plays a crucial role in fields such as science, technology, economics, and engineering, shaping the future of innovation and addressing real-world challenges. The interdisciplinary nature of mathematics continues to unlock new frontiers, making it indispensable in the modern era.

On behalf of the organizing committee, I extend a heartfelt welcome to our esteemed speakers and respected attendees. We are honored to have such a distinguished gathering of experts and scholars. Your participation and insights will undoubtedly enrich this conference, fostering meaningful discussions and knowledge exchange.

Thank you for being here and for sharing your expertise.
Warmest regards for the success of PDMTSE-2K25.

G. Andrews.

Dr.G. ANDREWS
PRINCIPAL

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MESSAGE FROM THE CHAIRPERSON

It is my pleasure to welcome you to "A National Conference on Pure and Discrete Mathematics: Exploring the Technological Era's Applications in Science and Engineering (PDMTSE-2K25)," scheduled for February 28th, 2025. This conference highlights the commitment of our Department of Mathematics to promoting research, innovation, and collaboration in this ever-evolving field.

As science and technology advance at an unprecedented pace, mathematical sciences continue to play a crucial role. Pure mathematics provides a strong theoretical foundation for exploring scientific truths, while discrete mathematics offers essential tools for solving real-world problems, particularly in today's digital landscape. PDMTSE-2K25 aims to facilitate insightful discussions and the exchange of innovative ideas that will drive progress in mathematics and its applications.

On behalf of the PG and Research Department of Mathematics, John's College, Palayamkottai,

I encourage all participants to actively engage in the sessions, share their expertise, and build meaningful connections that will pave the way for future research and development.

Wishing everyone a successful and enriching conference experience.

Dr. J. VIJAYA XAVIER PARTHIPAN
HEAD OF THE DEPARTMENT

Since 1878

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MESSAGE FROM THE CO-CHAIRPERSON

Greetings to all.Mathematics is one of the most beautiful and powerful creations of the human mind. It is the gateway to the sciences, offering us the hope that every problem has a solution. The study of mathematics starts with small details but leads to great discoveries.

At St. John's College, Palayamkottai, the PG and Research Department of Mathematics is organizing "A National Conference on Pure and Discrete Mathematics: Exploring the Technological Era's Applications in Science and Engineering." This initiative is truly exciting, as mathematics plays a vital role in our daily lives—it even teaches us to be mindful of the signs we encounter, both in equations and in life.On behalf of the organizing committee, I am honored to welcome you all to this conference. It is inspiring to see so many brilliant minds gathered here, eager to exchange ideas, engage in discussions, and contribute to the future of mathematics.

We are privileged to have renowned speakers sharing their expertise with us today.

Your insights and contributions are invaluable, and we look forward to thought-provoking presentations that will challenge, inspire, and expand our understanding.
Thank you for being part of this event. Wishing everyone a successful and enriching conference.

S. Vijila

Mrs. VIJILA VELVET DAICY
ASSOCIATE PROFESSOR

Since 1878

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MESSAGE FROM THE CONVENOR

Mathematics is often seen as a subject confined to textbooks, a discipline we either loved or dreaded in school. But in reality, Mathematics is far more than that it is the very language of the universe the silent architect of our reality.

Technology has revolutionized both the practice and education of Mathematics, enhancing computational capabilities, fostering deeper understanding, and introducing innovative branches of study. The smartphones in our hands, the vast expanse of the internet, and the computers we rely on—all are built upon intricate mathematical principles. Fields like engineering, telecommunications, and computer science owe their very existence to the power of Mathematics.

Yet, Mathematics extends beyond technology and scientific discovery. It is a powerful tool for deciphering the world around us. It enables us to quantify, analyze, and predict patterns and trends, from weather forecasting to economic modeling. It helps us make sense of complex systems and empowers us to navigate the intricacies of modern life with informed decision-making.

Mathematics is more than a subject—it is a way of seeing, thinking, and understanding the very fabric of our existence.

Beyond its practical applications, it cultivates critical thinking and problem-solving skills, teaching us to reason logically, identify patterns, and approach challenges systematically.

This publication stands as a testament to the collaborative efforts and intellectual contributions of all participants. I trust that this collection of works will serve as a valuable resource for the Mathematics community and inspire further research and collaboration.

May this conference be a meaningful platform for learning, discovery, and networking.

DR.J.SUBHASHINI
ASSISTANT PROFESSOR

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MESSAGE FROM THE ORGANIZING SECERATARIES

Dr. P. GINO METILDA
ASSISTANT PROFESSOR

Warm greetings to one and all.

On behalf of the organizing committee of Pure and Discrete Mathematics: Exploring the Technological Era's Applications in Science and Engineering (PDMTSE-2K25), we extend a heartfelt welcome to everyone gathered for what promises to be an exciting and intellectually stimulating event.

The book of nature is written in the language of mathematics. Mathematics has long been at the heart of scientific advancement, and through the exchange of ideas, research, and collaboration, we continue to push the boundaries of knowledge. This conference provides a unique platform to explore the latest developments in various fields of mathematics, share insights, and engage in discussions that will inspire future research and discovery.We are honored to have such an esteemed group of speakers and researchers with us. Each of you brings a wealth of knowledge and expertise, and we eagerly anticipate your thought-provoking presentations and discussions. Your contributions not only deepen our understanding of mathematical concepts but also inspire new ways of thinking, leading to breakthroughs and innovative applications of mathematics in the world around us.

To all participants, we encourage you to take full advantage of the sessions, panel discussions, and networking opportunities throughout the conference. The conversations we have here will help shape the future of mathematical research and foster collaborations with lasting impact in both academia and industry. Thank you once again for being here. We wish you a productive, enriching, and inspiring conference.

Dr. R. ARUL ANANTHAN
ASSISTANT PROFESSOR

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MESSAGE FROM THE RESOURCE PERSON

Any professional needs to know the current trends in his/her discipline in Order to cope with the fast-moving scientific advancements. Also, the vital part of Human Resource Development is the `Knowledge Exchange'. This is done by organizing workshops/thematic conferences or meetings to train the young minds. This way, the experts can exchange/decipate their ideas to others.

The development of Mathematics is done through good libraries, and `Knowledge exchange'. In this spirit, the PG and Research Department of Mathematics, St. John's College is organizing a national conference on `Pure and Discrete Mathematics, exploring the technological era's Applications in Science and Engineering'. This way PG and the Research Department of Mathematics at St. John's College create an opportunity for the young minds working/studying there. The theme of the conference is a mixture of Pure and Applied Mathematics. Unless Pure and Applied Mathematics go hand in hand, one cannot invent real-time applications.

R. THANGADURAI, PH.D.
PROFESSOR
HARISH-CHANDRA RESEARCH INSTITUTE,
PRAYAGRAJ, (U.P).

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MESSAGE FROM THE RESOURCE PERSON

Dr. K Thiyagu
Associate Professor,
Department of Education,
Central University of Karnataka
Kadaganchi. Aland Road,
Kalaburagi District - 585 367,
India.

I am honored to be a part of the National Conference on Pure and Discrete Mathematics, organized by the PG & Research Department of Mathematics, St. John's College, Palayamkottai, Tirunelveli. This conference serves as a remarkable platform for academicians, researchers, and professionals to engage in discussions on the profound impact of mathematics in science, engineering, and technology.

In today's technological era, mathematics plays a pivotal role in various fields, providing the foundation for advancements in computing, artificial intelligence, cryptography, data science, and numerous other domains. The theme of this conference is particularly significant, as it highlights the integration of mathematical theories with modern technological applications. The fusion of pure and discrete mathematics with contemporary innovations continues to shape the future of scientific research and industrial development.I extend my sincere congratulations to the entire organizing committee for selecting such a relevant and thought-provoking topic. The meticulous planning and dedication of the team in organizing this conference are truly commendable. By bringing together experts, scholars, and enthusiasts, the conference fosters an environment of knowledge-sharing, collaboration, and intellectual growth.

One of the key aspects of this conference is the exploration of technological tools for teaching and learning mathematics. As education undergoes rapid digital transformation, leveraging technology to enhance mathematical comprehension and application is of paramount importance. Digital platforms, artificial intelligence-based learning tools, and interactive mathematical software are revolutionizing traditional teaching methods, making mathematics more accessible and engaging for learners at all levels. Such discussions and insights gained from this conference will undoubtedly contribute to the evolution of mathematical education and research.

Wishing the National Conference on Pure and Discrete Mathematics a resounding success!

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MESSAGE FROM THE RESOURCE PERSON

It is with great pleasure that I acknowledge the organization of the National Conference on Pure and Discrete Mathematics by the PG & Research Department of Mathematics, St.Jone's College, Palayamkottai. This conference serves as a platform to explore the technological applications of mathematics in science and engineering, thereby highlighting the significance of the subject.

Mathematics forms the foundation of various disciplines, providing a framework for logical thinking, problem-solving, and precision. The impact of mathematics is evident in everyday life, education, industry, and research.I extend my heartfelt congratulations to the entire faculty of the Department of Mathematics and all the participants for their valuable contributions. This compilation of research articles reflects the dedication, innovation, and scholarly excellence of the contributors, enriching the field of mathematics with contemporary insights and applications.I am confident that the contributions will advance knowledge in mathematics. I believe that this kind of collection of papers from various fields of mathematics is valuable and important for teachers and young researchers working in the domain of research and teaching.

I am sure that readers will find the contributions by various authors in the book valuable for advancing knowledge in the field of mathematics.
Best wishes for the entire organizing team.

Sincerely,

Sunil Kumar Sharma.
Department of Mathematics
NSCBM Govt.College
Hamipur
Himachal Pradesh

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CORDIAL AND PRODUCT CORDIAL LABELING OF CYCLE-BASED STRUCTURES

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Abstract

A graph G is said to be *cordial* if there exists a vertex labeling $f: V(G) \to \{0,1\}$ that induces an edge labeling $f^*: E(G) \to \{0,1\}$defined by$f^*(uv) = |f(u) - f(v)|$and satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Here $v_f(x)$ and $e_f(x)$are the number of vertices and edges respectively with label x (x = 0 or 1). A graph G is said to be the *product cordial* if there exists binary vertex labeling $f$defined on the set $V(G)$ and inducing an edge labeling$f^*: E(G) \to \{0,1\}$defined by$f^*(uv) = f(u)f(v)$ and satisfying the same conditions. Herein, we demonstrate that the n-triangular snake, uniform theta and odd-quasi uniform theta graphs admit cordial and Mongolian tent, n-triangular snake and series parallel graphs admit product cordial.

Keywords: Cordial labeling, product cordial labeling, theta graph, series parallel graph, n-triangular snake graph.

AMS Subject Classification: 05C78

SOFT GRAPHS AND LINKED MATRICES: A STRUCTURAL STUDY

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Abstract

Molodtsov introduced the soft set theory as a general mathematical strategy for dealing with uncertain data. Soft set theory is currently being used by many academics to address decision-related problems. A parameterized point of view for graphs is provided using the concept of a soft graph. The soft graph uses parameterization to create several descriptions of a complex relation that is represented by a graph. In this article, we introduce various matrices associated with soft graphs, such as the S-adjacency matrix and S-incidence matrix, and we look at some of their characteristics.

Keywords: S-Adjacency Matrix; S-Incidence Matrix.

Mathematics Subject Classification 2020*:* 05C99

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CONTRIBUTION OF MODIFIED HOMOTOPY ANALYSIS METHOD ON HYDROMAGNETIC LAYER OF CASSON FLUID

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Abstract

This study looks at how a non-Newtonian Casson fluid moves via porous inclined surfaces while taking thermal radiation and magnetic fields into account. To fully understand their influence on boundary layer temperature profiles, complex connections among the fluid, magnetic field and radiation are examined. The governing equations reduced to ordinary differential equations using a similarity transformation and these equations are then analytically solved using the Modified Homotopy Analysis Method (MHAM). This approach provides a comprehensive understanding of the non-linear flow behaviour, revealing the significant influence of surface magnetization and fluid properties on flow dynamics, heat transfer and skin friction. On comparing our analytical results with numerical solution, a satisfactory agreement is reached. The physical parameters involved in the model are illustrated graphically to show their impacts on the system. The result demonstrates the potential of magnetized surfaces in controlling fluid flow and heat transfer with simplifications for various engineering applications, including aerospace, chemical processing and bio-medical devices. This study contributes to the development of innovative thermal management and fluid flow control solutions.

Keywords: Non-Newtonian Casson fluid, Heat transfer, Magnetic field, Boundary layer, Modified Homotopy analysis method (MHAM), Numerical solution.

2020 Mathematics Subject Classifications: 14F35, 14F42, 55P05

ON FAIR DETOUR DOMINATION POLYNOMIAL OF GRAPHS

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Abstract

A Fair detour dominating (FDD) set F ⊆ V (G) is a set that is detour dominating and any pair of vertices not in F has same number of neighbours in F. A Fair detour domination polynomial of a graph G is defined as $F\gamma_D(G,x) = \sum_{i=f\gamma_d}^{|V(G)|} f(G,i)x^i$ where $f(G,i)$ is the number of FDD sets of cardinality i. In this work, we have investigated FDD polynomial of some standard graphs.

Keywords: Dominating Polynomial, Detour domination, Fair domination, Fair detour domination polynomial

AMSC 2020: 05C12, 05C69

DETOUR PI AND DETOUR CO-PI INDEX OF HARMONIC GRAPHS

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Abstract

Graph theory is a delightful playground for the exploration of proof techniques in Discrete Mathematics and its results have applications in many areas of sciences. The field of graph theory is rich in its theoretical and application area. If every vertex of G has equal average degree, then the Graph is called Pseudo-regular graph. Otherwise, it is named as Harmonic graphs. The Detour Padmakar-Ivan (DPI) of a graph G is defined as $DPI(G) = \sum_{e=uv\in E(G)} [N_u(e) + N_v(e)]$ and edges equidistant from both ends of the edges e = uv are not counted. Similar to the vertex version of Detour PI index, another important index Detour Co-PI index of G which is defined as $DCo-PI(G) = \sum_{e=uv\in E(G)} |N_u(e) - N_v(e)|$. In this paper computation of the Detour Pi, Detour Co-Pi Index in Harmonic graphs are proposed.

Keywords: Pseudo-regular graph, Detour Pi Index, Detour Co-Pi Index

AMSC 2020: 05C12, 05C69

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4 - TOTAL GEOMETRIC MEAN CORDIAL LABELING OF PATH RELATED GRAPHS

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Abstract

In this paper we investigate 4 - total geometric mean cordial (4 - TGMC) labeling behavior of path related graphs like T_s, Q_s, $M(P_s)$, L_s, SL_s.

Keywords: Triangular snake, Quadrilateral snake, Middle Graph, Ladder, Slanting Ladder, 4 - Total geometric mean cordial labeling, 4 - (TGMC) graph.

AMS Subject Classification: 05C78

TOPOLOGICAL INDICES FOR COPRIME GRAPH OF GROUPS

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Abstract

Let G be a finite group with identity e. The coprime of G, Γ_G is a graph with G as the vertex set and two distinct vertices u and v are adjacent if and only if $(|u|, |v|) = 1$. In this paper, we investigate the topological indices for coprime graph of groups such as Wiener index, Harrary index, first Zagreb index.

Keywords: Coprime graph, Wiener, Zagreb

AMS subject classification (2010): 05C25, 05C35, 05C69

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APPLICATION OF LINEAR ALGEBRA IN TRAFFIC FLOW ANALYSIS

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Abstract

Linear algebra is essential in traffic flow analysis, using matrices and equations to model and optimize transportation networks. It helps determine vehicle flow, predict congestion, and improve traffic management through techniques like eigenvalues, Markov chains, and optimization. This enhances efficiency and sustainability in urban mobility.

Keywords: Linear Algebra, Traffic Flow, Matrices, Network Modeling.

AMS subject classification: 15A06

2 - PRIME IDEAL OF A LATTICE

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Abstract

In this paper, I introduce a novel ideal called 2 - prime ideal I within the structure of lattices L, aiming to expand the theoretical foundation of lattice theory and its associated algebraic frameworks. The newly defined ideal is constructed by imposing specific conditions on the order and algebraic relations of elements within a lattice. I begin by establishing the formal definition of the 2 - prime ideal and identifying its key properties. The paper provides explicit examples to illustrate the construction and behaviour of the 2 - prime ideal, highlighting its differences from traditional lattice ideals. In addition, I derive a series of theorems that characterize the 2 - prime ideal in terms of lattice-theoretic operations. This study lays the groundwork for further research into generalized ideals and their role in understanding complex algebraic systems.

Keywords: 2 - prime ideal of a commutative ring, Lattice, Ideal of a lattice, Prime ideal of a lattice, Zorn's lemma.

AMS Subject Classification: 13A15; 16H20; 06A11.

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TOTAL DOMINATION DECOMPOSITION OF COMB GRAPHS

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Abstract

This theory is based upon comb graphs which admits Total Domination Decomposition (TDD). Here a graph is decomposed into parts accordingly that the total domination number of parts ranges from $\mathbf{2}$ to $\boldsymbol{m+1}$. The preliminaries and propositions of total domination are used on the graph to get the consecutive results. We have introduced TDD of Graphs [1] and is defined as a family $\psi = \{G_1, G_2, \dots, G_m\}$ of subgraphs of G in such a way G has all its edges belongs to exact one G_j, each G_j is connected and atleast one edge contained in it and $\gamma_t(G_j) = j+1, 1 \leq j \leq m$. By the concept of isolate domination and decomposition [2,3], here we combine total domination with decomposition to categorize the comb graphs which admits TDD and to find their range of vertices.

Keywords: Total Domination Decomposition, isolate domination, decomposition

AMSC 2020: 05C12, 05C69

EDGE COLORING OF GRAPH USING EDGE ADJACENCY MATRIX

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Abstract

Graph coloring is the most popular area of research in graph theory. Graph coloring includes, vertex coloring, edge coloring, region coloring etc in a graph. Proper edge coloring of a graph

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means, coloring of edges of a graph such that no two adjacent edges have the same color. In this paper a new method is proposed for proper edge coloring of a graph with the minimum number of colors using edge adjacency matrix. This method is explained with example and it helps to find the edge chromatic number of a graph.

Keywords: Graph Coloring, edge Coloring, edge Chromatic number.

2010 AMS Subject Classification: 05C15

MULTIFACTORIZATION IN COMPLETE BIPARTITE GRAPHS INTO CYCLES AND STARS

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Abstract

Let $Kn,n\,(\lambda)$ denote a complete bipartite graph with multiplicity (λ). A (G,H) - Multifactorization of a given graph into G- factors and H- factors with atleast one copy of G-factor and H- factor. In this paper, we studied about the (G,H) - Multifactorization of $Kn,n(\lambda)$, where G is a cycle and H is a Star.

Keyword: Multifactorization, Cycles and Stars.

2000 Mathematics Subject Classification Number: 05C69

PAIR QUOTIENT CORDIAL LABELING OF SOME TRIANGULAR SNAKE GRAPHS

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Abstract

Consider a (p,q) graph $G=(V,E)$. Define $\gamma=\begin{cases}\frac{p}{2}, & if\ p\ is\ even\\ \frac{p-1}{2}, & if\ p\ is\ odd\end{cases}$ and $A=\{\pm1,\pm2,\pm3,\ldots,\pm\gamma\}$ called the set of labels. Consider a map $f:V\to A$. When γ is even, assign

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different labels in L to the p elements of V and when γ is odd, assign different labels in A to the $p-1$ elements of V and repeat a label for the lone vertex. The above labeling is said to be a **pair quotient cordial labeling** if for each edge uv of G, there exists a labeling $\left|\frac{f(u)}{f(v)}\right|$ or $\left|\frac{f(v)}{f(u)}\right|$ based on $f(u) \geq f(v)$ or $f(v) > f(u)$ such that $\left|\delta_{f_1} - \delta_{f_1^c}\right| \leq 1$ where δ_{f_1} and $\delta_{f_1^c}$ denote the number of edges labeled with 1 and number of edges not labeled with 1 respectively. A graph which admits pair quotient cordial labeling is called **a pair quotient cordial graph**. In this paper, we look into the behaviour of pair quotient cordial labeling of triangular snake graphs.

Keywords: Triangular snake, Alternate Triangular snake, Double Triangular snake, Alternate Double Triangular snake and Pair Quotient Cordial.

AMS (2010): 05C78

TOTAL DOMINATION INTEGRITY OF SOME GRAPH CLASSES

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Abstract

The concept of domination has been extensively applied in various fields such as Science, Technology, Engineering, Communication Networks etc. Total domination, a well-known variant of domination, plays a significant role in these applications. Integrity is another crucial parameter in network design. A subset S of (G) is called a total dominating set (TD - set) if every vertex of G is adjacent to at least one vertex in S, and S contains no isolated vertices. The total domination integrity of a graph G without isolated vertices is denoted by $TDI(G)$ and is defined as $TDI(G) = min\{|S| + m(G - S) : S \subseteq V(G)\}$ where S is a dominating set of G and $m(G - S)$ is the order of the largest component of $G - S$. In this paper, the total domination integrity of various graph classes, such as windmill graphs, ladder graphs, helm graphs, pan graphs etc. were determined. Additionally, a Python program for computing the total domination integrity of any graph is provided.

Keywords: Integrity, domination integrity, total domination, total domination integrity.

Mathematics Subject Classification Number: 05C70

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STRONG CO-SECURE DOMINATION NUMBER OF SILICATES FAMILY CHEMICAL GRAPH STRUCTURES

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Abstract

A graph $G = (V, E)$ is finite, connected and the set $D \subseteq V$ is *dominating set,* if all vertices V in D guards remaining vertices $V - D$ of G with minimum cardinality. A set $D \subseteq V(G)$ is said to be *strong dominating set* if *e* is an edge with end vertices *u* and *v* and degree of *u* is greater than or equal to degree of *v*, then *u strongly dominates v.* If every vertex of V - D is strongly dominated by some vertex of D, then D is strong dominating set. A graph G is *strong co - secure dominating set* $SCSD$, if for each $u \in D$ there exists a vertex $v \in V - D$ such that $v \in N(u)$, $uv \in E(G) and\ (D \setminus \{u\} \cup \{v\})$ is a dominating set of G with minimum cardinality. The chemical graphs here used are organized from silicate family where nodes are ions and bonds are edges. Various uses are invented when silicates combine with many other materials or chemicals.

2D view of silicates considered in general and the strong co - secure domination number of chain silicate and cyclic silicate is studied in this paper.

Keywords: Domination, Strong Domination, Strong co - secure Domination, Chain and Cyclic silicates.

AMSC 2020: 05C12, 05C69

SOME PROPERTIES OF B# - FRONTIER IN INTUITIONSTIC TOPOLOGICAL SPACES

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Abstract

Using the idea of intuitionistic b#-open sets and b#-closed sets, the concepts of b#-Interior and b#-Closure in intuitionistic topology have been studied and several of their properties are proved.

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Moreover, the attributes of intuitionistic b# Frontier has been presented and studied. Also introduced the properties of intuitionistic b# Frontier has been studied. Many counter examples have been pointed out for the applicable classifications. The relation between intuitionistic b#-Interior, b#-Closure and b#-Frontier have been studied.

Keywords: Intuitionistic $b^{\#}$-Interior, Intuitionistic $b^{\#}$-Closure and Intuitionistic $b^{\#}$-Frontier

AMS Subject Code: 54A40

STUDY ON TOPOLOGICAL INDICES OF PAXLOVID DRUG USED FOR THE TREATMENT OF COVID – 19

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Abstract

Paxlovid is a drug made up of Nirmatrelvir and Ritonavir drugs which is used for the treatment of Covid-19. In this paper we calculated topological indices for Paxlovid. We targeted on the evaluation of standard degree based topological indices for General Randic Index, Atom-Bond connectivity index, Geometric arithmetic index, Forgotten index, Balaban index, symmetric division index and recently introduced reduced reciprocal Randic index and reduced second Zagreb index. These indices are particularly used for analysing and comparing the structures of different molecules. Also, we computed the application part.

Keywords: Randic Index, Atom-Bond connectivity index, Geometric arithmetic index, Forgotten index, Balaban index, symmetric division index.

AMS Classification Code: 54A05

APPLICATIONS OF SOME TOPOLOGICAL INDICES FOR RALOXIFENE DRUG

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Abtract

The drug Raloxifene has been widely used in the prevention and treatment of osteoporosis and breast cancer in postmenopausal disease for women. In this paper we focused on the calculation of degree based topological indices of first and second Zagreb Indices, Atom-Bond connectivity

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Index, Hyper Zagreb Index, updated second Zagreb Index and Max-Min Rodeg Index. These indices are effective tools for analysing and understanding the structure of molecules. Also, we computed some polynomials for some index drawn 3-dimensional graph using Python software.

Keywords: Zagreb Indices, Atom-Bond connectivity Index, Hyper Zagreb Index, updated second Zagreb Index and Max-Min Rodeg Index.

AMS Classification Code: 54A05

GAME CHROMATIC NUMBER OF DEGREE-SPLITTING GRAPHS

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Abstract

Let $G = (V, E)$ be a graph with n vertices. Alisa and Ben are the two players. They alternately color all the vertices with k sets of colors with proper coloring. If all the vertices are colored, Alisa wins. If not, Ben wins. The minimum number of colors needed to color all the vertices of G used by Alisa is the game chromatic number (GCN) and it is denoted by $\chi_g(G)$. After several studies, in this paper, we investigated the game chromatic number of degree-splitting graphs of path Pn and cycle Cn for $n \geq 3$.

Keywords: Graph, Game Chromatic number, Degree-splitting graph

AMS Classification Code: 05C15

ON THE BETWEENNESS CENTRALITY OF VERTEX – WEIGHTED GRAPHS

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Abstract

Betweenness is a centrality measure based on the number of shortest paths. It is assumed that information is primarily routed along shortest paths. A vertex is said to be more central if more shortest paths run through it. Hence a vertex with higher betweenness centrality would have more

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control over the network, because more in formation passes through it. In this paper we define the betweenness centrality of vertex-weighted graphs and find n - ary corona product of some graphs.

Keywords: Betweenness centrality, Weighted graph, n - ary corona product.

AMS Classification Code: 05C15

THE CHROMATIC RESTRAINED DOMINATION ON STRONG PRODUCT OF GRAPHS

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Abstract

Let $G = (V, E)$ be the graph. A subset D of V is said to be a chromatic restrained dominating set (or crd-set) if (i) D is a restrained dominating set and (ii) $\chi(< D >) = \chi(G)$. The minimum cardinality taken over all minimal chromatic restrained dominating sets is called the chromatic restrained domination number of G and is denoted by $\gamma_r^c(G)$. In this paper, the chromatic restrained domination number on the strong product of certain standard graphs were obtained.

Keywords: Domination, Restrained Domination, Chromatic Number, Strong product

AMS Subject Classification: 05C15, 05C69

A SEMI-ANALYTICAL STUDY ON VELOCITY AND THERMAL SLIPS IMPACT ON THE WILLIAMSON FLUID FLOW

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Abstract

In the occurrence of radiation and a magnetic field, a semi-analytical investigation is carried out on the flow of the Williamson fluid through a stretching sheet under the impact of thermal and velocity slip effects. The semi-analytical expressions for non-dimensional velocity and dimensionless temperature are attained by adopting the Modified Homotopy analysis approach. The system's numerous physical aspects are represented through graphical diagrams that illustrate their impact. Furthermore, the dimensionless skin friction factor and non-dimensional Nusselt number are depicted graphically and shown numerically in tables. The error percentage is computed to establish the efficiency and precision of the procedure.

Keywords: Williamson fluid; Thermal slip; Magnetohydrodynamic (MHD); Numerical solution; Modified Homotopy analysis method.

2020 Mathematics Subject Classifications: 14F35, 55P05

RESULT RELATED TO FRACTIONAL ORDER INTEGRAL EQUATION

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Abstract

In this research Paper, we study the result for solution for a fractional order integral equation in Banach Algebras. We make use of some of the standard tools of the fixed Point theory for two operators to establish the main result. We shall prove our result in four to five different steps.

Keywords: Banach Algebras, Integral Equation, Existence Result, Fixed Point Theorem.

2020 Mathematics Subject Classifications: 46H05, 46J10, 46J15

SOLVING QUADRATIC DIOPHANTINE EQUATION OF THE FORM $au^2 + bv^2 = (a + b\alpha^2)^n$

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Abstract

This paper proposes a simple and effective method to solve the quadratic Diophantine equation of the form $au^2 + bv^2 = (a + b\alpha^2)^n$, where n, a, α are positive integers and b is any positive non-square integer, whose solution is a positive integer. The solution is obtained by an analytical method, and the answers are verified by using a python program.

Keywords: Quadratic Diophantine equation, Euler's formula, Polar form, Positive integer Solutions

2020 Mathematics Subject Classifications: 11D04, 11D09, 11D72

SOME PAIR QUOTIENT CORDIAL GRAPHS

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Abstract

Consider a (p, q) graph $G = (V, E)$. Define $\gamma = \begin{cases} \frac{p}{2}, & if\ p\ is\ even \\ \frac{p-1}{2}, & if\ p\ is\ odd \end{cases}$ and $A = \{\pm 1, \pm 2, \pm 3, \ldots, \pm\gamma\}$ called the set of labels. Consider a map $f : V \to A$. When γ is even, assign different labels in A to the p elements of V and when γ is odd, assign different labels in A to the $p - 1$ elements of V and repeat a label for the lone vertex. The above labeling is said to be a **pair quotient cordial labeling** if for each edge uv of G, there exists a labeling $\left|\frac{f(u)}{f(v)}\right|$ or $\left|\frac{f(v)}{f(u)}\right|$ based on $f(u) \geq f(v)$ or $f(v) > f(u)$ such that $\left|\delta_{f_1} - \delta_{f_1^c}\right| \leq 1$ where δ_{f_1} and $\delta_{f_1^c}$ denote the number of edges labeled with 1 and number of edges not labeled with 1 respectively. A graph which admits pair quotient cordial labeling is called a pair quotient cordial graph. A graph which admits pair quotient cordial labeling is called a **pair quotient cordial graph**. In this paper,

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we look into the behaviour of pair quotient cordial labeling of Star, Bistar, Complete, Ladder and $K_2 + mK_1$ graphs.

Keywords: Star, Bistar, Complete, Ladder and Pair Quotient Cordial .

AMS (2010): 05C78

THE SIMPLICITY AND 0 - SUPER MODULARITY OF $CS[S(S(B_n))]$

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Abstract

In this paper, the simplicity of $CS(S(S(B_n)))$ is discussed and we prove that it is simple. and the 0-super modularity of $CS(S(S(B_n)))$ is discussed and we prove that it is not 0-super modular for any $n \geq 1$.

Keywords: Simple lattices, Modular lattices

2020 Mathematics Subject Classifications: 00A69

FIXED POINT RESULTS FOR A PAIR OF SUZUKI TYPE MAPPINGS IN B-METRIC SPACE

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Abstract

This research paper delves into the fascinating realm of fixed point theory within the framework of b-metric space. In this manuscript, we explore the presence and uniqueness of coincidence and common fixed point for two self maps in the context of b-complete b-metric space via triangular -admissible function. Also, several results within the frame of b-metric space are established.

Key words: Triangular β-admissible map, b-complete b-metric space, co-incident point, common fixed point.

2010 Mathematics Subject Classification: 47H10, 54H25

ISBN: 978-93-48505-75-0

ANALYTICAL APPROXIMATIONS FOR THERMAL RADIATION ON MAGNETOHYDRODYNAMIC BOUNDARY LAYER FLOW OF WILLIAMSON NANOFLUID

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ABSTRACT

A study of analytical methods is conducted in order to attain an approximate solution for the MHD Williamson nanofluid boundary layer flow through heat and mass transport with velocity, thermal slip effects and radiation in a porous medium. The similarity transformation is used to transform the governing PDEs into ODEs. The approximate analytical expressions for the corresponding velocity, concentration and temperature gradients in dimensionless form are derived via the Modified Homotopy Analysis Method (MHAM). A very excellent fit can be achieved when the analytical findings are compared to the numerical solution. Numerous implications of the physical components involved in the problem are graphically demonstrated. The coefficient of local skin friction, the Sherwood number, and the Nusselt number are all determined and highlighted in the table. Furthermore, the results exhibit how quickly and effectively the approach converges.

Keywords: Homotopy, Sherwood number, Nusselt number

2020 Mathematics Subject Classifications: 55P05, 55P15

A MATHEMATICAL STUDY ON MEASLES DISEASE IN PAKISTAN

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Abstract

This study uses actual data from Pakistan to investigate an established mathematical framework that describes the dynamics and epidemiology of measles transmission. Sanitation and

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immunization are seen as mitigating strategies in the model. The six primary model components that can be addressed analytically using the homotopy analysis approach are Susceptible, Recovered, Infected, Exposed, Hospitalised, and Vaccinated. Specific analytical formulae are given for each of the six compartments. To show the influence of various model parameter categories including the frequency with which hospitalised persons with measles visit due to complications, rate of vaccinating susceptible class and recruitment rate into susceptible class, graphics are provided. The outcomes show that this approach is the most practical, easy to use, and efficient. A satisfactory match is obtained by comparing the findings with the numerical simulation (MATLAB). This technique will be extended to tackle epidemic models especially, SIR, SEIR, SVIR, SVEIR, SLVEIR based on malaria, chikungunya, tuberculosis, HIV, hepatitis A virus, typhoid, Ebola, Cholera etc.

Keywords: Epidemic model, Measles infection, Non-linear initial value problem, Homotopy analysis method (HAM), Numerical simulation.

2020 Mathematics Subject Classifications: 55P05, 76F65

A RETRIAL INVENTORY SYSTEM WITH BERNOULLI'S TRAIL FOR MULTIPLE SERVER VACATION

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Abstract

In this article we consider a continuous review inventory system with Markovian demand. The operating policy is (s, S) policy, with order quantity Q (= S-s). Lead time is assumed to be exponential distribution. The server assumes Bernoulli's trialed multiple vacation. Whenever the inventory level is zero server had a choice to stay in the system or take another vacation. At the end of each vacation server assumes the Bernoulli's trialed multiple vacation. The demands that occur during stock out period or during the server vacation period enter into orbit of infinite size. These orbiting demands retry for their demand after a random time, which is assumed to be exponential distribution. The joint probability distribution of the number of demands in the orbit, status of the server and the inventory level is obtained in the steady state case. Various system

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performance measures in the steady state are derived and the long run total expected cost rate is calculated. Several instances of a numerical example, which provide insight into the behavior of the system, are presented.

Keywords: Continuous review inventory system, Multiple vacation, Lead time, (s, S) policy.

2020 Mathematics Subject Classifications: 90B05

MATHEMATICAL MODELING AND OPTIMIZATION OF IMMOBILIZED GLUCOSE ISOMERASE REACTORS USING HILL KINETICS

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ABSTRACT

Glucose isomerization is a critical bioprocess for producing high-fructose corn syrup (HFCS), a widely used sweetener in the food industry. This process, catalyzed by immobilized glucose isomerase (GI), enhances enzyme reuse and reactor efficiency. However, optimizing the performance of these reactors remains challenging due to factors such as enzyme efficiency, mass transfer limitations, and complex reaction dynamics. In this study, a mathematical model is developed that integrates Hill kinetics with reaction-diffusion principles to simulate the transient behavior of glucose and fructose concentrations in immobilized enzyme reactors. The model accounts for enzyme cooperativity and the interplay between enzyme kinetics and mass transfer effects. Simulation results provide concentration profiles over time and along the reactor, and are used to optimize reactor parameters, such as enzyme concentration and flow rate, to maximize fructose yield. The findings offer valuable guidance for improving the efficiency of glucose isomerization in industrial applications.

Keywords: Hill kinetics, optimization

2020 Mathematics Subject Classifications: 76B75

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MATHEMATICAL ANALYSIS OF CHEMICALLY REACTIVE MHD FLOW WITH INDUCED MAGNETIC FIELD THROUGH CHANNEL

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ABSTRACT

In this paper, a mathematical analysis is performed on the MHD flow across a channel made by two infinite vertical plates. A source of heat/sink is placed near one of the plates and first order chemical reaction is observed in the flow. Induced magnetic field arises due to applied magnetic field of strength is taken in the fluid motion equation to study its impact. The system of ordinary differential equations expressing the fluid model are solved analytically under the appropriate boundary conditions after going through non dimensional process. The solutions obtained for fluid velocity, Induced magnetic field, temperature distribution and concentration distribution in the system are plotted graphically w.r.t to various fluid parameters. In addition to this, the behavior of drag at the plates and induced current density are shown via tables. The significant impression of induced magnetic field is seen on velocity and drag.

Keywords: Induced magnetic field, fluid model, fluid parameters

2020 Mathematics Subject Classifications: 34A06

WEAKLY ϕ-SYMMETRIC LP KENMOTSU MANIFOLDS

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Abstract

The aim of this paper is to investigate weakly ϕ - symmetric LP - Kenmotsu manifolds. The investigation reveals that both ϕ - symmetric and weakly ϕ - symmetric LP - Kenmotsu manifolds are η - Einstein.

Keywords: ϕ-symmetric, weakly ϕ-symmetric, LP-Kenmotsu manifolds

2020 Mathematics Subject Classifications: 58A50, 05E05

FORECASTING INFLATION USING CONSUMER PRICE INDEX IN INDIA

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ABSTRACT

Inflation, a crucial economic indicator, reflects the rise in the general price level of goods and services, impacting purchasing power and economic stability. This study aims to forecast inflation trends in India using Consumer Price Index (CPI) data and time series models. It identifies key CPI categories influencing inflation, compares rural and urban trends, and analyzes relationships among CPI components using regression.

Keywords: Inflation, Consumer Price Index (CPI), Urban and Rural, Time Series Analysis, Forecasting.

2020 Mathematics Subject Classifications: 62M10, 37M10

MATHEMATICAL ANALYSIS OF TUMOR-IMMUNE AND HOST CELLS INTERACTIONS WITH CHEMOTHERAPY

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Abstract

A mathematical model of tumor-immune and host cells interaction with chemotherapy is analyzed. The model involves the system of non-linear equations that contain Michaelis–Menten term with resting immune cells. Analytical expressions pertaining to concentrations of tumor cells, hunting immune cells, resting immune cells, normal cells, and drug concentration are presented. We have employed Homotopy Perturbation Method (HPM) to evaluate the approximate analytical solutions of the non-linear initial value problem. Different parameters involved in this model are analyzed using the analytical result obtained.

Keywords: Homotopy, Homotopy Perturbation Method

2020 Mathematics Subject Classifications: 58J47, 65L05

DOMINATING ENERGY IN NEUTROSOPHIC SOFT GRAPHS

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Abstract

The idea of soft sets which was first proposed by Molodtsov has gained importance due to its applications in various domains. Combining neutrosophic soft sets with graphs, domination in neutrosophic soft graphs were defined based on the strength of connectedness. This manuscript deals with dominating energy in neutrosophic soft graphs. Further, the bounds of dominating energy and the properties of eigen values of dominating matrix of neutrosophic soft graphs are analysed.

Keywords: neutrosophic graph, neutrosophic soft sets, neutrosophic soft graph, dominating matrix, dominating energy.

AMS Subject Classification: 05C72, 05C69, 94D05

METRIZABILITY ON TOPOLOGICAL SIMPLE ROUGH GROUPS

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ABSTRACT

In this paper, we focus on the concepts of rough pseudonorm and metrizability in topological simple rough groups. Further, we discuss some results related to these concepts. In particular, we explore the interplay between rough pseudonorms and metrizability, and also highlighting the necessary and sufficient condition to ensure a topological simple rough group is metrizable. These findings contribute to a deeper understanding of the structural dynamics of topological simple rough groups.

Keywords: Simple rough group, Topological simple rough group, Rough pseudonorm, Rough pseudometric, Metrizable, Symmetric neighbourhoods.

2020 Mathematics Subject Classification:22A05, 54E35, 20D05.

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ON NEUTROSOPHIC p*gp-CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract

Aim of these **c**oncepts, we introduced a new class of neutrosophic p*gp-closed sets on neutrosophic topological space X, known as neutrosophic p*gp-closed (briefly Np*gp-closed) sets. Also, inspect this new set is relate with other existing sets. Further, we discuss about their characterizations of neutrosophicp*gp-closed sets.

Keywords: Neutrosophic Closed Sets, Neutrosophic Open Sets, Neutrosophic Pre Closed Set

AMS Subject Classification: 18B30, 03E72, 11B05

SHADOW SOFT SEPARATION AXIOMS: A THEORETICAL FRAMEWORK VIA SHADOW SOFT POINTS

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Abstract

Shadow soft set improves the accuracy of modelling complex systems characterized by partial or ambiguous information. In this work, we introduce the concept of shadow soft point, which serves as the fundamental element for developing shadow soft topology, accompanied by illustrative examples. Furthermore, explore neighborhood system of shadow soft point and its properties. Additionally, we formulate shadow soft T_i-spaces (i =0,1,2) in terms of shadow soft points and provide a detailed analysis of relationships among the spaces.

Key words: Shadow soft set, Shadow soft topological space, Shadow soft point, Shadow soft separation axioms.

2020 Mathematics Subject Classifications: 54A40, 03E72, 94D05

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APPLICATIONS OF GENERALIZED V - OPEN SETS IN TOPOLOGICAL SPACES

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Abstract

In this article the concepts of gv-open set, we introduce and study topological properties such as gv-limit points and gv-derived set. Also, we investigate the operators namely, gv-border, gv-frontier, gv-exterior using gv closure and the gv interior operators. In addition, we study the concept of gv-kernal using gv-open sets. Further, we provide the relations between them. Finally, some of their fundamental properties are discuss.

Keywords: gv-limit points, gv-derived set, gv-border, gv-frontier, gv-exterior and gv-kernal.

2020 Mathematics Subject Classifications: 11B05, 18B05, 26A21

THEORETICAL INSIGHTS INTO TYPES OF TURIYAM SOFT RELATIONS

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Abstract

In this paper we study the Types of Turiyam Soft Relations. We establish the Equivalence Class of Turiyam Soft Relations. We also introduce the concepts of Turiyam Soft Partition and Turiyam Soft Block and the theorems related to this topic has also been discussed.

Keywords: Symmetric Turiyam Soft Relation, Transitive Turiyam Soft Relation, Reflexive Turiyam Soft Relation, Equivalence Class, Partition of Turiyam Soft Set.

2020 Mathematics Subject Classifications: 03B52, 03E72, 90C70

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ADDRESSING FUZZY TRANSPORTATION PROBLEMS USING HEXAGONAL INTUITIONISTIC FUZZY NUMBERS WITH A NOVEL APPROACH

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Abstract

Economic strategy, information analysis, and dynamics all heavily rely on fuzzy numbers. Finding out where different fuzzy numbers are located is an important milestone in many numerical models. Using our recommended positioning method, we in this work turn the Hexagonal Intuitionistic Fuzzy transportation problem into a sophisticated and well-respected model. To get a fuzzy optimal solution, this transformation is then applied to another suggested method. The statistical example demonstrates how well the suggested approach uses fuzzy-based computations to solve transportation-related problems

Keywords: Fuzzy set, ranking of hexagonal fuzzy numbers, intuitionistic fuzzy numbers, fuzzy transportation problem, centroid method

2020 Mathematics Subject Classifications: 94D05, 03B52, 03E72

DISTANCE BASED SIMILARITY MEASURES OF INTUITIONISTIC FUZZY BINARY SOFT SETS AND ITS APPLICATIONS

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Abstract

Distance based similarity measures of Intuitionistic fuzzy binary soft sets is defined and their basic properties are discussed. The distance-based similarity measure is calculated for different Intuitionistic Fuzzy Binary Soft Sets. Additionally, we defined an algorithm to compute the

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similarities of Intuitionistic Fuzzy Binary Soft Set (IFBSS), and use the proposed algorithm for decision making problem in the real-life.

Keywords: Soft set, Intuitionistic, Fuzzy, Binary, Similarity measure, Distance based similarity.
2020 Mathematics Subject Classifications: 03D45, 03F55, 03E72

FIXED POINT THEOREM IN GREY METRIC SPACE

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Abstract

In 2020, grey metric space was introduced and all its properties have been studied in detail. In this paper, fixed point of a function in grey metric space has been defined and Banach's fixed point theorem has also been proved for complete grey metric space.

Keywords: Grey metric space, complete grey metric space, compact grey metric space.
2020 Mathematics Subject Classifications: 11J83, 32C18, 54A05

ON NEUTROSOPHIC ϒ − REGULAR AND ϒ − NORMAL SPACES

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Abstract

The focus of this paper is to explore the concepts of regular and normal spaces in neutrosophic topology by means neutrosophic ϒ −open sets. We further establish their stronger forms and discuss their attributes.

Keywords: neutrosophic ϒ −open, neutrosophic ϒ −closed, neutrosophic ϒ −regular, neutrosophic ϒ −normal
2020 Mathematics Subject Classifications: 18B30, 03E72, 11B05

ISBN: 978-93-48505-75-0

A STUDY ON KINDS OF PAIRWISE MICRO (NEAR, CLOSER) RELATIONS IN MICRO BITOPOLOGICAL SPACE

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Abstract

Pairwise micro near and Pairwise micro closer are the names given to the relationships. We discuss different types of pairwise micro near and pairwise micro closer relations in micro bitopological space in this study. In addition to reviewing and providing examples, we also looked into some properties.

Keywords: Micro bitopological space, Pairwise micro near and Pairwise micro closer.
2020 Mathematics Subject Classifications: 54A10, 54E55.

FURTHER STUDY ON DERIVATIVE TOPOLOGICAL SPACES

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Abstract

A derivative topology on a differential ring R is defined by means of differential subsets of a differential ring. The derivative topology on R is a collection τ of differential subsets of R, which has the structure that $\emptyset$, R are in τ and is closed under arbitrary union and arbitrary intersection. The primary objective of this paper is to compare the derivative topology to various topologies such as general topology, supra topology, generalized topology, filter topology and interior

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topology. It is observed that the structure of derivative topology differs from that of other types of topologies. Finally, we prove that the intersection of any family of derivative topologies is again a derivative topology on a differential ring.

Key words: Derivative topology, differential subset, supra topology, filter topology, interior topology.

2020 Mathematics Subject Classifications: 54H13, 54A05, 54A10, 54B05.

ENTROPY MEASURES ON INTUITIONISTIC FUZZY BINARY SOFT SETS AND ITS APPLICATIONS

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Abstract

Entropy is a fundamental concept for measuring uncertainty in various mathematical models. Entropy measures on Intuitionistic fuzzy binary soft sets is defined and their basic properties are discussed. Entropy measure is calculated for different Intuitionistic Fuzzy Binary Soft Sets. Additionally, we defined an algorithm to compute the Entropy on Intuitionistic Fuzzy Binary Soft Set (IFBSS), and use the proposed algorithm for decision making problem in the real-life.

Keywords: Soft set, Intuitionistic, Fuzzy, Binary, Entropy.

AMS Mathematics Subject Classification: 03D45, 03F55, 03E72

ON (1,2)*- WEAKLY DELTA GENERALIZED BETA CLOSED SETS AND CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

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Abstract

In this paper, we introduce a new class of sets called (1,2)*-w$\delta g\beta$-closed sets and explore their fundamental properties. We investigate how these sets relate to other well-known types of $\tau_{1,2}$-closed sets in bitopology, highlighting both similarities and differences. Additionally we define and study (1,2)*-w$\delta g\beta$-continuous functions, focusing on their basic characteristics and behavior under various conditions. Furthermore, we provide several characterizations of these functions, establishing their significance within the broader framework of bitopological spaces. Lastly, we discuss potential applications of (1,2)*-w$\delta g\beta$-closed sets and (1,2)*-w$\delta g\beta$-continuous functions, demonstrating their utility in both theoretical and applied contexts.

Keywords: (1,2)*-w$\delta g\beta$-closed sets, (1,2)*-w$\delta g\beta$-continuous functions

AMS Mathematics Subject Classification: 54A10, 54E55

(1,2)*- WEAKLY DELTA GENERALIZED BETA CLOSED AND OPEN MAPS IN BITOPOLOGICAL SPACES

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Abstract

This paper aims to establish the concepts of (1,2)*-w$\delta g\beta$-closed maps and (1,2)*-w$\delta g\beta$ open maps. The interconnections between these maps and other types of closed maps are examined. It is demonstrated that the composition of (1,2)*-w$\delta g\beta$-closed maps msy not necessarily be (1,2)*-w$\delta g\beta$-closed. Furthermore, characterizations and practical applications of these maps are provided.

Keywords: (1,2)*-w$\delta g\beta$-closed maps, (1,2)*-w$\delta g\beta$ open maps, (1,2)*-w$\delta g\beta$-closed maps

AMS Mathematics Subject Classification: 54A10, 54E55

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MEASURES OF THE BATCH FUZZY QUEUING MODEL FOR THE PENTAGONAL FUZZY NUMBER USING PASCAL'S TRIANGLE GRADED MEAN AND GRADED MEAN

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Abstract

We suggested a method using Pascal's triangle graded mean and graded mean to convert pentagonal fuzzy numbers into clear figures to evaluate the performance measures of a batch fuzzy queuing model of size "b" of a single server model with pentagonal fuzzy numbers, which are arrival rate and service rates.

We can successfully defuzzify the Pentagonal Fuzzy Numbers with the aid of this suggested way. Furthermore, we present instances that demonstrate the use of the graded mean and Pascal's triangle graded mean in batch "b" size Pentagonal fuzzy queuing models.

We have included numerical examples to support our findings.

Keywords: Membership Function, Pentagon Fuzzy Number, Pascal's triangle graded mean and graded mean

AMS Mathematics Subject Classification (2010): 03E72; 60K25; 68M20; 90B22.

TOPOLOGICAL POLYNOMIAL ANALYSIS OF IMATINIB USING DEGREE-BASED INDICES

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Abstract

The imatinib is an inhibitor used for the treatment of various types of cancer and bone marrow diseases. In this paper we used degree based topological indices to draw different polynomials like GA polynomial, ABC polynomials, Hyper Zagreb polynomials, Harmonic polynomials, Randic polynomials, Sum connectivity polynomials, first, second and third Zagreb polynomials, Augmented polynomials, Harmonic polynomials, Forgotten topological polynomials, inverse sum Indeg polynomials and Somber polynomials for $C_{29}H_{31}N_7O$.

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Keywords: GA polynomial, ABC polynomials, Hyper Zagreb polynomials, harmonic polynomials, Randic polynomials, Sum connectivity polynomials.

2020 Mathematics Subject Classifications: 54A10, 54E55

FUZZY ALTERNATIVE RANKING ORDER METHOD ACCOUNTING FOR TWO - STEP NORMALIZATION BASED MULTI-CRITERIA DECISION - MAKING APPROACH TO CHEMICAL SUPPLIER SELECTION

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Abstract

Multi-criteria decision making is an interdependent process comprising the core elements of alternatives, criteria and their inter associations. The intricacy of decisioning depends on the inter relational impacts between the elements of decision making. This research work addresses the problem of supplier selection with special reference to chemical industries. This study employs Fuzzy Alternative Ranking Order Method Accounting for Two-Step Normalization (FAROMAN) to rank eight suppliers using six criteria of cost, quality, delivery time, reliability, safety and flexibility. The ranking results using FAROMAN are compared with deterministic AROMAN to exhibit the efficacy of fuzzy integrated ranking approach. Sensitivity analysis is performed to determine the effects of the key parameters over ranking results. This decision approach shall be applied to other decisioning situations to design optimal solutions to real-world problems.

Keywords: Fuzzy AROMAN, Chemical Industries, Supplier Selection, Optimal Ranking

2020 Mathematics Subject Classifications:

ANALYZING MARKET OUTCOME USING GAME THEORY: A STUDY OF NASH EQUILIBRIUM IN THE COURNOT DUOPOLY MODEL

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Abstract

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The paper uses game theory to look at market outcomes using the Cournot duopoly model, with a focus on Nash equilibrium. The model, which is a basic idea in industrial organization, looks at how any firms decide how much to produce by considering how each firm will react to what its competitors do. This model looks at how companies consciously act to make the most money, taking into account how they are all linked. Asymmetric cost structures, similar cost structures, and product differentiation are some of the big changes that have been made to the model that help us understand in what way markets really work. The paper looks at the policy consequences and stresses how important the Cournot framework is for figuring out market efficiency, welfare effects, and regulatory measures. This research shows how important game theory is for understanding how competition works and how to act strategically in oligopolistic markets.

Keywords: Game theory, Cournot duopoly model, Nash equilibrium, Oligopoly Markets

2020 Mathematics Subject Classifications: 90C33, 91B24

ON TESTING OF STATISTICAL HYPOTHESIS FOR VARIANCE BASED ON REVISED SIGNED DISTANCE UNDER FUZZY ENVIRONMENT

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Abstract

In this paper, we observe the difficulties of testing statistical hypotheses in a fuzzy environment, specifically for trapezoidal fuzzy data. We extend the revised signed distance for testing the statistical hypotheses for variance with trapezoidal fuzzy data. The method introduced in this paper is also to be employed for testing the hypothesis of variance. Finally, some numerical examples are given to illustrate the plausibility of the revised signed distance for statistical hypothesis testing.

Keywords: Statistical hypothesis, fuzzy data, triangular fuzzy numbers, testing hypothesis, signed distance.

2020 Mathematics Subject Classifications: 03B52, 62B10, 62B86

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DIFFERENTIAL NEIGHBORHOODS AND DIFFERENTIAL LIMIT POINTS IN DERIVATIVE TOPOLOGICAL SPACES

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Abstract

In recent years, some generalized structures of topologies were introduced. In this way, derivative topological space was introduced. To contribute in this orientation, we introduce and investigate the properties of differential neighborhoods and differential limit points in derivative topological spaces. We explore many properties of them and discuss their behaviour on derivative topological spaces. A differential limit point is a point in a differential ring that can be approached arbitrarily close by the elements that same set. These concepts serve as the building blocks for defining other key properties in derivative topological spaces. The relation between differential open sets, differential closure, differential neighborhoods and differential derived sets were obtained.

Key words: Derivative topology, differential open, differential neighborhood, differential limit point and differential derived set.

2020 Mathematics Subject Classifications: 54A10

A STUDY ON VARIOUS DOMINATION NUMBERS RELATED TO SECURITY PROCESS OF HEXAGONAL STAR NETWORK AND COMPARISON AND NON – COMPARISON OF THIS NETWORK DOMINATION NUMBERS

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Abstract

Graph theory on networks plays a momentous role composed of operators (vertices) and its linked connections (edges). To scrutinize the properties, pathways and connections of networks, the tool like graph related parameters are imperative. Dominating set of a graph is subset of vertices, whose nodes are adjacent to remaining nodes belong to graph such that capturing these dominating nodes helps to find the total number of nodes by their degrees.

Securing such dominating nodes is more essential to protect the entire network from attackers like viruses, unstable current supplies, natural calamities, errors in arrangements etc. The dominating set basics are circumstantial [5]. Nodes on support of its neighbors guard the whole co – nodes within a fraction of second promptly. The requirement of defenders with minimum cardinality for standard graphs, equality between secure and co – secure dominating sets and their bounds are studied in [4]. In this paper, we enclosed the minimum cardinality of secure, co – secure, perfect secure and perfect co – secure domination number of some dimensional networks on base of increase in level. Network structures are discussed below with their commercial and confidential usage.

Key words: secure, co – secure, strong, perfect secure, perfect co – secure domination number, Hexagonal star Networks.

Mathematics subject classification number: 05C69, 05C92, 92E10

DEVELOPMENT OF KALMAN FILTER-BASED SENSOR FUSION ALGORITHM FOR 360-DEGREE ENVIRONMENT PERCEPTION SYSTEM IN AUTONOMOUS RURAL ENVIRONMENTS

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Abstract

Kalman filter based sensor fusion algorithm is capable of offering an unbiased approximation of the status of moving machine which is competently operate in scenarios when the sensor errors and the functioning machine dynamics is well within control. In order to that Self-Driving cars' perception systems must have excellent accuracy and reliability in order to detect a 360-degree environment for safe and effective driving in rural areas. In existing system machine learning based sensor fusion algorithm used for predicting system states based on sensor data. The limitation of this algorithm is not adapting well to the environmental changes which leads to inaccurate forecasting of data. However, the varying environmental conditions, ecosystems, and weather conditions of rural locations create a challenge to sensor fusion systems, which are complicated and difficult to operate, so that enhanced Kalman filter-based sensor fusion algorithm is proposed. The goal of this research is to design a sensor fusion technique based on Kalman filters for 360-degree environment perception systems in rural autonomous situations. To provide accurate and reliable environmental evaluations, the suggested algorithm will gather information from all cameras, LiDAR (Light Detection and Ranging), Radars (Radio detection and ranging), and GPS (Global Positioning System) devices. Because the environment is always changing, it enables Kalman filter-based algorithm adaptation in situations including noisy or unclear data. The performance of the suggested algorithm is evaluated using actual data that was collected from the rural area and the results demonstrate a higher degree of accuracy and reliability than existing machine learning sensor fusion methods.

Keywords: machine dynamics, sensor fusion algorithm

2020 Mathematics Subject Classifications: 00A69

ENHANCED MULTIVARIATE TIME SERIES FORECASTING USING MODIFIED SOFTS ARCHITECTURE

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Abstract

This study proposes a novel enhancement to the Series-cOre Fused Time Series forecaster (SOFTS) for improving multivariate time series forecasting. The modified architecture incorporates convolutional layers, attention mechanisms, and advanced pooling techniques better to capture intricate patterns and relationships within the data. These enhancements aim

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to address the limitations of traditional forecasting models and provide a more robust and accurate solution. The methodology involves pre-processing the collected network traffic data, feature extraction, and implementing the modified SOFTS model. Performance evaluation against existing models such as Particle Swarm Optimization (PSO), Whale Optimization Algorithm (WOA), and Ridge Regression with Autoencoder (RSA) demonstrates superior results across critical metrics, including Mean Absolute Error (MAE), Mean Absolute Scaled Error (MASE), and R-squared (R^2). The proposed modifications significantly enhance the model's predictive accuracy and generalizability, providing a valuable tool for forecasting service quality in various applications.

Keywords: Multivariate Time Series Forecasting, SOFTS, Convolutional Layers, Attention Mechanism, Advanced Pooling, Mean Absolute Error, Mean Absolute Scaled Error, R-squared, Network Traffic Data

2020 Mathematics Subject Classifications: 62M10, 37M10

Construction of Quality Interval Skip Lot Sampling Plan through Tangent Angle

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Abstract

Acceptance sampling systems are advocated when small sample size are necessary and desirable towards costlier testing for product quality. In this paper we introduce a new design for Quality Interval Skip Lot Sampling Plan. This paper developing a Skip Lot Sampling Plan based on Trigonometric ratios and hypotenuse ratios along with decision region (d 1) and probabilistic region (d 2) which is more applicable in practical situations. Maximum Allowable Percent Defective (MAPD) is also considered for the selection of parameters for sampling plan which is representative of quality interest of all involved in production. Tables were presented and examples were illustrated.

Keywords: Skip Lot Sampling Plan, Quality Probability of Acceptance, Decision Regions, Operating Characteristic Curve, Trigonometric ratios, Hypotenuse ratios.

2020 Mathematics Subject Classifications: 97G60

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CHROMATIC POLYNOMIALS IN VARIOUS GRAPHS

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Abstract

The chromatic polynomial of a graph is a powerful concept in graph theory that counts the number of ways to color the vertices of a graph using specified number of colors, subject to the condition that adjacent vertices must have different colors. This paper explores the mathematical properties, calculation methods, and applications of chromatic polynomials in real-world scenarios. This paper deals with the fundamental principles of chromatic polynomials, including the basic definitions and formulas used to compute them for common graph structures such as trees, cycles, and complete graphs. Furthermore, the project delves into practical applications of chromatic polynomials, illustrating how this mathematical tool is used to optimize resources, such as in the scheduling of tasks or designing communication networks.

Keywords: Graph coloring, Chromatic Number, Chromatic Polynomial, planar graph, complete graph

MSC 2020 Classification: 05C07, 05C15, 05C31, 05C38, 05C90

RESTRAINED STAR CHROMATIC INDEX IN THE GRAPH OPERATION: KRONECKER PRODUCT

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Abstract

Graph coloring is a mathematical concept that assigns labels (colors) to the elements of a graph, such as vertices or edges in a way that avoids conflicts between adjacent elements. One of the main goals of graph coloring is to minimize the total requirement in achieving a proper coloring, and that minimum requirement is called as its respective chromatic number or index. Restrained star edge coloring is a special type of graph coloring of edges, according to which in addition to

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the basic condition for a proper coloring, all its bichromatic subgraphs must be in the form of a galaxy. Here, the minimum requirement is referred to as its restrained star chromatic index and is denoted as χ'_{rs}. The purpose of this paper is to analyze the restrained star edge coloring and its corresponding restrained star chromatic index, χ'_{rs} of some graphs obtained using the graph operation - "kronecker Product". In addition, the exact value of its χ'_{rs} and bounds are also examined with the usage of suitable illustrations.

Keywords: Restrained star edge coloring, Restrained star chromatic index, Kronecker product of graphs, path, cycle, star.

MSC 2020 Classification: 05C07, 05C15, 05C38, 05C76

STATISTICAL MODELING FOR GLAUCOMA DETECTION AND PREDICTING ITS DIAGNOSIS

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Abstract

Glaucoma is a leading cause of irreversible blindness, requiring early detection and accurate diagnosis for effective management. This project leverages statistical and machine learning techniques to identify critical risk factors, assess the diagnostic performance of tools like visual field tests and optical coherence tomography (OCT), and develop predictive models for precise diagnosis. Additionally, it explores interactions between variables, such as age and family history, and classifies glaucoma types to gain deeper insights into disease progression. The findings aim to improve clinical decision-making, enhance early detection, and support better patient outcomes through data-driven approaches.

Keywords: Glaucoma Risk Factors, Clinical Decision Support, Diagnostic Sensitivity and Specificity, Glaucoma Prediction

2020 Mathematics Subject Classifications: 62C86, 62C05, 62C86

TOPOLOGICAL INDICES OF CHOLESTERIN: MATHEMATICAL INSIGHTS INTO MOLECULAR STRUCTURE

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Abstract

Topological indices are essential tools in mathematical chemistry, offering valuable insights into the structural and molecular graphs. In this study, we focus on calculating novel degree-based topological indices for cholesterin, including the modified Y index, modified S index, sigma Y index, sigma S index, modified minus Y index, modified minus S index, reduced Y index, reduced S index, reduced modified Y index, and reduced modified S index. These indices extend traditional degree-based descriptors and provide deeper insights into the molecular topology of cholesterin.

Keywords: Modified Y & S index, sigma Y & S index, modified minus Y & S index, reduced Y & S index, reduced modified Y & S index.

2020 Mathematics Subject Classifications: 57M15

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SOFT GRAPHS AND LINKED MATRICES: A STRUCTURAL STUDY

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Abstract

Molodtsov introduced the soft set theory as a general mathematical strategy for dealing with uncertain data. Soft set theory is currently being used by many academics to address decision-related problems. A parameterized point of view for graphs is provided using the concept of a soft graph. The soft graph uses parameterization to create several descriptions of a complex relation that is represented by a graph. In this article, we introduce various matrices associated with soft graphs, such as the S-adjacency matrix and S-incidence matrix, and we look at some of their characteristics.

Keywords: S-Adjacency Matrix; S-Incidence Matrix.

Mathematics Subject Classification 2020*:* 05C99

1. Introduction

The novel idea of soft set theory was introduced by Molodtsov [8] in 1999. This is a mathematical method for addressing uncertainty. Soft set theory can be used to solve a variety of real-world problems. Authors like Biswas, Maji, and Roy [6], [7] have studied the concept of soft sets in great depth and used it to address a variety of decision-making circumstances. Thumbakara and George originally introduced the idea of the soft graph in 2014 [13]. Additionally, they proposed ideas like soft semigraphs [4] and soft hypergraphs [3] and talked about soft graph operations [14]. In 2015, Akram and Nawas [1] modified the idea of soft graphs first forth by Thumbakara and George. Many concepts, like soft trees, soft bridges, soft cut vertex, soft cycle, etc., were also defined by them [2] in terms of soft graphs. Thenge, Jain, and Reddy all added to the body of knowledge regarding connected soft graphs [10]. They also [11] examined the concepts of degree and the radius, diameter, and centre of a soft graph. In 2019, Sarala and Manju [9] introduced the idea of soft bipartite graphs and talked about some of their characteristics. Thenge, Jain, and

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Reddy addressed the ideas of incidence and adjacency matrices of a soft graph in 2020 [12]. In this study, we describe various kinds of matrices related to soft graphs and look at some of their characteristics.

2. Preliminaries

2.1. Soft Graph

Thumbakara and George [11] introduced the concept of soft graphs as follows. "Let $F= (T, D)$ be a simple graph and C be any nonempty set. Let R' be an arbitrary relation between elements of C and elements of T. That is $R' \subseteq CXT$. A mapping $I: C \rightarrow P(T)$ can be defined as $I(c) = \{t \in T | cR't\}$. The pair (I, C) is a soft set over T. Then (I, C) is said to be a Soft Graph of F if the subgraph induced by $I(c)$ in F is a connected subgraph of F for all c in C."

Example 2.1.

Consider the following graph $F= (T, D)$.

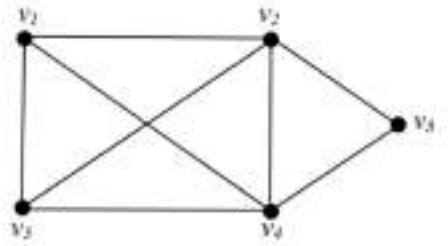

Figure 1. $F= (T, D)$

Let C = $\{v_1, v_3\}$. Define a function I by $I(c) = \{t \in T | d(c, t) < 2\}$. Then $I(v_1) = \{v_1, v_2, v_4, v_5\}$, $I(v_3) = \{v_2, v_3, v_4\}$. Here the subgraph induced by $I(c)$ in F is a connected subgraph of F, for all $c \in C$. These subgraphs are shown in figure 2.

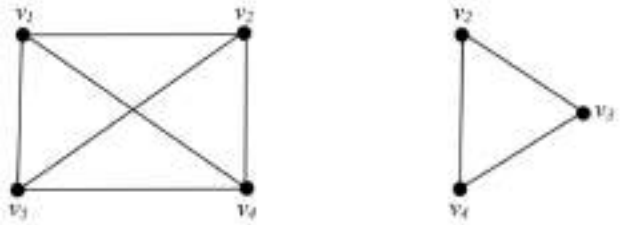

Figure 2. Subgraphs induced by $I(v_1)$ and $I(v_3)$

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Here, subgraphs induced by $I(v_1)$ and $I(v_3)$ are connected subgraphs of *D.* Therefore (*I*, *C*) is a soft graph.

Definition 2.2.

Muhammad Akram and Saira Nawas [1] defined the soft graph as follows. "Let F^*= (*T, D*) be a simple graph and *C* be any nonempty set. Let *R'* be an arbitrary relation between elements of *C* and elements of *T.* That is $R' \subseteq CXT$. A mapping *I:* $T \to P(T)$ can be defined as $I(c) = \{t \in T | cR't\}$. Also, define a mapping *J:* $C \to P(D)$ by $J(c) = \{uv \in D | \{u, v\} \subseteq I(c)\}$. The pair (*I*, *C*) is a soft set over *T* and the pair (*J*, *C*) is a soft set over *D*.

Then the 4-tuple *F* = (*F*, I, J, C*) is called a soft graph if it satisfies the following conditions:

1) F^*= (*T, D*) is a simple graph,
2) *C* is a nonempty set of parameters,
3) (*I, C*) is a soft set over *T*,
4) (*J, C*) is a soft set over *D,*
5) (*I*(c), *J*(c)) is a subgraph of F^* for all $c \in C$

If we represent (I(c), J(c)) by *B(c),* then soft graph *G* is also given by $\{B(c): c \in T\}$. Then *B(c)* corresponding to a parameter *c* in *C* is called a part of the soft graph *F*. The part degree of the vertex *v* in *B(c)* denoted by *d* (*v*) [*B(c)*] is the degree of the vertex *v* in that part *B(c).*"

Example 2.2.

Consider a graph F^*= (*T, D*) given below.

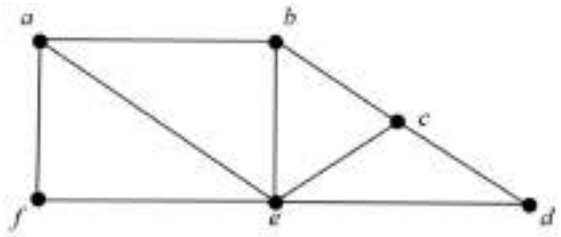

Figure 3. F^*= (*T, D*)

Let *C* = {*a, c*}⊆*T* be a parameter set. Define *I:* $C \to P(T)$ by $I(c) = \{t \in VT | d(c, t) < 2\}$, for all *c* ∈*C.* That is, *I(a)* = {*a, b, f, e*} and *I* (*c*) = {*b, c, d, e*}. Then, (I, C) is a soft set over T. Define *J:* $C \to P(D)$ by *J(c)* = $\{uv \in D | \{u, v\} \subseteq I(c)\}$ for all *c* ∈*C.* That is, *J* (*a*) = *{ab, af, ae, be, ef*} and *J (c)* = {*bc, cd, be, ed, ce*}. Then, (J, C) is a soft set over D. Thus, *B* (*a*) = (*I* (*a*), *J* (*a*)) and *B*(*c*) =

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(I (c), J (c)) are subgraphs of F^* as shown in figure 4. Hence, F= {B (a), B (c)} is a soft graph of F^*.

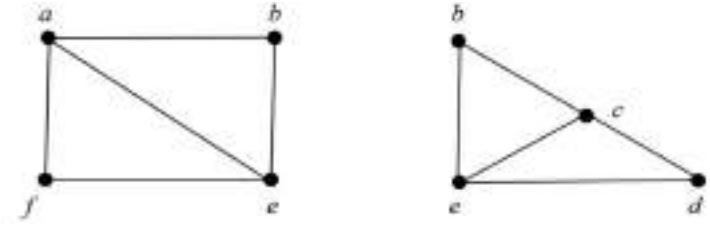

Figure 4. F= {B (a), B (c)}

3. S – Adjacency Matrix of Soft Graphs

Definition 3.1.

Let F^*= (T, D) be a graph and F = (F^*, I, J, C) be a soft graph of F^* represented by {$B(c)$: c∈C} where B (c) = (I (c), J(c)). Suppose that B (c) is a part of F for some c∈C having m vertices say $v_1, v_2 \ldots v_m$. Then, the s-adjacency matrix of this part B (c) denoted by SA[B (c)] is an m x m matrix [a_{ij}] where a_{ij} is the number of edges joining the vertex v_i to v_j in the part B (c).

Definition 3.2.

Let F^*= (T, D) be a graph and F = (F^*, I, J, C) be a soft graph of F^* represented by {$B(c)$: c∈C}. Then s-adjacency matrix of F denoted by SA(F) is given by SA(F) = {SA[B (c)] : c∈C} where SA[B (c)] denotes the s-adjacency matrix of the part B (c) of F.

Example 3.1.

Consider a graph F^*= (T, D) as shown in the following figure 5.

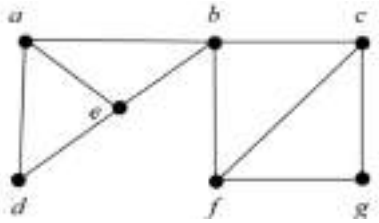

Figure 5. F= (T, D)

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Let $C = \{a, c\} \subseteq T$ be a parameter. Define $I: C \to P(T)$ by $I(c) = \{t \in T | d(c,t) < 2\}$, for all $c \in C$. That is, $I(a) = \{a, b, d, e\}$ and $I(c) = \{b, c, f, g\}$. Then, (I, C) is a soft set over T. Define $J: C \to P(D)$ by $J(c) = \{uv \in D | \{u, v\} \subseteq I(c)\}$ for all $c \in C$. That is, $J(a) = \{ab, ad, ae, be, de\}$ and $J(c) = \{bc, bf, cg, cf, fg\}$. Then, (J, C) is a soft set over D. Thus, $B(a) = (I(a), J(a))$ and $B(c) = (I(c), J(c))$ subgraphs of F^* as shown in figure 6. Hence, $F = \{B(a), B(c)\}$ is a soft graph of F^*.

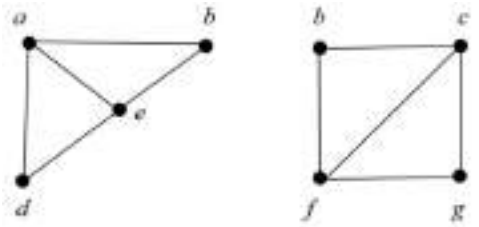

Figure 6. $F = \{B(a), B(c)\}$

S-Adjacency matrix of this soft graph is given by $SA(F) = \{SA[B(a)], SA[B(c)]\}$ where $SA[B(a)]$ and $SA[B(c)]$ are 4 x 4 matrices given below.

$$SA[B(a)] = \begin{array}{c} \\ a \\ b \\ d \\ e \end{array} \begin{array}{c} \begin{array}{cccc} a & b & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}, \quad SA[B(c)] = \begin{array}{c} \\ b \\ c \\ f \\ g \end{array} \begin{array}{c} \begin{array}{cccc} b & c & f & g \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

3.1. Properties of S – Adjacency Matrix of a Soft Graph

1. $SA[B(c)]$ contains only 0 and 1 as its elements, $\forall\ c \in C$.

2. For all $c \in C$, $SA[B(c)]$ is a square matrix of order m where m is the number of vertices in $I(c)$.

3. $SA[B(c)]$ is a symmetric matrix for all $c \in C$.

4. Diagonal entries of $SA[B(c)]$ are 0's since F is a soft graph of a simple graph F^*.

5. If n is any positive integer and matrix multiplication of n copies of $SA[B(c)]$ is given by $[SA[B(c)]]^n$, then the $(i,j)^{th}$ entry of $[SA[B(c)]]^n$ is the number of different $v_i v_j$ walks in the part $B(c)$ of length n where v_i and v_j are i^{th} and j^{th} vertices of $B(c)$.

4. S – Incidence Matrix of Soft Graphs

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Definition 4.1.

Let F^*= (*T, D*) be a graph and *F* = (F^*, *I, J, C*) be a soft graph of F^* given by {*B(c)* : c∈*C*} where *B* (*c*) = (*I* (*c*), *J*(*c*)). Suppose that *B* (*c*) is a part of *f* for some *c*∈*C* having *n* vertices say $v_1, v_2 \ldots v_n$ and *q* edges $e_1, e_2 \ldots e_q$. Then, the *s*-incidence matrix of this part *B* (*c*) denoted by *SI*[*B* (*c*)] is an *n* x *q* matrix [m_{ij}] where $(i,j)^{th}$ entry m_{ij} is the number of times the vertex v_i is incident with the edge e_j in the part *B(c)*. i.e. $m_{ij} = 0$ if v_i is not an end of e_j and $m_{ij} = 1$ if v_i is an end of the edge e_j.

Definition 4.2.

Let F^*= (*T, D*) be a graph and *F* = (F^*, *I, J, C*) be a soft graph of F^* given by {*B(c)* : c∈*C*}. Then *s*-incidence matrix of F denoted by *SI*(*F*) is given by *SI*(*F*) = {*SI*[*B(c)*] : c∈*C*} where *SI*[*B(c)*] denotes the *s*-incidence matrix of the part *B* (*c*) of *F*.

Example 4.1.

Consider a graph F^*= (*T, D*) given below.

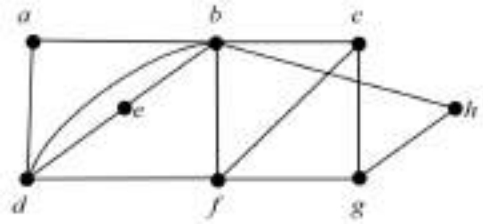

Figure 7. F^*= (*T, D*)

Let *C* = {*d, c*}⊆*T* be a parameter set. Define *I: C*→*P*(*T*) define by $I(c) = \{t \in T | cR't \Leftrightarrow d(c,t) \leq 1\}$ for all *c* ∈*C*. That is, *I (d)* = {*a, b, d, e, f*} and *I* (*c*) = {*b, c, f, g*}. Then, (I, C) is a soft set over T. Define *J: C*→*P (D)* defined by *J(t)* = $\{uv \in D | \{u,v\} \subseteq I(c)\}$ for all *c* ∈*C*. That is, *J*(*d*) = {*ab, ad, be, bd, bf, de, df*} and *J (c)* = {*bc, bf, cg, cf, fg*}. That is, (J, C) is a soft set over D. Thus, *B* (*d*) = (*I* (*d*), *J* (*d*)) and *B*(*c*) = (*I* (*c*), *J* (*c*)) are subgraphs of F^* as shown in figure 8.

Hence *F* = {*B* (*d*), *B* (*c*)} is a soft graph of F^*.

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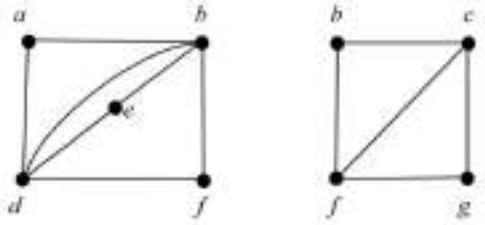

Figure 8. $F = \{B(d), B(c)\}$

S-incidence matrix of this soft graph is given by $SI(F) = \{SI[B(d)], SI[B(c)]\}$ where $SI[B(d)]$ is the 5 x 7 s-incidence matrix and $SI[B(c)]$ is the 4 x 5 s-incidence matrix of the parts $B(d)$ and $B(c)$ respectively which are given below.

$$SI[B(d)] = \begin{array}{c} \\ a \\ b \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{ccccccc} ab & ad & bd & be & bf & de & df \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array},$$

$$SI[B(c)] = \begin{array}{c} \\ b \\ c \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccc} bc & bf & cf & cg & fg \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}.$$

4.1. Properties of S – Incidence Matrix of a Soft Graph

1. $SI[B(c)]$ contains only 0 and 1 as its elements, $\forall\ c \in C$.

2. For all $c \in C$, $SI[B(t)]$ is a square matrix of order n x q where n is the number of vertices and q is the number of edges in $B(c)$.

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3. Sum of the elements in each column of $SI[B(c)]$ is 2.

4. Sum of the elements in the row corresponding to the vertex v (say) in the part $B(c)$ is the part degree $d(v)[B(c)]$ of that vertex v, $\forall\ c \in C$.

Conclusion

By utilizing the idea of a soft set in a graph, the soft graph was first introduced. Soft graph uses parameterization to create a number of descriptions of a complex relation that is represented by a graph. Due to its capacity to deal with the parameterization tool, theory of soft graphs is a branch of graph theory that is rapidly growing. We introduced many forms of matrices connected to soft graphs in this study and looked into some of their characteristics.

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CORDIAL AND PRODUCT CORDIAL LABELING OF CYCLE-BASED STRUCTURES

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Abstract

A graph G is said to be *cordial* if there exists a vertex labeling $f: V(G) \to \{0,1\}$ that induces an edge labeling $f^*: E(G) \to \{0,1\}$defined by$f^*(uv) = |f(u) - f(v)|$and satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Here $v_f(x)$ and $e_f(x)$are the number of vertices and edges respectively with label x (x = 0 or 1). A graph G is said to be the *product cordial* if there existsbinary vertex labeling $f$defined on the set $V(G)$ and inducing an edge labeling$f^*: E(G) \to \{0,1\}$defined by$f^*(uv) = f(u)f(v)$ and satisfying the same conditions. Herein, we demonstrate that the n-triangular snake, uniform theta and odd-quasi uniform theta graphs admit cordial and Mongolian tent, n-triangular snake and series parallel graphs admit product cordial.

Keywords: Cordial labeling, product cordial labeling, theta graph, series parallel graph, n-triangular snake graph.

AMS Subject Classification: 05C78

1. Introduction

Cordial labeling has customizable applications in network designing, scheduling, circuit designing, information network, coding theory and social network grouping. It is a remarkable tool for designing efficient systems in both practical and theoretical perspective. Understanding how cordial labeling operates on different graph families can provide intuition into general properties of graph and support in designing efficient algorithms for applications in computing, data processing and combinatorics. The product cordial is a versatile graph labeling technique for analysis of combination of network. Determining whether a graph is cordial can lead to develop efficient algorithm for networking proficiently [14].

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For a graph $G = (V(G), E(G))$, the notations $v_f(x)$ and $e_f(x)$ indicates the number of vertices and edges respectively, labeled with x ($x \in 0, 1$) in G. A binary labeling f defined on a vertex set $V(G)$ is cordial if f induces a map f^* defined by $f^*(uv) = |f(u) - f(v)|$ for every pair of vertices $u,v \in V(G)$. When the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1, and the number of edges labeled 0 and the number of edges labeled 1 differ at most by 1, then label f is cordial labeling of G [4].

A variation of cordial labeling, *product cordial* of a graph G introduced by Sundaram et al. [9] is a binary vertex labeling f defined on $V(G)$ which induces an edge function f^* defined by $f^*(uv) = f(u)f(v)$ and satisfying the same conditions for vertex and edge label counts. Refer articles [4], [5], [7], [6], [3], [8], [2], [1], [13], [11] for a survey on cordial and product cordial labeling.

In this article, cordial and product cordial labeling of graphs with cycles as subgraphs are studied.

2. *n*-Triangular Snake graph

A *triangular cactus* is a connected graph in which every block (maximal biconnected subgraph) is a triangle and a *triangular snake* is a special type of triangular cactus where the block-cut pointof the graph forms a path. A *n-triangular snake*$TS^n(m)$ is constructed from a path P_m and attaching n number of trianglessequentially. Figure 1 depicts an n-triangular snake graph$TS^n(m)$.

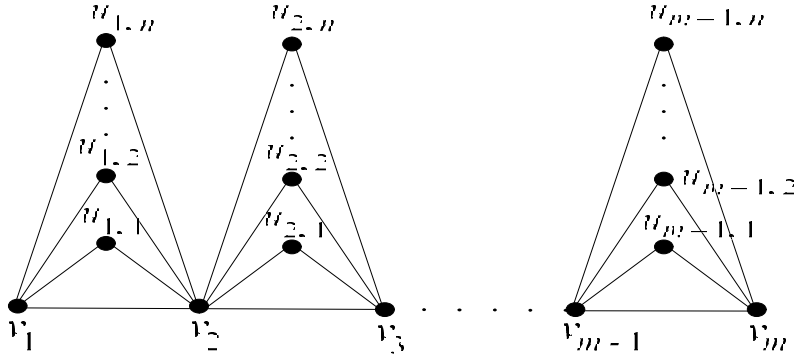

Figure 1: Generalized *n*-Triangular Snake $TS^n(m)$

Theorem 2.1. *An n-triangular snake graph G is cordial.*

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Proof. For $1 \leq i \leq m$, define a binary vertex labeling $f$on $V(G)$by,

$f(v_{4i}) = 0, f(v_{4i-1}) = 1, f(v_{4i-2}) = 1, f(v_{4i-3}) = 0.$

For $1 \leq i \leq n$and $1 \leq j \leq m-1$,

$$f(u_{i,2j}) = \begin{cases} 1, & if\ n\ is\ odd \\ 0 & if\ n\ is\ even \end{cases}$$

$$f(u_{i,2j-1}) = \begin{cases} 0, & if\ n\ is\ odd \\ 1 & if\ n\ is\ even \end{cases}$$

The assignment of labels to the edge *uv* in *G* is the absolute difference between $f(u)$ and $f(v)$.

In the instance that m and n are odd; m is odd and n is even; m, n are even, we observe that$v_f(0) = \left\lfloor \frac{(n+1)m-n}{2} \right\rfloor$and$v_f(1) = \left\lceil \frac{(n+1)m-n}{2} \right\rceil$.

But when m is even and n is odd,$v_f(0) = \left\lceil \frac{(n+1)m-n}{2} \right\rceil$and$v_f(1) = \left\lfloor \frac{(n+1)m-n}{2} \right\rfloor$.

Based on this observation, we have $\left| v_f(0) - v_f(1) \right| \leq 1$ and $\left| e_f(0) - e_f(1) \right| \leq 1$. Hence *G* is cordial.

Using the labeling $f$defined in Theorem 2.1 and considering the cases n is even, m is odd and both are odd, assign for each edge uv in *G* with label $f(u)f(v)$. We determine that

$v_f(0) = \left\lfloor \frac{(n+1)m-n}{2} \right\rfloor$and$v_f(1) = \left\lceil \frac{(n+1)m-n}{2} \right\rceil$.

Furthermore,$e_f(0) = e_f(1) = \frac{(2n+1)(m-1)}{2}$.

So that, $\left| v_f(0) - v_f(1) \right| \leq 1$ and $\left| e_f(0) - e_f(1) \right| \leq 1$.Thus, we have Theorem 2.2.

Theorem 2.2. *For n is even, m is odd, and both are odd, the graph G = TS^n(m) is product cordial.*

3. Uniform Theta Graph

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A *generalized theta* graph with l longitudes $L_1, L_2 \ldots L_l$ is said to be *uniform theta* graph if $|L_1| = |L_2| = \cdots = |L_l|$ where $|L_i|$ denotes the number of internal vertices of L_i. A *uniform theta* graph $\theta^*(l,k)$ with l longitudes and $|L_i| = k$ [12].

Theorem 3.1. *With l longitudes and k vertices, let G = θ*(l, k). Then G is cordial.*

Proof. Take $1 \leq i \leq k$. Define a binary labeling f on $V(G)$ by

$f(v_{4i}^1) = 1, f(v_{4i-1}^1) = 1, f(v_{4i-2}^1) = 0, f(v_{4i-3}^1) = 0$;

$f(v_{4i}^j) = 1, f(v_{4i-1}^j) = 1, f(v_{4i-2}^j) = 0, f(v_{4i-3}^j) = 0$ for $1 \leq j \leq l$, j is even;

$f(v_{4i}^j) = 0, f(v_{4i-1}^j) = 0, f(v_{4i-2}^j) = 1, f(v_{4i-3}^j) = 1$ for $1 \leq j \leq l$, j is odd;

$f(u_1) = 0$ if $\frac{k+1}{2}$ is even or odd and

$$f(u_2) = \begin{cases} 0, & if\ \dfrac{k+1}{2}\ is\ even \\ 1, & if\ \dfrac{k+1}{2}\ is\ odd \end{cases}$$

By definition of cordial labeling, each edge uv in G is assigned a label considering the absolute difference between the labels of its endpoints u, v.

Suppose that both l and k can be odd, or l can be even and k is odd.

From the definition of f, we deduce that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| = 0$. Thus, G admits cordial labeling.

The graph $\theta^*(6,7)$ with cordial labels is shown in Figure 2.

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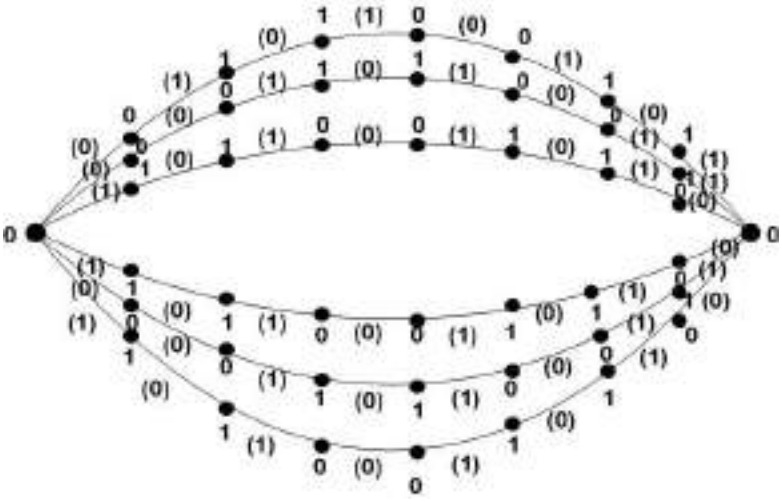

Figure 2: Cordial labeling of θ^*(6,7)

4. Odd Quasi- Uniform Theta graph

A *generalized theta* graph G with *l* longitudes is said to be to be *quasi-uniform theta* graph if $|L_1| = |L_2| = \cdots = |L_{l-1}| \geq |L_l|$. A quasi-uniform theta graph is said to be odd if $|L_{l-1}| \geq |L_l|$ is odd [12].

Theorem 4.1. *An odd quasi-uniform theta graph G is cordial.*

Proof. We check for the instances, when both k and l are odd; k is odd while l is even.

For each i, $1 \leq i \leq k$, define a function f that labels each vertex of *G* with either 0 or 1 by

$f(v_{4i}^j) = 0, f(v_{4i-1}^j) = 1, f(v_{4i-2}^j) = 1, f(v_{4i-3}^j) = 0$ for $1 \leq j \leq l$, j is odd;

$f(v_{4i}^j) = 1, f(v_{4i-1}^j) = 0, f(v_{4i-2}^j) = 0, f(v_{4i-3}^j) = 1$ for $1 \leq j \leq l$, j is even;

$f(u_1) = 0$ if $\frac{k+1}{2}$ is even or odd and

$$f(u_2) = \begin{cases} 0, & if\ \dfrac{k+1}{2}\ is\ even \\ 1, & if\ \dfrac{k+1}{2}\ is\ odd \end{cases}$$

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Now f induces an edge labeling function f^* on $E(G)$ defined by $|f(u) - f(v)|$ for every pair $u, v \in V(G)$. Utilizing the map f, we obtain that $v_f(0) = \left\lfloor \frac{lk+1}{2} \right\rfloor$ and $v_f(1) = \left\lceil \frac{lk+1}{2} \right\rceil$ and $e_f(0) = \left\lceil \frac{l(k+1)-1}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{l(k+1)-1}{2} \right\rfloor$.

This implies that the absolute difference between the number of vertices with label 0 and 1 is less than or equal to 1. The same is true for the absolute difference between the number of edges with labeled 0 and 1.

Thus, the labeling *f* satisfies the conditions of cordial labeling and the graph *G* is cordial.

The graph shown in Figure 3 is $\theta^*(6,7)$ with its cordial labels.

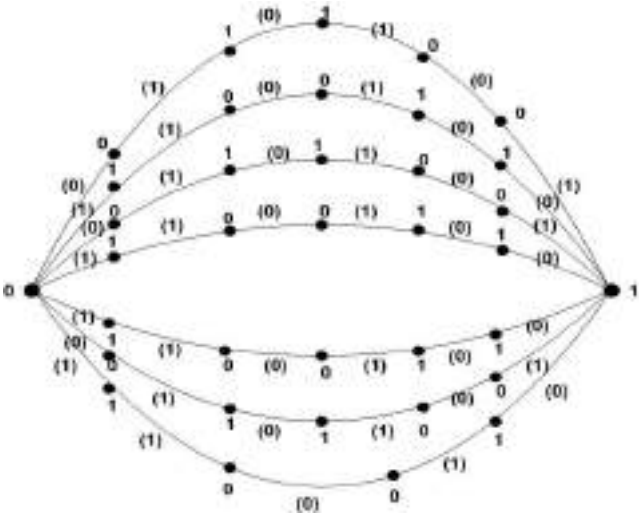

Figure 3: Cordial labeling of odd quasi-uniform theta θ^*(6,7)

5. Mongolian Tent graph

A *Mongolian TentM*(*m,n*) is a graph obtained from $P_m \times P_n$ by inserting an additional vertex and joining every vertex in the first row of $P_m \times P_n$ to the additional vertex. The additional vertex is represented as v_1 and the vertices of $P_m \times P_n$ are represented as $v_2, v_3, \ldots, v_{mn+1}$.

Theorem 5.1. *If G = M(m, n) with m, n ≥ 4, then G admits product cordial.*

Proof. Define a map f on $V(G)$ that assigns to every vertex of *G* 0 or 1 by

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$$f(v_i) = \begin{cases} 1, & 1 \le i \le \left\lceil \frac{mn+1}{2} \right\rceil \\ 0, & \left\lceil \frac{mn+1}{2} \right\rceil < i \le mn+1 \end{cases}$$

The edges v_iv_j in G for $1 \le i, j \le mn$ and $i \ne j$ are assigned with the label $f(v_i)f(v_j)$.

Using the definition of f, the implication for the cases m and n can be such that m is odd and n is even, m is even and n is odd, or both m, n are even is that

$v_f(0) = \left\lfloor \frac{mn+1}{2} \right\rfloor$ and $v_f(1) = \left\lceil \frac{mn+1}{2} \right\rceil$.

Further, $e_f(0) = \frac{2mn-}{2}$ and $e_f(1) = \frac{2mn-n}{2}$.

Thus, the conditions of product cordial labeling are satisfied and G is a product cordial.

6. Series Parallel graph

A *series parallel* graph is a *chain* graph in which each block is a *generalized theta graph*. The series parallel graph is denoted by $sp(l,k,r)$ for $l,k \ge 3$, $r \ge 2$ where l,k,r are the number of *longitudes*, *vertices* and *blocks respectively* [10].

Theorem 6.1. If *G is a series parallel graph, then G is product cordial.*

Proof. Consider the cases where l is even, then m and n can be both odd, one odd and one even, or both even. Define a map f from the vertex set of G to the set $\{0,1\}$ by

$$f(x_{m,i}) = 1 \text{ for } 1 \le i \le \left\lceil \frac{l(mr+1)+1}{2} \right\rceil, f(y_{m,i}) = 0 \text{ for } i > \left\lceil \frac{l(mr+1)+1}{2} \right\rceil$$

$$f(z_i) = 1 \text{ for } 1 \le i \le \left\lceil \frac{l+1}{2} \right\rceil, f(z_i) = 0 \text{ for } \left\lceil \frac{l+1}{2} \right\rceil \le i \le l.$$

It can be inferred that, $v_f(0) = \left\lfloor \frac{r(lk+1)+1}{2} \right\rfloor$ and $v_f(1) = \left\lceil \frac{r(lk+1)+1}{2} \right\rceil$.

Also, $e_f(0) = \frac{rl(k+1)}{2}$ and $e_f(1) = \frac{rl(k+1)}{2}$.

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The values $v_f(0), v_f(1), e_f(0), e_f(1)$ points out that the absolute difference between the number of vertices with label 1 and the number of vertices with label 0 is 1. In the same way, the absolute difference between the number of edges with the label 1 and the number of edges with the label 0 is 0. Thus, *G* admits product cordial.

Product cordial labeling of *sp*(4, 5, 2) is illustrated in Figure 4.

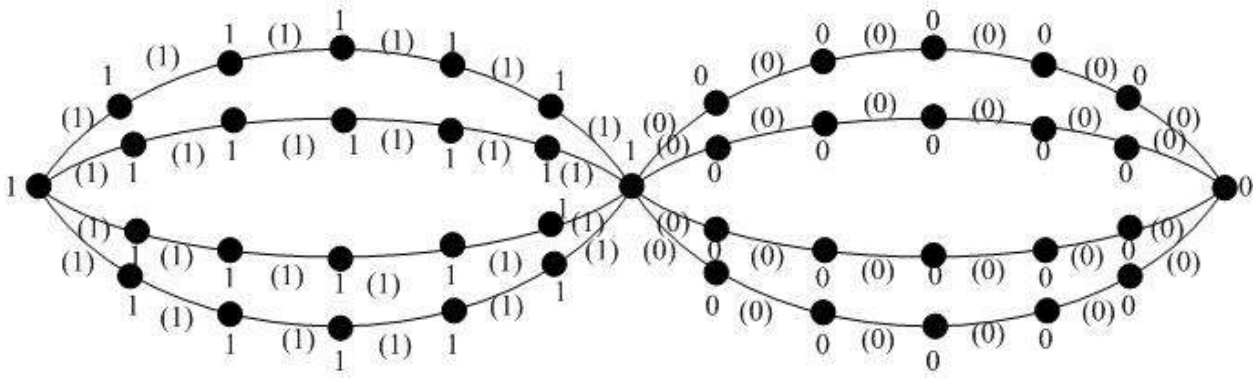

Figure 4: Product Cordial labeling of *sp*(4,5,2)

Conclusion

In this article, we report that the graphs *n*-triangular snake, uniform theta and odd-quasi uniform theta are cordial and the graphs Mongolian tent, *n*-triangular snake and series parallel are product cordial.

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ON FAIR DETOUR DOMINATION POLYNOMIAL OF GRAPHS

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Abstract

A Fair detour dominating (FDD) set F ⊆ V (G) is a set that is detour dominating and any pair of vertices not in F has same number of neighbours in F. A Fair detour domination polynomial of a graph G is defined as $F\gamma_D(G,x) = \sum_{i=f\gamma_d}^{|V(G)|} f(G,i)x^i$ where f(G,i) is the number of FDD sets of cardinality i. In this work, we have investigated FDD polynomial of some standard graphs.

Keywords: Dominating Polynomial, Detour domination, Fair domination,

Fair detour domination polynomial

AMSC 2020: 05C12, 05C69

1. Introduction

Throughout this paper, we consider G=(V,E) to be the finite simple connected graph with atleast two vertices. We refer to [1, 2] for basic definitions and terminologies.

The Detour distance D(x,y) is the length of the longest path joining two vertices x and y. A x − y detour is x − y path of length D(x.y)[3]. The neighbours of a vertex u, N(u)={w ∈ V (G)/uw ∈ E(G)}.

Definition 1.1 [4, 5] A set F ⊆ V (G) is said to be a detour dominating set ($\gamma_d$set) if F is the set with minimum cardinality among those sets that satisfies the subsequent two conditions

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i. **Detour set**. If all the vertices of G lies on a detour joining some pairs of vertices of F.

ii. **Dominating set.** If every vertex of G is either in F or neighbour to a vertex in F.

Definition 1.2 A set F ⊆ V (G) is said to be a Fair detour dominating (FDD) set

(i) If F is γ_d- set.

(ii) Fair dominating set[6]. If for any pair of vertices u,w not in F, the number of neighbours of these two vertices in F are the same. To be precise, $|N(u) \cap F| = |N(w) \cap F|$, u,w not in F.

For $n \geq 1$ and u not in F, if $|N(u) \cap F| = n$, then F is a n-FDD set. Among

FDD sets, the set with least number of vertices is defined to be the $f\gamma_d$-set and its cardinality is defined to be fair detour domination number ($f\gamma_d(G)$).

Theorem 1.3 [5] Every Pendant vertex must lie in every detour dominating set.

Theorem 1.4 If the collection of all pendant vertices of a graph G forms a FDD set then it is the unique minimum FDD set.

2. Fair Detour Domination Polynomial

Definition 2.1 A FDD polynomial of G is defined as $F\gamma_D(G,x) = \sum_{i=f\gamma_d}^{|V(G)|} f(G,i)x^i$ where f(G,i) is the number of FDD sets of cardinality i.

Example 2.2 For the graph in Fig. 2.1, the vertex set V(G) forms $f\gamma_d$ set of cardinality 6 and so f(G,6) = 1. There are six $f\gamma_d$ sets of cardinality 5. They are

$\{v_1,v_2,v_3,v_4,v_5\},\{v_1,v_2,v_3,v_4,v_6\},\{v_1,v_2,v_3,v_5,v_6\},\{v_1,v_2,v_4,v_5,v_6\},\{v_1,v_3,v_4,v_5,v_6\}$,

$\{v_2,v_3,v_4,v_5,v_6\}$. The $f\gamma_d$ sets of cardinality 4 are $\{v_1,v_2,v_3,v_4\},\{v_1,v_2,v_3,v_5\}$,

$\{v_1,v_3,v_4,v_5\},\{v_1,v_3,v_4,v_6\},\{v_1,v_3,v_5,v_6\},\{v_2,v_4,v_5,v_6\}$. The $f\gamma_d$ sets of cardinality 3 are $\{v_1,v_3,v_5\},\{v_1,v_2,v_4\},\{v_1,v_2,v_5\},\{v_1,v_2,v_6\},\{v_1,v_3,v_4\},\{v_2,v_3,v_4\},\{v_3,v_4,v_5\}$, $\{v_3,v_4,v_6\},\{v_1,v_5,v_6\}$. Hence $F\gamma_D(G,x) = x^6 + 6x^5 + 6x^4 + 9x^3$.

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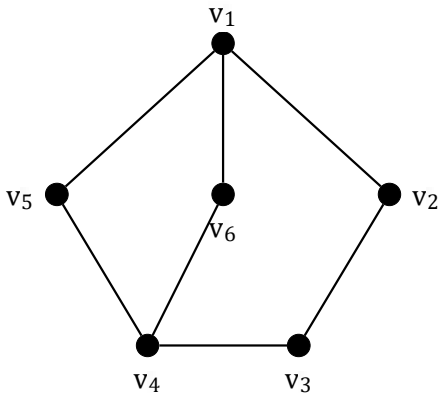

Fig. 2.1: Graph with $f\gamma_d = 3$

Observation 2.3 For a connected graph G of order n

1)The coefficient of x^n is always 1 and hence it is a monic polynomial.

2)There will be no constant and linear term.

3)Let G be a graph with p end vertices and q internal vertices such that p+q = n

then

(i) If $p \neq 0$ then $f(G,n-1) = n - p$.

(ii) If $p = 0$ then $f(G,n-1) = n$

Theorem 2.4 For any complete graph $K_n, n \geq 2$, $F\gamma_D(K_n,x) = (1+x)^n - nx - 1$.

Proof: For a complete graph, $f\gamma_d(K_n) = 2$ for all $n \geq 2$. Therefore, every subset of $V(K_n)$ with atleast 2 vertices forms a FDD set. That is, every vertex of K_n will be located on a detour connecting pair of vertices of FDD set of cardinality $i \geq 2$ and every vertex not in FDD set of cardinality $i \geq 2$ has equal number of neighbours in the FDD set. Hence there are $\binom{n}{i}$ FDD sets of cardinality $i, 2 \leq i \leq n$.

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$$\begin{aligned} F\gamma_D(K_n, x) &= \sum_{i=2}^{n} f(K_n, i)x^i \\ &= \sum_{i=2}^{n} \binom{n}{i} x^i \\ &= \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots + \binom{n}{n}x^n \\ &= (1+x)^n - nx - 1 \end{aligned}$$

Theorem 2.5 For any star graph $K_{1,n}$, $n \geq 2$, $F\gamma_D(K_{1,n},x) = x^n(1+ x)$.

Proof: Let {u} be the central vertex and F be the n end vertices adjacent to u in a star graph $K_{1,n}$. Every pendant vertex must belong to FDD-set. Hence these n pendant vertices of $K_{1,n}$ forms a detour dominating set and u has n neighbours in F. Hence F is the minimum FDD set of cardinality n. The vertex set of $K_{1,n}$ is FDD set of cardinality n +1. Hence

$$\begin{aligned} F\gamma_D(K_{1,n}, x) &= \sum_{i=2}^{n} f(K_{1,n}, i)x^i \\ &= x^n + x^{n+1} \\ &= x^n(1+x) \end{aligned}$$

Definition 2.6 Each edge of the star graph $K_{1,n}$ is subdivided to obtain the Spider graph $S_{1,n}$. One or more but not all edges of the star graph are subdivided to obtain the the Wounded Spider graph $W_{1,n}$

Lemma 2.7 The coefficients of spider graph in the FDD polynomial are given by i) $f(S_{1,n}, n+1) = \binom{n+1}{1}$

ii) $f(S_{1,n}, 2n) = \binom{n+1}{n}$

iii) $f(S_{1,n}, n+i+1) = \binom{n}{i}, 1 \leq i \leq n-2$

Proof:

(i) The collection of all end vertices along with any other vertex v of the spider

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graph forms fγ_d of cardinality n +1. The vertex v can be chosen in $\binom{n+1}{1}$.

(ii) Any subset F of V ($S_{1,n}$) of cardinality 2n forms a FDD set. Since the pendant vertices of cardinality n will always be in FDD set, the other n vertices can be chosen from the remaining n +1 internal vertices. Hence f($S_{1,n}$,2n) =

$\binom{n+1}{n}$

(iii) Let F be the FDD set of cardinality n + i +1,1 ≤ i ≤ n − 2. All the end vertices and the vertex with maximum degree will always belong to F. Then the remaining i vertices can be chosen from the n vertices that are used to subdivide the n edges of $K_{1,n}$ to obtain the spider graph. Hence $f(S_{1,n}, n+i+1) = \binom{n}{i}, 1 \leq$i≤ n − 2

Theorem 2.8

For the spider graph $S_{1,n}$ with 2n +1 vertices and 2n edges, Fγ_D($S_{1,n}$,x)

$$= \binom{n+1}{1}x^{n+1} + \binom{n}{1}x^{n+2} + \binom{n}{2}x^{n+3} + \ldots + \binom{n}{n-2}x^{n+(n-1)} + \binom{n+1}{n}x^{2n} + x^{2n+1}$$

Proof: The proof follows from the above lemma.

Theorem 2.9 Let i, 1 ≤ i ≤ n − 1 be the number of vertices used to subdivide the edges of the star graph $K_{1,n}$ to obtain the wounded spider graph $W_{1,n}$. The wounded spider graph has n + i +1 vertices and n + i edges.

i)If $i = 1$ *then* $F\gamma_D(W_{1,n}, x) = 2x^{n+i} + x^{n+1+i}$

ii)If $2 \leq i \leq n-2$ *then* $F\gamma_D(W_{1,n}, x) = x^{n+1} + \binom{i}{1}x^{n+2} + \ldots + \binom{i}{i-2}x^{n+(i-1)} + \binom{i+1}{i}x^{n+i} + x^{n+1+i}$

iii)If $i = n-1$ *then* $F\gamma_D(W_{1,n}, x) = x^n + x^{n+1} + \binom{i}{1}x^{n+2} + \ldots + \binom{i}{i-2}x^{n+(i-1)} + \binom{i+1}{i}x^{n+i} + x^{n+1+i}$

Proof: The vertex set of $W_{1,n}$ forms a FDD set of cardinality n + i +1.

i) Let i = 1. Let v be the vertex used to subdivide a edge of the star graph and u be the head vertex of the star graph. There are n end vertices and n−i of these end vertices are adjacent to the head vertex

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and one end vertex is adjacent to v. Let F be the set of all these end vertices. Then F is a detour dominating set but i = 1 = |N(v)∩F|≠ |N(u)∩F| = n−1 = n−i. Hence there will be no FDD

set of cardinality n. Now, F along with u forms a 2- FDD set and F along with v forms a n-FDD set and so there are two FDD sets of cardinality n +1 = n + i. Hence $F\gamma_D(W_{1,n},x) = 2x^{n+i}+x^{n+1+i}$

(ii) Let $2 \leq i \leq n-2$. Let F be the set consists of n end vertices and the head vertex of the star graph. Then F forms the unique $2-f\gamma_d$-set of cardinality n+1. Now, F along with any other vertex from the remaining i vertices again forms a 2-FDD set. Hence $f(W_{1,n}, n+1+j) = \binom{i}{j}, j = 1,2..i-2$. Let the set F consists of n end vertices and any i number of vertices chosen from the set D where D is the set of cardinality i +1 that consists of the head vertex and the i vertices used to subdivide the edges of the star graph. Then F forms a FDD set of cardinality n + i. This implies

$$\begin{aligned} F\gamma_D(W_{1,n},x) &= \sum_{i=n+1}^{n+1+i} f(W_{1,n},i)x^i \\ &= x^{n+1} + \sum_{j=1}^{i-2}\binom{i}{j}x^{n+1+j} + \binom{i+1}{i}x^{n+i} + x^{n+1+i} \\ &= x^{n+1} + \binom{i}{1}x^{n+2} + \ldots + \binom{i}{i-2}x^{n+(i-1)} + \binom{i+1}{i}x^{n+i} \\ &\quad + x^{n+1+i} \end{aligned}$$

iii)Let i = n − 1. The set of all end vertices forms a unique $f\gamma_d$-set of cardinality n. By a similar argument as in the above case we get $F\gamma_D(W_{1,n},x)$ =

$x^n + x^{n+1} + \binom{i}{1}x^{n+2} + \ldots + \binom{i}{i-2}x^{n+(i-1)} + \binom{i+1}{i}x^{n+i} + x^{n+1+i}$.

Definition 2.10 The Friendship graph F_n is obtained by taking n copies of $K_1 + P_2$ with a common vertex $K_1 = \{u\}$.

Theorem 2.11 For the friendship graph,

$F\gamma_D(F_n,x) = 2^n x^{n+1} + \sum_{i=1}^{n-2}\binom{n}{i}2^{n-i}x^{n+i+1} + (2n+1)x^{2n} + x^{2n+1}$.

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Proof. Since the common vertex and a vertex from n copies of P_2 forms a minimum FDD set, we have $f\gamma_d(F_n) = n + 1$. There are $\binom{2}{1}^n$ choices for this set. Also, all but one vertex of the friendship graph forms FDD set of cardinality 2n and there are 2n + 1 ways for such a set. The vertex set forms FDD set of cardinality 2n+1. For $1 \leq i \leq n-2$, we have to fix n-i of n paths P_2 and have to choose one of the two vertices in the remaining i paths P_2 and the common vertex to form FDD set of cardinality n + i +1.

$$
\begin{aligned}
F\gamma_D(F_n, x) &= \sum_{i=n+1}^{2n+1} f(F_n, i)x^i \\
&= \binom{2}{1}^n x^{n+1} + \sum_{i=1}^{n-2} \binom{n}{i}\binom{2}{1}^{n-i} x^{n+i+1} + (2n+1)x^{2n} + x^{2n+1} \\
&= 2^n x^{n+1} + \binom{n}{1}(2^{n-1})x^{n+2} + \ldots + \binom{n}{n-2}(2^2)x^{n+(n-2)} + \\
&\qquad (2n+1)x^{2n} + x^{2n+1}.
\end{aligned}
$$

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PAIR QUOTIENT CORDIAL LABELING OF SOME TRIANGULAR SNAKE GRAPHS

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Abstract

Consider a (p,q) graph $G = (V,E)$. Define $\gamma = \begin{cases} \frac{p}{2}, & if\ p\ is\ even \\ \frac{p-1}{2}, & if\ p\ is\ odd \end{cases}$ and

$A = \{\pm 1, \pm 2, \pm 3, \ldots, \pm\gamma\}$ called the set of labels. Consider a map $f : V \to A$. When γ is even, assign different labels in L to the p elements of V and when γ is odd, assign different labels in A to the $p-1$ elements of V and repeat a label for the lone vertex. The above labeling is said to be a **pair quotient cordial labeling** if for each edge uv of G, there exists a labeling $\left|\frac{f(u)}{f(v)}\right|$ or $\left|\frac{f(v)}{f(u)}\right|$ based on $f(u) \geq f(v)$ or $f(v) > f(u)$ such that $\left|\delta_{f_1} - \delta_{f_1^c}\right| \leq 1$ where δ_{f_1} and $\delta_{f_1^c}$ denote the number of edges labeled with 1 and number of edges not labeled with 1 respectively. A graph which admits pair quotient cordial labeling is called **a pair quotient cordial graph**. In this paper, we look into the behaviour of pair quotient cordial labeling of triangular snake graphs.

Keywords: Triangular snake, Alternate Triangular snake, Double Triangular snake, Alternate Double Triangular snake and Pair Quotient Cordial.

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1. Introduction

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In 1987, the concept of cordial labeling was introduced by I. Cahit [2]. This concept was evolved into quotient cordial labeling in 2016 and pair difference cordial labeling in 2021 [5]. In this work, by considering finite, simple and undirected graphs. We examine the pair quotient cordial labeling behaviour of Triangular snake, Alternate Triangular snake, Double Triangular snake, Alternate Double Triangular snake graphs.

2. Preliminaries

Definition 2.1. [7]

The Triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3. *Let* $V(T_n) = \{x_i, y_j / 1 \le i \le n+1, 1 \le j \le n\}$ and $E(T_n) = \{x_i x_{i+1} / 1 \le i \le n\} \cup \{x_i y_i, x_{i+1} y_i / 1 \le i \le n\}$. There are $2n + 1$ vertices and $3n$ edges.

Definition 2.2. [7]

The Alternate Triangular snake $A(T_n)$ is obtained from the path $x_1 x_2 \ldots x_n$ by joining x_i and x_{i+1} (alternatively) to new vertex y_i. (i.e) Every odd edge of a path is replaced by C_3. The edge $x_1 x_2$ lies on the triangle and the edge $x_{2n-1} x_{2n}$ lies on the triangle. Let $V(A(T_n)) = \{x_i, y_j / 1 \le i \le 2n, 1 \le j \le n\}$ and $E(A(T_n)) = \{x_i x_{i+1}, x_{2i} y_j, x_{2i-1} y_j / 1 \le i \le 2n, 1 \le i \le n\}$. There are $3n$ vertices and $4n - 1$ edges.

Definition 2.3. [8]

A Double Triangular Snake DT_n is the graph obtained from the path $x_1 x_2 \ldots x_n x_{n+1}$ by joining x_i, x_{i+1} with the new vertices y_i and z_i, $1 \le i \le n$.

Definition 2.4. [8]

An Alternate Double triangular snake $A(DT_n)$ is the graph obtained from the path $x_1 x_2 \ldots x_{2n}$ by joining x_i x_{i+1} (alternatively) with two new vertices y_i and z_i, $1 \le i \le n$.

3. Pair Quotient Cordial Labeling

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Definition 3.1. [6]

Consider a (p, q) graph $G = (V, E)$. Define $\gamma = \begin{cases} \frac{p}{2}, & if\ p\ is\ even \\ \frac{p-1}{2}, & if\ p\ is\ odd \end{cases}$ and

$A = \{\pm 1, \pm 2, \pm 3, \ldots, \pm\gamma\}$ called the label set. Consider a map $f : V \to A$. When γ is even, assign different labels in A to the p elements of V and when γ is odd, assign different labels in A to the $p-1$ elements of V and repeat a label for the lone vertex. The above labeling is said to be a **pair quotient cordial labeling** if for each edge uv of G, there exists a labeling $\left|\frac{f(u)}{f(v)}\right|$ or $\left|\frac{f(v)}{f(u)}\right|$ based on $f(u) \geq f(v)$ or $f(v) > f(u)$ such that $\left|\delta_{f_1} - \delta_{f_1^c}\right| \leq 1$ where δ_{f_1} and $\delta_{f_1^c}$ denote the number of edges labeled with 1 and number of edges not labeled with 1 respectively. A graph which admits pair quotient cordial labeling is called a **pair quotient cordial graph**.

Theorem 3.1.

The Triangular Snake graph T_n is pair quotient cordial only if $n = 3, 4, 5$.

Proof:

Consider the vertex set and edge set as in definition 2.1.

Case (i): $n = 1$.

Since $T_1 \cong C_3$[6], it is not pair quotient cordial.

Case (ii): $n = 2$.

Assign the labels x_1, x_2 as $1, 2$, y_1, y_2 as $-1, -2$ and assign the label x_3 as 2.

Then $\delta_{f_1} = 4$, $\delta_{f_1^c} = 2$

$\Rightarrow \left|\delta_{f_1} - \delta_{f_1^c}\right| = 2 > 1$, a contradiction.

Case (iii): $n \geq 3$.

Define $f(x_i) = i, 1 \leq i \leq n, f(y_i) = -i, 1 \leq i \leq n$ and $f(x_{n+1}) = n$.

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Here $\delta_{f_1} = n+2$, $\delta_{f_1^c} = 3n-(n+2) = 2n-2$

$\Rightarrow \ \left|\delta_{f_1} - \delta_{f_1^c}\right| = |n+2-2n+2| = n-4.$

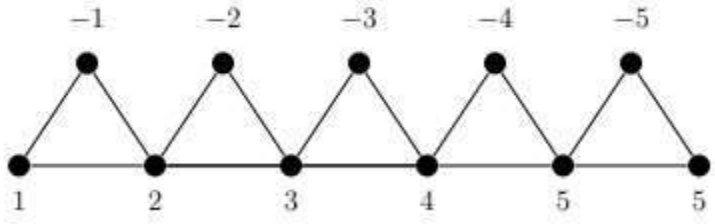
Suppose T_n is Pair Quotient cordial, then $n - 4 \leq 1$.

$$\Rightarrow n \leq 5$$

Hence the graph T_n is pair quotient cordial only if $3 \leq n \leq 5$.

Illustration:

The pair quotient cordial labeling of T_5.



Theorem 3.2.

The alternate triangular snake graph $A(T_n)$ is pair quotient cordial only if $n = 2, 3, 5$.

Proof:

Consider the vertex set and edge set as in definition 2.2.

Case (i): $n = 1$.

Since $A(T_1) \cong C_3[6]$, it is not pair quotient cordial.

Case (ii): n is odd and $n > 1$.

Define $f(x_{4i-3}) = 3i-2, 1 \leq i \leq \frac{n+1}{2}, f(x_{4i-2}) = 3i-1, 1 \leq i \leq \frac{n-1}{2}$,

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$f(x_{4i-1}) = -3i+1, 1 \le i \le \frac{n-1}{2}$, $f(x_{4i}) = -3i, 1 \le i \le \frac{n-1}{2}$,

$f(x_{2n}) = 3\left(\frac{n+1}{2}\right) - 2, f(y_{2i-1}) = 2-3i, 1 \le i \le \frac{n+1}{2}$ and $f(y_{2i}) = 3i, 1 \le i \le \frac{n-1}{2}$.

Then $\delta_{f_1} = \frac{3(n+1)}{2}$ and $\delta_{f_1^c} = 4n-1-\frac{3n+3}{2} = \frac{5(n-1)}{2}$

$\Rightarrow \left|\delta_{f_1} - \delta_{f_1^c}\right| = \left|\frac{3(n+1)}{2} - \frac{5(n-1)}{2}\right| = n-4.$

Suppose $A(T_n)$ is pair quotient cordial.

Then $n-4 \le 1 \Rightarrow n \le 5$.

Since n is odd and $n > 1$, it is true for $n = 3, 5$.

Case (iii): n is even.

Define $f(x_{4i-3}) = 3i-2, 1 \le i \le \frac{n}{2}, f(x_{4i-2}) = 3i-1, 1 \le i \le \frac{n}{2}$,

$f(x_{4i-1}) = -3i+1, 1 \le i \le \frac{n}{2}$, $f(x_{4i}) = -3i, 1 \le i \le \frac{n}{2}$,

$f(y_{2i-1}) = 2-3i, 1 \le i \le \frac{n}{2}$ and $(y_{2i}) = 3i, 1 \le i \le \frac{n}{2}$.

Then $\delta_{f_1} = \frac{3n}{2}$ and $\delta_{f_1^c} = 4n-1-\frac{3n}{2} = \frac{5n-2}{2}$

$\Rightarrow \left|\delta_{f_1} - \delta_{f_1^c}\right| = \left|\frac{3n}{2} - \frac{5n-2}{2}\right| = n-1.$

Suppose $A(T_n)$ is pair quotient cordial.

Then $n-1 \le 1 \Rightarrow n \le 2$.

Since n is even, it is true for $n = 2$.

Illustration:

The pair quotient cordial labeling of $A(T_3)$.

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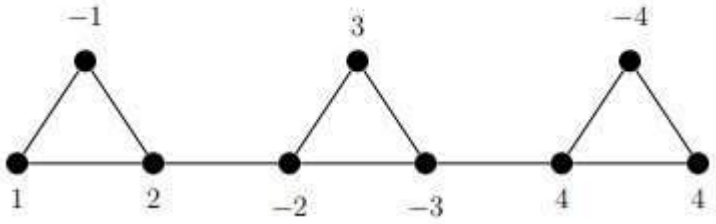

Theorem 3.3.

The Double Triangular Snake graph DT_n is not pair quotient cordial for $n > 1$.

Proof:

Case (i): $n = 1$.

Label the vertices x_1, x_2 as $1, 2$ and y_1, z_1 as $-1, -2$.

Here $\delta_{f_1} = 2$ and $\delta_{f_1^c} = 3$.

Then $\left|\delta_{f_1} - \delta_{f_1^c}\right| = 1$.

$\therefore DT_1$ is pair quotient cordial.

Case (ii): n is odd and $n > 1$.

Define $f(x_i) = i, 1 \leq i \leq n+1, f(y_i) = -i, 1 \leq i \leq n$ and

$f(z_{2i-1}) = -(i+n), 1 \leq i \leq \frac{n+1}{2}, f(z_{2i}) = i+n, 1 \leq i \leq \frac{n-1}{2}$.

Here $\delta_{f_1} = n+1$ and $\delta_{f_1^c} = 5n - n - 1 = 4n - 1$.

Then $\left|\delta_{f_1} - \delta_{f_1^c}\right| = 4n - 1 > 1$, a contradiction.

Therefore, DT_n is not pair quotient cordial for n is odd and $n > 1$.

Case (iii): n is even.

Assign the label for x_i and y_i as in case (ii).

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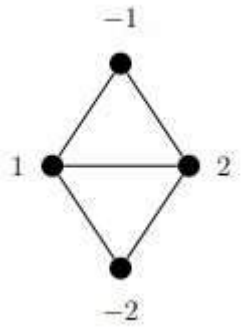
Then define $f(z_{2i-1}) = -(i+n), 1 \leq i \leq \frac{n}{2}, f(z_{2i}) = i+n, 1 \leq i \leq \frac{n-2}{2}$ and $f(z_n) = 1$.

Here $\delta_{f_1} = n+2$ and $\delta_{f_1^c} = 5n - n - 2 = 4n - 2$.

Then $\left|\delta_{f_1} - \delta_{f_1^c}\right| = |n+2-4n+2| = 3n-4 > 1$, a contradiction.

Therefore, DT_n is not pair quotient cordial for n is even.

Illustration: The pair quotient cordial labeling of DT_1.



Theorem 3.4.

The Alternate Double Triangular Snake graph $A(DT_n)$ is not pair quotient cordial for $n > 1$.

Proof:

Case (i): $n = 1$.

Define $f(x_1) = 1, f(x_2) = 2, f(y_1) = -1, f(z_1) = -2$.

Here $\delta_{f_1} = 2$ and $\delta_{f_1^c} = 3$.

Then $\left|\delta_{f_1} - \delta_{f_1^c}\right| = 1$.

$\therefore A(DT_1)$ is pair quotient cordial.

Case (ii): $n > 1$.

Define $f(x_i) = i, 1 \leq i \leq 2n, f(y_i) = -(2i-1), 1 \leq i \leq n$ and $f(z_i) = -2i, 1 \leq i \leq n$.

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Here $\delta_{f_1} = 2n$ and $\delta_{f_1^c} = 6n - 1 - 2n = 4n - 1$.

Then $\left|\delta_{f_1} - \delta_{f_1^c}\right| = |2n - 4n + 1| = 2n - 1 > 1$, a contradiction.

Hence $A(DT_n)$ is not pair quotient cordial for $n > 1$.

Conclusion:

The behavior of Triangular Snake, Alternate Triangular Snake, Double Triangular Snake and Alternate Double Triangular snake graphs was examined in this article.

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TOPOLOGICAL INDICES FOR COPRIME GRAPH OF GROUPS

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Abstract

Let G be a finite group with identity e. The coprime of G, Γ_G is a graph with G as the vertex set and two distinct vertices u and v are adjacent if and only if (|u|,|v|) = 1. In this paper, we investigate the topological indices for coprime graph of groups such as Wiener index, Harrary index, first Zagreb index.

Keywords: Coprime graph, Wiener, Zagreb

AMS subject classification (2010): 05C25,05C35,05C69

1 Introduction

Algebraic graph theory has close deals with group theory. Algebraic graphs can be constructed in many ways, some special graphs are Cayley graph, Zero divisor graph, Annihilating ideal graph, etc. Here we investigate about coprime graph of groups. Paul Erdos and N. Sarkozy [2] was constructed the coprime graph in 1997. Let G be a finite group with identity e. The coprime of G, Γ_G is a graph with G as the vertex set. Adjacent is exist in Γ_G, whenever order of the two distinct vertices are relatively coprime.

In Chemistry, Topological indices have been applied extensively. The study of topological indices is a fascinating field that is receiving a lot of interest. Several research works have concentrated on different topological indices. For instance, Alimon et al. shown the Szegeb index and Wiener index of the coprime graph of dihedral group in 2020 [1]. Zahidah et al. discovered some topological indices of coprime graphs of generalized quaternion groups in 2021 [7] and Husni et al. explored the Harmonic index and Gutman index of coprime graphs in 2022. Recently in 2023, Abdul Gazir Syarifudin et al. discovered the topological indices of the relative coprime graph of dihedral group [8]. On the continuation, we investigate topological indices such as Wiener index, Harrary index, first Zagreb index.

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The sum of the squares of the degrees of all vertices in a given graph is called the first Zagreb index $M_1(G)$. Its introduced by Mihalic and Gutman in 2000 and defined as $M_1(G) = \sum_{z\in G} \big(\deg(\mathrm{x})\big)^2$ by Jahandideh et al. [5] in 2015. The sum of the distances between all pairs of vertices in a given graph is called the Wiener index $W.I(G)$. It is defined as $W.I(G) = \sum_{x,y\in V(G)} \deg(\mathrm{x},\mathrm{y})$ by Alimon et al. [1] in 2020. The sum of the reciprocals of the distances between all pairs of vertices in a given graph is called the Harary index $H.I(G)$. It was introduced by Frank Harary in 1976. It is defined as $H.I(\Gamma_G) = \sum_{u,v\in V(G)} \frac{1}{d(u,v)}$ by Zahidah et al. [7] in 2021.

One can refer [3] for the graph terminology. Let $\Gamma = (V, E)$ be a graph with a vertex set V =V(Γ) and an edge set $E = E(\Gamma)$. One can refer [6] for the group terminology. All groups are considered as finite in this paper. The number of elements of G is called its order and is denoted by $|G|$. The order of an element x of G is the smallest positive integer n such that $x^n = e$. The order of an element x is denoted by $|x|$. The set $\mathbb{Z}_n = \{0,1,2,\dots,n-1\}$ for $n \geq 1$ is a group under addition modulo n. Minimum number of edges between two vertices is called distance. Number of edges which are adjacent to a vertex v is called degree of the vertex v, it is denoted as $deg(v)$.

The following results are used in the subsequent section;

Proposition 1.1: [4] Let a_1 be the number of distance 1 which is between every vertices of graph G. Then number of distance 2 which is between every two vertices is

$a_2 = \frac{a_0(a_0+1)}{2} - a_0 - a_1$, where $a_0 = |V(\Gamma_G)|$.

Proposition 1.2: [9] Let G be a finite additive abelian group and $p_1, p_2, \dots, p_n$ be the distinct

prime numbers. Assume that $G \cong \prod_{i=1}^{n} G_i$ where $|G_i|$ is p_i-group with $\left|G_{i_j}\right| < \left|G_{i_{j+1}}\right|$ for $i_j = 1,2,\dots,n$ and $j = 1,2,3,\dots,n-1$. Then the number of the edges in Γ_G is

$$|E(\Gamma_G)| = \sum_{k=1}^{n} 2^{n-k} \left[\sum_{i_1=1(i_1<i_j)}^{k} \prod_{j=1}^{n-(k-1)} \left(p_{i_j} - 1\right) \right]$$

2 Main Results

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Theorem 2.1: Let G be a finite additive abelian group and $p_1, p_2, \dots, p_n$ be the distinct prime numbers. Assume that $G \cong \prod_{i=1}^{n} G_i$ where $|G_i|$ is p_i-group with $\left|G_{i_j}\right| < \left|G_{i_{j+1}}\right|$ for $i_j = 1,2,\dots,n$ and $j = 1,2,3,\dots,n-1$. Then the Wiener Index and Harary Index of Γ_G are

(i) $W.I(\Gamma_G) = a_0(a_0 - 1) - a_1$ (ii) $H.I(\Gamma_G) = \frac{a_0(a_0-1)-2a_1}{4}$

where $a_1 = |E(\Gamma_G)|$ and $a_0 = |V(\Gamma_G)|$

Proof: Let $G \cong \prod_{i=1}^{n} Z_{p_i^{\alpha_i}}$. Observe that the coprime graph of G has at most distance 2 for any two vertices. Let a_1 and a_2 are the number of pair of adjacent vertices which make the distance 1 and distance 2 in Γ_G respectively.

From the Proposition 1.1 and 1.2 we have,

$$a_2 = \frac{a_0(a_0+1)}{2} - a_0 - a_1, \text{ where } a_0 = |V(\Gamma_G)|$$

$$a_1 = \sum_{k=1}^{n} 2^{n-k} \left[\sum_{i_1=1(i_1<i_j)}^{k} \prod_{j=1}^{n-(k-1)} \left(p_{i_j} - 1\right) \right]$$

Case 1: Wiener Index

By the definition, $W.I(\Gamma_G) = \sum_{u,v\in\Gamma_G} d(u,v)$

$$W.I(\Gamma_G) = a_1 + 2a_2$$

$$= a_1 + 2\left[\frac{a_0(a_0+1)}{2} - a_0 - a_1\right]$$

$$W.I(\Gamma_G) = a_0(a_0 - 1) - a_1$$

Case 2: Harary Index

By the definition and observation from the graph Γ_G, $H.I(\Gamma_G) = \sum_{u,v\in V(G)} \frac{1}{d(u,v)}$

$$H.I(\Gamma_G) = a_1 + \frac{a_2}{2}$$

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$$= a_1 + \left[\frac{\frac{a_0(a_0+1)}{2} - a_0 - a_1}{2}\right]$$

$$H.I(\Gamma_G) = \frac{a_0(a_0-1) + 2a_1}{4}$$

Remark 2.2: Let G be a finite additive abelian group and $p_1, p_2, \dots, p_n$ be the distinct prime numbers. Assume that $G \cong \prod_{i=1}^{n} G_i$ where $|G_i|$ is p_i-group with $|G_i| < |G_{i+1}|$ for $i = 1,2, \dots, n-1$.

Then the number of vertices which has the order $\prod_{i=1}^{n} p_i$ is $\prod_{i=1}^{n}(p_i - 1)$

Lemma 2.3: Let G be a finite additive abelian group and $p_1, p_2, \dots, p_n$ be the distinct prime numbers. Assume that $G \cong \prod_{i=1}^{n} G_i$ where $|G_i|$ is p_i-group with $|G_i| < |G_{i+1}|$ for $i = 1,2, \dots, n-1$.

$$deg(1) = \prod_{i=1}^{n} p_i - 1$$

$$deg\left(\prod_{i=1}^{n-1} p_i\right) = \prod_{i \neq j=1}^{n} p_j$$

$$deg\left(\prod_{i=1}^{n} p_i\right) = 1$$

where $\prod_{i=1}^{n} p_i$ is the order of the vertices in $V(\Gamma_G)$.

Proof: Let $G \cong \prod_{i=1}^{n} \mathbb{Z}_{p_i^{\alpha_i}}$.

Partition the vertex set $V(\Gamma_G)$ into 2^n subsets by their order, say $P(\Gamma_G)$ such that, $P(\Gamma_G) =$ $\{\{1\}, \{p_1\}, \{p_2\}, \dots, \{p_n\}, \{p_1p_2\}, \{p_1p_3\}, \dots, \{p_{n-1}p_n\}, \{p_1p_2p_3\},$

$$\{p_1p_2p_4\}, \dots, \{p_{n-2}p_{n-1}p_n\}, \dots, \{p_1p_2, \dots, p_{n-1}p_n\}\}$$

Here order of the identity vertex is 1 and also adjacent to all other vertices.

Observe that Γ_G $deg(1) = \prod_{i=1}^{n} p_i - 1$.

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Let choose a vertex $u \in P(\Gamma_G) - [\{e\} \cup \{p_1p_2, \dots, p_{n-1}p_n\}]$. Then the vertex u is adjacent to set of all vertices in $W = \{V \in P(\Gamma_G), \gcd(u, v) = 1, v \in V\}$

Let p_1 is adjacent to $W = \{V \in P(\Gamma_G), \gcd(p_1, v) = 1, v \in V\} = \{V \in P(\Gamma_G), p_1 \notin V\}$

If p_1 is adjacent to p_2, then the number of edges between $u \in p_1$ and p_2 are $p_2 - 1$

Similarly $u \in p_1$ to p_3 is $p_3 - 1$ and $u \in p_1$ to p_4 is $p_4 - 1$ and so on.

$$\deg(p_1) = 1 + (p_2 - 1) + (p_3 - 1) + \cdots + (p_n - 1) + (p_2 - 1)(p_3 - 1) + (p_2 - 1)(p_4 - 1)$$

$$+ \cdots + \left(p_{\{n-1\}} - 1\right)(p_n - 1) + (p_2 - 1)(p_3 - 1)(p_4 - 1) + (p_2 - 1)(p_3 - 1)(p_5 - 1)$$

$$+ \cdots + \left(p_{\{n-2\}} - 1\right)\left(p_{\{n-1\}} - 1\right)\left(p_{\{n\}} - 1\right) + \cdots + (p_2 - 1)(p_3 - 1)(p_4 - 1)\cdots(p_n - 1)$$

$$deg(p_1) = \prod_{i \neq 1, j=2}^{n} p_j$$

Similarly,

$deg(p_2) = \prod_{i \neq 2, j=1}^{n} p_j$, $deg(p_3) = \prod_{i \neq 3, j=1}^{n} p_j \cdots deg(p_n) = \prod_{i \neq n, j=1}^{n} p_j$

$$\text{Hence } deg(p_i) = \prod_{i \neq j=1}^{n} p_j, \qquad where\ 1 \leq i \leq n$$

Similarly, we can derive for all vertices which has the order $\left(\prod_{i=1}^{n-1} p_i\right)$.

By the Γ_G , vertices has the order $\prod_{i=1}^{n-1} p_i$ is adjacent to the vertices which has the order $\prod_{i \neq j=1}^{n-1} p_j$.

$$\text{Hence } deg\left(\prod_{i=1}^{n-1} p_i\right) = \prod_{i \neq j=1}^{n} p_j$$

Vertices has the order $\prod_{i=1}^{n} p_i$ is adjacent only with the identity vertex.

$$\text{Hence } deg\left(\prod_{i=1}^{n} p_i\right) = 1$$

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Theorem 2.4: Let G be a finite additive abelian group and $p_1, p_2, \dots, p_n$ be the distinct prime numbers. Assume that $G \cong \prod_{i=1}^{n} G_i$, where $|G_i|$ is p_i-group with $\left|G_{i_j}\right| < \left|G_{i_{[j+1]}}\right|$ for $i_j = 1,2,\dots,n$ and $j = 1,2,3,\dots,n-1$. Then the first Zagreb index of Γ_G is

$$M_1 = \prod_{i_1=1}^{n}(p_{i_1}-1) + \sum_{i_1=1}^{n}\left[(p_{i_1})^2 \prod_{i_1 \neq i_l \in i_j}(p_{i_l}-1)\right] + \sum_{i_1=1}^{n-1}\left[\left(\prod_{k=1}^{2} p_{i_k}\right)^2 \prod_{i_l \neq i_k \in i_j}(p_{i_l}-1)\right]$$
$$+ \sum_{i_1=1}^{n-2}\left[\left(\prod_{k=1}^{3} p_{i_k}\right)^2 \prod_{i_l \neq i_k \in i_j}(p_{i_l}-1)\right] + \cdots + \sum_{i_1=1}^{2}\left[\left(\prod_{k=1}^{n-1} p_{i_k}\right)^2 \prod_{i_l \neq i_k \in i_j}(p_{i_l}-1)\right]$$
$$+ \left(\prod_{i_1=1}^{n} p_{i_1} - 1\right)^2$$

where $i_1 < i_j, i_k$

Proof: Let $G \cong \prod_{i=1}^{n} \mathbb{Z}_{p_{i_1}^{\alpha_i}}$ and $i_j = \{1,2,3,\dots,n\}, j \in \mathbb{N}$.

Based on their degree the vertices of $V(\Gamma_G)$ can be partitioned into 2^n vertex sets.

$$V_1 = \{x_1 : \deg(x_1) = 1\}$$

$$V_2 = \{x_2 : \deg(x_2) = p_1\}$$

$$V_3 = \{x_3 : \deg(x_3) = p_2\}$$

$$\vdots$$

$$V_{n+1} = \{x_{n+1} : \deg(x_{n+1}) = p_n\}$$

$$V_{n+2} = \{x_{n+2} : \deg(x_{n+2}) = p_1 p_2\}$$

$$V_{n+3} = \{x_{n+3} : \deg(x_{n+3}) = p_1 p_3\}$$

$$\vdots$$

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$$V_{2n+1} = \{x_{2n+1} : \deg(x_{2n+1}) = p_{n-1}p_n\}$$

$$\vdots$$

$$V_{2^n} = \left\{x_{2^n} : \deg(x_{2^n}) = \prod_{i=1}^{n} p_i - 1\right\}$$

In Γ_G, the number of vertices with order $\{e\}$ is 1 and by Remark 2.2 number of the vertices with order $\prod_{i=1}^{n} p_i$ is $\prod_{i=1}^{n}(p_i - 1)$.

By the definition, $M_1(G) = \sum_{x \in \Gamma_G} \big(\deg(x)\big)^2$

Now we have to find it for each vertex set. To find for a set V, we have to product the number of vertices in V and square of the degree of a vertex in V.

For V_1, $V_1 = \deg(x_1) = 1$

By Lemma 2.3 and Remark 2.2, $x_1 = \prod_{i=1}^{n} p_{i_1}$ and $|V_1| = \prod_{i_1=1}^{n}(p_{i_1} - 1)$

$$M_{1_1} = 1^2 \times \prod_{i_1=1}^{n}(p_{i_1} - 1)$$

For V_2, $V_2 = \deg(x_2) = p_1$

By Lemma 2.3 and Remark 2.2, $x_2 = \prod_{j=2}^{n} p_j$ and $|V_2| = \prod_{j=2}^{n}(p_j - 1)$

$$M_{1_2} = p_1^2 \times \prod_{j=1}^{n}(p_j - 1)$$

Similarly, we can find it for $V_3, V_4, \ldots, V_n$.

For V_{n+1}, $V_{n+1} = \deg(x_{n+1}) = p_n$

By Lemma 2.3 and Remark 2.2, $x_{n+1} = \prod_{j=1}^{n-1} p_j$ and $|V_{n+1}| = \prod_{j=1}^{n-1}(p_j - 1)$

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$$M_{1_{n+1}} = p_n^2 \times \prod_{j=1}^{n}(p_j - 1)$$

$$\sum_{b=2}^{n+1} M_{1_b} = \sum_{i_1=1}^{n} \left[(p_{i_1})^2 \times \prod_{i_1 \neq i_l \in i_j}^{n} (p_{i_l} - 1) \right]$$

For V_{n+2} , $V_{n+2} = \deg(x_{n+2}) = p_1 p_2$

By Lemma 2.3 and Remark 2.2, $x_{n+2} = \prod_{j=3}^{n} p_j \ and \ |V_{n+2}| = \prod_{j=3}^{n}(p_j - 1)$

$$M_{1_{n+2}} = (p_1 p_2)^2 \times \prod_{j=3}^{n}(p_j - 1)$$

For V_{n+3} , $V_{n+3} = \deg(x_{n+3}) = p_1 p_3$

By Lemma 2.3 and Remark 2.2, $x_{n+3} = \prod_{j=1 \,\&\, j\neq 1,3}^{n} p_j \ and \ |V_{n+3}| = \prod_{j=1 \,\&\, j\neq 1,3}^{n}(p_j - 1)$

$$M_{1_{n+3}} = (p_1 p_3)^2 \times \prod_{j=1 \,\&\, j\neq 1,3}^{n} (p_j - 1)$$

Similarly, we can find it for $V_{\{n+4\}}, V_{\{n+5\}}, \ldots, V_{2n}$

For V_{2n+1} , $V_{2n+1} = \deg(x_{2n+1}) = p_{n-1} p_n$

By Lemma 2.3 and Remark 2.2,

$x_{2n+1} = \prod_{j=1 \,\&\, j\neq n-1,n}^{n} p_j$ and $|V_{2n+1}| = \prod_{j=1 \,\&\, j\neq n-1,n}^{n}(p_j - 1)$

$$M_{1_{2n+1}} = (p_{n-1} p_n)^2 \times \prod_{j=1 \,\&\, j\neq n-1,n}^{n} (p_j - 1)$$

$$\sum_{b=n+2}^{2n+1} M_{1_b} = \sum_{i_1=1}^{n-1} \left[\left(\prod_{k=1}^{2} p_{i_k} \right)^2 \times \prod_{i_l \neq i_k \in i_j}^{n} (p_{i_l} - 1) \right]$$

Similarly, we can derive it for V_b, where $2n + 2 \leq b \leq 3n + 1$

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$$\sum_{b=2n+2}^{3n+1} M_{1_b} = \sum_{i_1=1}^{n-2} \left[\left(\prod_{k=1}^{3} p_{i_k} \right)^2 \times \prod_{i_l \neq i_k \in i_j}^{n} (p_{i_l} - 1) \right]$$

Continuing this process until we get for V_{2^n-n}

For V_{2^n-n} , $V_{2^n-n} = \deg(x_{2^n-n}) = \prod_{k=1}^{n-1} p_k$

By Lemma 2.3 and Remark 2.2, $x_{2^n-n} = \mathrm{p_n}$ *and* $|V_{2^n-n}| = \mathrm{p_n} - 1$

$$M_{1_{2^n-n}} = \left(\prod_{k=1}^{n-1} p_k \right)^2 \times (\mathrm{p_n} - 1)$$

For V_{2^n-n+1} , $V_{2^n-n+1} = \deg(x_{2^n-n+1}) = \prod_{k=1,k\neq n-1}^{n-1} p_k$

By Lemma 2.3 and Remark 2.2, $x_{2^n-n+1} = \mathrm{p_{n-1}}$,$|V_{2^n-n+1}| = \mathrm{p_{n-1}} - 1$

$$M_{1_{2^n-n+1}} = \left(\prod_{k=1,k\neq n-1}^{n-1} p_k \right)^2 \times (\mathrm{p_{n-1}} - 1)$$

Similarly, we can find it for $V_{2^n-n+2}, V_{2^n-n+3}, \ldots, V_{2^n-2}$.

For V_{2^n-1}, $V_{2^n-1} = \deg(x_{2^n-1}) = \prod_{k=2}^{n} p_k$

By Lemma 2.3 and Remark 2.2, $x_{2^n-1} = \mathrm{p_1}$, $|V_{2^n-1}| = \mathrm{p_1} - 1$

$$M_{1_{2^n-1}} = \left(\prod_{k=2}^{n} p_k \right)^2 \times (\mathrm{p_1} - 1)$$

$$\sum_{b=2^n-n}^{2^n-1} M_{1_b} = \sum_{i_1=1}^{2} \left[\left(\prod_{k=1}^{n-1} p_{i_k} \right)^2 \times \prod_{i_l \neq i_k \in i_j}^{n} (p_{i_l} - 1) \right]$$

For V_{2^n}, $V_{2^n} = \deg(x_{2^n}) = \prod_{k=1}^{n} p_k - 1$

By Lemma 2.3 and Remark 2.2, $x_{2^n} = \{\mathrm{e}\}$ and $|V_{2^n}| = 1$

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$$M_{1_{2^n}} = \left(\prod_{k=1}^{n} p_k - 1\right)^2 \times 1$$

Hence $M_1 = \sum_{b=1}^{2^n} M_{1_b}$

$$M_1 = \prod_{i_1=1}^{n}(p_{i_1} - 1) + \sum_{i_1=1}^{n}\left[(p_{i_1})^2 \times \prod_{i_1 \neq i_l \in i_j}^{n}(p_{i_l} - 1)\right] + \sum_{i_1=1}^{n-1}\left[\left(\prod_{k=1}^{2} p_{i_k}\right)^2 \times \prod_{i_l \neq i_k \in i_j}^{n}(p_{i_l} - 1)\right]$$
$$+ \sum_{i_1=1}^{n-2}\left[\left(\prod_{k=1}^{3} p_{i_k}\right)^2 \times \prod_{i_l \neq i_k \in i_j}^{n}(p_{i_l} - 1)\right] + \cdots + \sum_{i_1=1}^{2}\left[\left(\prod_{k=1}^{n-1} p_{i_k}\right)^2 \times \prod_{i_l \neq i_k \in i_j}^{n}(p_{i_l} - 1)\right]$$
$$+ \left(\prod_{k=1}^{n} p_k - 1\right)^2$$

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TOPOLOGICAL INDICES OF CHOLESTERIN: MATHEMATICAL INSIGHTS INTO MOLECULAR STRUCTURE

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ABSTRACT:

Topological indices are essential tools in mathematical chemistry, offering valuable insights into the structural and molecular graphs. In this study, we focus on calculating novel degree-based topological indices for cholesterin, including the modified Y index, modified S index, sigma Y index, sigma S index, modified minus Y index, modified minus S index, reduced Y index, reduced S index, reduced modified Y index, and reduced modified S index. These indices extend traditional degree-based descriptors and provide deeper insights into the molecular topology of cholesterin.

KEYWORDS: Modified Y & S index, sigma Y & S index, modified minus Y & S index, reduced Y & S index, reduced modified Y & S index.

INTRODUCTION:

Molecular graphs are frequently characterised using topological and graph invariants based on the distances between graph vertices, which build connections between structural and characteristics of molecules, forecasting chemical compounds' biological functions, and creating chemical applications. Topological indices come in a variety of forms, such as degree-based topological indices, distance-based topological indices, and counting-related polynomials. Chemists now have an intriguing tool thanks to graph

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theory: topological indices. Molecular graphs are commonly used to depict molecules and molecular compounds. The structural formula of a chemical compound is represented graph-theoretically by a molecular graph, where atoms are represented by vertices and chemical bonds by edges.

When comparing the physical, chemical, or biological characteristics of molecules in Quantitative Structure Property Relationships (QSPR) and Quantitative Structure Activity Relationships (QSAR), topological indices are useful since they may be employed directly as straightforward numerical descriptors. The present trend of numerically coding chemical structures using topological indices or topological coincides has shown to be quite successful in the fields of medicinal chemistry and bioinformatics.

Drugs that lower cholesterol are used to treat elevated cholesterol and lower the risk of heart disease, stroke, and other cardiovascular diseases. These drugs aid in lowering bad cholesterol, or low-density lipoprotein (LDL), and increasing good cholesterol, or high-density lipoprotein (HDL). The two dimensional molecular structure are taken from PubChem. Two dimension

Molecular graph of Cholesterin have referred in PubChem.

BASIC DEFINITIONS:

Let's consider simple connected graph A, each with disjoint vertex and edge sets. The degree of a vertex n is the number of edges incident on the vertex n and is expressed as da(n)= $ﬥa(n)$ for every nϵN(A) a vertex set.

In 2015, B Furtula and I Gutman put forward the concept of the F Index[2] which is given by

$$F(A)= \sum_{mn\epsilon E(A)}[ﬥa(m)^2 + ﬥa(n)^2]$$

In 2020, Abdu Alameri and Noman AI-Naggar put forward the concept of the Y-index[3] which is defined as:

$$Y(A)=\sum_{mn\epsilon E(A)}[ﬥa(m)^3 + ﬥa(n)^3]$$

In 2021, S. Nagarajan and G. kayalvizhi and G. priyadarshini put forward the concept of the S-index[4], which is defined as:

$$S(A)=\sum_{mn\epsilon E(A)}[ﬥa(m)^4 + ﬥa(n)^4]$$

The modified Y-INDEX and modified S-index[1] are defined as:

$$MY(A)=\sum_{n\epsilon N(A)} \frac{1}{[ﬥa(m)^4]}$$

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$$MS(A)= \sum_{n\epsilon N(A)} \frac{1}{[ℳa(m)^5]}$$

Minus Y-index and Minus S-index[5] is defined as:

$$MiY(A)= \sum_{mn\epsilon E(A)}[|ℳa(m)^3 - ℳa(n)^3|]$$

$$MiS(A)= \sum_{mn\epsilon E(A)}[|ℳa(m)^4 - ℳa(n)^4|]$$

Sigma Y-index and Sigma S-index is defined by[6]:

$$\sigma Y(A) = \sum_{mn\epsilon E(A)} [ℳa(m)^3 - ℳa(n)^3]^2$$

$$\sigma S(A) = \sum_{mn\epsilon E(A)} [ℳa(m)^4 - ℳa(n)^4]^2$$

Modified Minus Y-index and Modified S-index is defined by[1]:

$$MMY(A)= \sum_{mn\epsilon E(A)} \frac{1}{|ℳa(m)^3 - ℳa(n)^3|}$$

$$MMS(A)= \sum_{mn\epsilon E(A)} \frac{1}{|ℳa(m)^4 - ℳa(n)^4|}$$

MAIN RESULT:

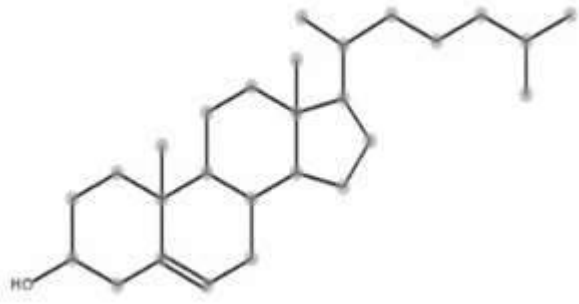

Fig 1: Cholesterin

Theorem 1: The F-index of cholesterin is 406

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Proof: The F-index of cholesterin is defined as:

$F(C_{27}H_{46}O)= \sum_{mn\epsilon E(A)}[ṁa(m)^2 + ṁa(n)^2]$

$F(C_{27}H_{46}O) = 4[1^2 + 3^2] + 2[1^2 + 4^2] + 6[2^2 + 2^2] + 10[2^2 + 3^2] + 3[3^2 + 3^2] + 4[3^2 + 4^2]$

$F(C_{27}H_{46}O) = 406$

Theorem 2: The Y-index of cholesterin is 1214.

Proof: The Y-index of cholesterin is defined as:

$Y(C_{27}H_{46}O)=\sum_{mn\epsilon E(A)}[ṁa(m)^3 + ṁa(n)^3]$

$Y(C_{27}H_{46}O) = 4[1^3 + 3^3] + 2[1^3 + 4^3] + 6[2^3 + 2^3] + 10[2^3 + 3^3] + 3[3^3 + 3^3] + 4[3^3 + 4^3]$

$Y(C_{27}H_{46}O) = 1214$

Theorem 3: The S-index of cholesterin is 4382.

Proof: The S-index of cholesterin is defined as:

$S(C_{27}H_{46}O)=\sum_{mn\epsilon E(A)}[ṁa(m)^4 + ṁa(n)^4]$

$$S(C_{27}H_{46}O) = 4[1^4 + 3^4] + 2[1^4 + 4^4] + 6[2^4 + 2^4] + 10[2^4 + 3^4] + 2[2^4 + 4^4] + 3[3^4 + 3^4] + 4[3^4 + 4^4]$$

$S(C_{27}H_{46}O) = 4382$

Theorem 4: The modified Y-index of cholesterin is 0.0008

Proof: The modified Y-index of cholesterin is defined as:

$KY(C_{27}H_{46}O)=\sum_{n\epsilon N(A)} \frac{1}{[ṁa(m)^3+ṁa(n)^3]}$

$KY(C_{27}H_{46}O) = 0.0008$

Theorem 5: The modified S-index of cholesterin is 0.0002.

Proof: The modified S-index of cholesterin is defined as:

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KS($C_{27}H_{46}O$)= $\sum_{n\epsilon N(A)} \frac{1}{[ṁa(m)^4+ṁa(n)^4]}$

$KS(C_{27}H_{46}O) = 0.0002$

Theorem 6: The minus Y-index of cholesterin is 680.00

Proof: The minus Y-index of cholesterin is defined as:

KiY($C_{27}H_{46}O$)= $\sum_{mn\epsilon E(A)}[|ṁa(m)^3 - ṁa(n)^3|]$

$$KiY(C_{27}H_{46}O) = 4[1^3 - 3^3] + 2[1^3 - 4^3] + 6[2^3 - 2^3] + 10[2^3 - 3^3] + 2[2^3 - 4^3] + 3[3^3 - 3^3] + 4[3^3 - 4^3]$$

$KiY(C_{27}H_{46}O) = 680.00$

Theorem 7: The minus S-index of cholesterin is 2660.

Proof: The minus S-index of cholesterin is defined as:

KiS($C_{27}H_{46}O$)= $\sum_{mn\epsilon E(A)}[|ṁa(m)^4 - ṁa(n)^4|]$

$$KiS(C_{27}H_{46}O) = 4[1^4 - 3^4] + 2[1^4 - 4^4] + 6[2^4 - 2^4] + 10[2^4 - 3^4] + 2[2^4 - 4^4] + 3[3^4 - 3^4] + 4[3^4 - 4^4]$$

$KiS(C_{27}H_{46}O) = 2660$

Theorem 8: The Sigma Y-index of cholesterin is 462400

Proof: The Sigma Y-index of cholesterin is defined as:

$$\sigma Y(C_{27}H_{46}O) = \sum_{mn\epsilon E(A)} [ṁa(m)^3 - ṁa(n)^3]^2$$

$$\sigma Y(C_{27}H_{46}O)=[4[1^3 - 3^3] + 2[1^3 - 4^3] + 6[2^3 - 2^3] + 10[2^3 - 3^3] + 2[2^3 - 4^3] + 3[3^3 - 3^3] + 4[3^3 - 4^3]]^2$$

$\sigma Y(C_{27}H_{46}O) = (680.00)^2$

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$\sigma Y(C_{27}H_{46}O) = 462400$

Theorem 9: The Sigma S-index of cholesterin is 7075600

Proof: The Sigma S-index of cholesterin is defined as:

$$\sigma S(C_{27}H_{46}O) = \sum_{mn\epsilon E(A)} [ṁa(m)^4 - ṁa(n)^4]^2$$

$\sigma S(C_{27}H_{46}O) = (2660)^2$

$\sigma S(C_{27}H_{46}O) = 7075600$

Theorem 10: The modified minus Y-index of cholesterin is 0.001470

Proof: The modified minus Y-index of cholesterin is defined as:

MMY($C_{27}H_{46}O$)= $\sum_{mn\epsilon E(A)} \frac{1}{|ṁa(m)^3 - ṁa(n)^3|}$

$$= \left(\frac{1}{|4[1^3 - 3^3] + 2[1^3 - 4^3] + 6[2^3 - 2^3] + 10[2^3 - 3^3] + 2[2^3 - 4^3] + 3[3^3 - 3^3] + 4[3^3 - 4^3]|}\right)$$

$$MMK(C_{27}H_{46}O) = \frac{1}{|680.00|}$$

$MMK(A) = 0.001470$

Theorem 11: The modified minus S-index of cholesterin is 0.0004

Proof: The modified minus S-index of cholesterin is defined as:

MMS($C_{27}H_{46}O$)= $\sum_{mn\epsilon E(A)} \frac{1}{|ṁa(m)^4 - ṁa(n)^4|}$

$$= \left(\frac{1}{|4[1^4 - 3^4] + 2[1^4 - 4^4] + 6[2^4 - 2^4] + 10[2^4 - 3^4] + 2[2^4 - 4^4] + 3[3^4 - 3^4] + 4[3^4 - 4^3]|}\right)$$

$$MMS(C_{27}H_{46}O) = \left(\frac{1}{|2660|}\right)$$

$MMS(C_{27}H_{46}O) = 0.0004$

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APPLICATIONS:

Drugs that lower cholesterol are used to treat elevated cholesterol and lower the risk of heart disease, stroke, and other cardiovascular diseases. These drugs aid in lowering bad cholesterol, or low-density lipoprotein (LDL), and increasing good cholesterol, or high-density lipoprotein (HDL). Knowing Cholesterol Metabolism: The reduced and modified indices provide a better understanding of how cholesterin affects cholesterol synthesis through its interactions with enzymes like HMG-CoA reductase. Cholesterol & Disease Prediction: These indices help connect differences in molecular structure to the risk of diseases such metabolic disorders, atherosclerosis, and Alzheimer's disease.

CONCLUSION:

Modified, sigma, reduced, and modified minus versions of the Y and S indices are among the new degree-based topological indices for cholesterin that we have investigated in this work. By going beyond conventional molecular descriptors, these indices provide more in-depth understanding of the topological and structural characteristics of cholesterin.

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MATHEMATICAL ANALYSIS OF TUMOR-IMMUNE AND HOST CELLS INTERACTIONS WITH CHEMOTHERAPY

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Abstract

A mathematical model of tumor-immune and host cells interaction with chemotherapy is analyzed. The model involves the system of non-linear equations that contain Michaelis–Menten term with resting immune cells. Analytical expressions pertaining to concentrations of tumor cells, hunting immune cells, resting immune cells, normal cells, and drug concentration are presented. We have employed Homotopy Perturbation Method (HPM) to evaluate the approximate analytical solutions of the non-linear initial value problem. Different parameters involved in this model are analyzed using the analytical result obtained.

Keywords**:** Mathematical model**,** Cancer cell, Chemotherapy, Micheaelis-Menten function, Homotopy perturbation method.

1. Introduction

Analyzing tumor cells is necessary as it helpful to identify how the normal cells convert into cancer cells. It helps the scientists to develop new treatments, discover and create new medicines. Massard et. al [1] discussed the discovery of the primary site poses a significant challenge due to a lack of effective detection methods. Empirical chemotherapy [2] treatment was done primarily which gave only poor progress. New equipment and technology [3] has extensively increased by scientist in this research field. Mathematical

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models play a vital role in the development of cancer research. Mathematical model [4] help to identify the issues connected with tumor growth. Simulation [5] of mathematical model is the more efficient and less expensive way of analyzing the problems. Determination of optimal dosage and maximize therapeutic efficiency will minimize side effects. Improving the mathematical model [6] is useful to test various hypotheses and treatment strategies virtually.

Experiential [7] and Mathematical [8] investigations are increase to get rid of this deadly disease. Sarkar and Banerjee et. al [9] used the stochastic approach to study the cancer self-remission model. El-Gohary et. al [10] improved the same model by providing an optimal control strategy. Gurpreet Kaur et. al [11] extended the model by introducing Michaelis-Menten function. Tarekegn Dinku et.al. [12] expand the model research by considering the host cell as the fighting factor in the dynamics. Stability analysis of the model was done by the author by considering normal cells, drug control and incorporating the Michael–Menten function in resting immunity, while regarding hunting immunity as a predator variable. But in this paper, we present the approximate analytical expressions which are easier to interpret allowing for deeper understanding of the system being analyzed. Using HPM method [13,14] we found the analytical expression for the concentrations of tumor cells, hunting immune cells, resting immune cells, normal cells, and drug concentration corresponding to all possible values of the parameters.

2. Mathematical formulation of the problem and analysis

2.1. Mathematical formulation

Tarekegen Dinku et.al[12] developed the mathematical model representing the concentration of tumor cells(T), immune cells(H), resting immune cells(R), normal cells (N) and drug (D) by considering normal cells, drug control and incorporating the Michael-Menten function in resting immunity, while regarding hunting immunity as a predator variable.

$$\frac{dT}{dt} = r_1 T\left(1 - \frac{T}{k_1}\right) - \alpha_1 TH - \alpha_2 TN - \alpha_3 TD \tag{1}$$

$$\frac{dH}{dt} = \beta RH - \sigma_1 H - \sigma_2 TH - \sigma_3 HD \tag{2}$$

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$$\frac{dR}{dt} = r_2 R\left(1 - \frac{R}{k_2}\right) - \rho_1 RH - \rho_2 R + \frac{\rho TR}{T+\eta} - \rho_3 RD \tag{3}$$

$$\frac{dN}{dt} = r_3 N\left(1 - \frac{N}{k_3}\right) - \delta_1 TN - \delta_2 N - \delta_3 ND \tag{4}$$

$$\frac{dD}{dt} = u_0 - \gamma D \tag{5}$$

For simplicity of calculation, we let $\frac{1}{k_1} = b_1, \frac{1}{k_2} = b_2 and \frac{1}{k_3} = b_3$. The model becomes

$$\frac{dT}{dt} = r_1 T(1 - b_1 T) - \alpha_1 TH - \alpha_2 TN - \alpha_3 TD \tag{6}$$

$$\frac{dH}{dt} = \beta RH - \sigma_1 H - \sigma_2 TH - \sigma_3 HD \tag{7}$$

$$\frac{dR}{dt} = r_2 R(1 - b_2 R) - \rho_1 RH - \rho_2 R + \frac{\rho TR}{T+\eta} - \rho_2 RD \tag{8}$$

$$\frac{dN}{dt} = r_3 T(1 - b_3 N) - \delta_1 TN - \delta_2 N - \delta_3 ND \tag{9}$$

$$\frac{dD}{dt} = u_0 - \gamma D \tag{10}$$

with initial conditions

$$T(0) = A \geq 0 \tag{11}$$

$$H(0) = B \geq 0 \tag{12}$$

$$R(0) = E \geq 0 \tag{13}$$

$$N(0) = F \geq 0 \tag{14}$$

$$D(0) = G \geq 0 \tag{15}$$

The parameters involved in this model are explained in Appendix C.

2.2. Solution of initial value problem using HPM

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Exact solutions for the nonlinear problems are not possible. At this situation our deliberate need is asymptotic approximate solution [14] which gives the closed form solution to the differential equation models. The perturbation method is a well-established method which acts as the basic tool for the development of other asymptotic methods. There are many approximate solutions are available in literature. variational iteration method [15], Homotopy analysis method [16], Adomian decomposition method [17] and so on. In this paper we used Homotopy Perturbation Method to solve this nonlinear mathematical model. The basic principle of this method in described in Appendix A and detailed derivation of the concentration of the equation from Eq.(6) to Eq.(10) with their initial conditions Eq.(11) to Eq.(15) are described in Appendix B. We obtain the concentration of Tumor cells T(t), hunting immune cells H(t), resting immune cells R(t),normal drug and drug concentration D(t) respectively as follows.

$$T = Ae^{M_2 t} + \frac{\alpha_1 AB}{M_3} e^{M_2 t}(1 - e^{-M_3 t}) + \frac{\alpha_2 AF}{M_5} e^{M_2 t}(1 - e^{-M_5 t}) + \frac{\alpha_3 AM_1}{\gamma} e^{M t}(1 - e^{-\gamma t}) + \frac{r_1 b_1 A^2}{M_2} e^{M_2 t}(1 - e^{M_2 t}) \quad (16)$$

$$H = Be^{-M_3 t} + \frac{\beta BE}{M_4} e^{-M_3 t}(1 - e^{-M_4 t}) + \frac{\sigma_2 AB}{M_2} e^{-M_3 t}(1 - e^{M_2 t}) + \frac{\sigma_3 M_1 B}{\gamma} e^{-M_3 t}(e^{-\gamma t} - 1), \quad (17)$$

$$R = Ee^{-M_4 t} + \frac{AE}{\eta M_2}(M_4 + r_2) e^{-M_4 t}(e^{M_2 t} - 1) + \frac{AE^2 r_2 b_2}{\eta(M_2 - M_4)} e^{-M_4 t}(1 - e^{(M_2 - M_4)t}) + \frac{r_2 b_2 E^2}{M_4} e^{-M_4 t}(e^{-M_4 t} - 1) +$$
$$\frac{AEB\rho_1}{\eta(M_2 - M_3)} e^{-M_4 t}(1 - e^{(M_2 - M_3)t}) + \frac{\rho_1 BE}{M_3} e^{-M_4 t}(e^{-M_4 t} - 1), \quad (18)$$

$$N = Fe^{-M_5 t} + \frac{r_3 A}{(M_2 + M_3)}(e^{M_2 t} - e^{-M_3 t}) + \frac{AF(r_3 b_3 + \delta_1)}{(M_2 + M_3 - M_5)}(e^{-M_3 t} - e^{(M_2 - M_5)t}) + \frac{\delta_3 FM_1}{(M_5 + \gamma - M_3)}$$
$$(e^{-(M_5 + \gamma)t} - e^{-M t}) \quad (19)$$

$$D = \frac{u_o}{\gamma} + M_1 e^{-\gamma t} \qquad \text{where } M_1 = G - \frac{u_0}{\gamma} \quad (20)$$

3. Results and discussion

Controlling the spread of tumor cells is now challenging and important issue to study. Analytical results [Eq.(16) to Eq. (20)] have been found for the nonlinear system of equations [Eq. (6) to Eq. (15)] representing tumor-immune and host cells with chemotherapy. These analytical results give insight into some of the consequences of the public health policies. The drug concentration given by Eq.(10) is a linear equation. We

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can find the exact solution of the drug concentration. From the result obtained in Eq.(20) we can easily identify that drug concentration tends to the constant value $\frac{u_0}{\gamma}$ as $t \to \infty$.

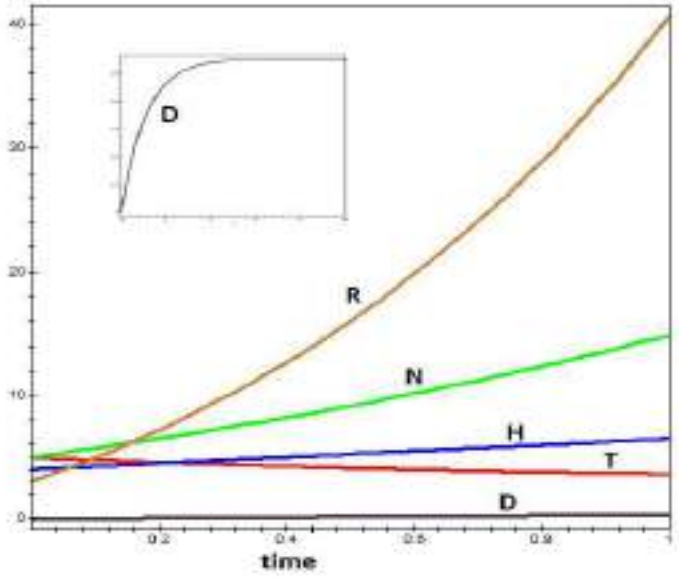

Figure 1. Plot of the two-dimensional comparative case diagram of the concentrations of tumor cells, hunting immune cells, resting immune cells, normal cells, and drug concentration for the values of $\alpha_1 = 0.014$, $\alpha_2 = 4.34\text{x}10^{-14}$, $\alpha_3 = 9\text{x}10^{-1}$, $r_1 = 0.005045$, $r_2 = 0.9$, $r_3 = 0.6169$, $\beta = 0.0937$, $\sigma_1 = 0.0937$, $\sigma_2 = 0.0003, \sigma_3 = 5\text{x}10^{-4}$, $\rho = 0.05$, $\rho_1 = 0.0122$, $\rho_2 = 4.388\text{x}10^{-14}$, $\eta = 1, \delta_1 = 0.0155$, $\delta_2 = 0.00005$, $\delta_3 = 9\times10^{-4}$, $\gamma = 0.9$, $G = 0, b_1 = 0.2\times10^{-8}, b_2 = 0.3601\times10^{-5}$, $b_3 = 0.183\times10^{-8}, u_0 = 0.5$

Resting immune cells which are not able to attack and wipe out tumor cells directly, but it helps to increase the hunting immune cells which attack and demolish tumor cells. Figure 1 explains chemotherapy effect on other cells on the body. Chemotherapy drug increases to a certain level, and reaches a stable position. As drug concentration increases normal cells increases and tumor cells decreases. Though stability of the tumor

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equilibrium with a enormous value means that the diminution of tumor through chemotherapy alone is not possible to reduce tumor cells.

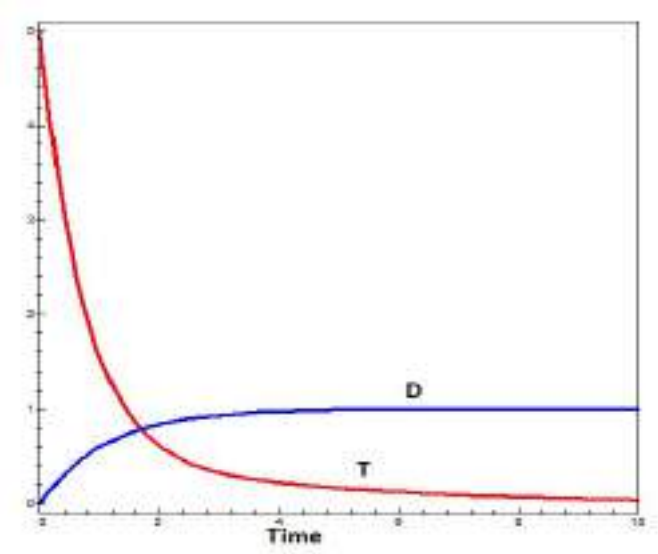

Figure 2. Plot of the two-dimensional comparative case diagram of the concentrations of tumor cells and drug concentration for the values of $\alpha_1 = 0.014,\ \alpha_2 = 4.34\text{x}10^{-14}$, $\alpha_3 = 9\text{x}10^{-1}, r_1 = 0.5045,, r_2 = 0.9$, $r_3 = 0.6169, \beta = 0.0937, \sigma_1 = 0.0937, \sigma_2 = 0.0003,\ \sigma_3 = 5\text{x}10^{-4},\ \rho = 0.05, \rho_1 = 0.0122,$ $\rho_2 = 4.388\text{x}10^{-14},\ \eta = 1\ ,\ \delta_1 = 0.0155,\ \delta_2 = 0.005,\ \delta_3 = 9\times10^{-4},\ \gamma = 0.9,$ $G = 0, b_1 = 0.2\times10^{-8}, b_2 = 0.3601\times10^{-5},\ b_3 = 0.183\times10^{-8}, u_0 = 0.5$.

Figure 2 illustrate the zero tumor number by modifying the model parameter with chemotherapy. Cancer professional can adjust the parameters of cancer-immune system connection by applying different treatment methods.

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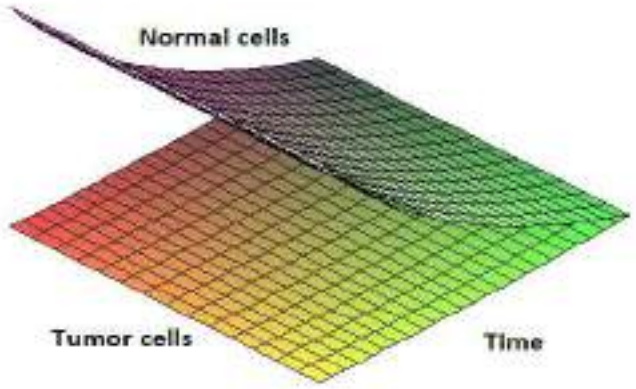

Figure 3. Plot of the three dimensional comparative case diagram of the concentrations of tumor cells and normal cells versus time.

It is observed from the figure 3 that normal cells and tumor cells are inversely proportional. If normal cells are increasing with the increase of time, then the tumor will decrease. Increasing the drug concentration may lead to more side effects. So according to the outcomes found from the mathematical models, it might be probable to stabilize the tumor cells at a definite value or to dismiss the cancer.

4. Conclusions

In this study, a mathematical model of concentrations of tumor cells, hunting immune cells, resting immune cells, normal cells, and drug concentration is solved using HPM method. The information gained from this theoretical model can be useful for the scientist analyzing cancer by fixing the parameter value. This may be useful to cancer experts to decide the drug level and choose the way of treatment needed for the patient. This mathematical model can be extended by modifying the drug concentration as a nonlinear equation involving other concentrations like tumor and normal cells.

Appendix A. Basic concepts of the HPM

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The HPM method has eliminated limitations of the traditional perturbation methods. On the other hand it can take full advantage of the traditional perturbation techniques, so there has been a considerable deal of research in applying homotopy technique for solving various strong non-linear equations [13,14]. To explain this method, let us consider the following function:

$$D_o(u) - f(r) = 0, \qquad r \in \Omega \tag{A1}$$

with the boundary conditions of $B_o(u, \frac{\partial u}{\partial n}) = 0, \qquad r \in \Gamma$ (A2)

where D_o is a general differential operator, B_o is a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω. Generally speaking, the operator D_o can be divided into a linear part L and a nonlinear part N. Eq. (A1) can therefore, be written as

$$L(u) + N(u) - f(r) = 0 \tag{A3}$$

By the homotopy technique, we construct a homotopy $v(r, p) : \Omega \times [0,1] \to \Re$ which satisfies

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[D_o(v) - f(r)] = 0. \tag{A4}$$

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0. \tag{A5}$$

where $p \in [0,1]$ is an embedding parameter , u_0 is an initial approximation of Eq.(A1), which satisfies generally the boundary conditions . From Eqs. (A4) and (A5), we will have

$$H(v,0) = L(v) - L(u_0) = 0 \tag{A6}$$

$$H(v,1) = D_o(v) - f(r) = 0. \tag{A7}$$

When p=0, Eq. (A4) or Eq. (A5) becomes a linear equation; when p =1 it becomes a non-linear equation. So the changing process of p from zero to unity is just that of $L(v) - L(u_0) = 0$ to $D_o(v) - f(r) = 0$. We can first use the embedding parameter p as a "small parameter", and assume that the solutions of Eqs. (A4) and (A5) can be written as a power series in p

$$v = v_0 + pv_1 + p^2 v_2 + ... \tag{A8}$$

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Setting $p = 1$ results in the approximate solution of Eq. (A1):

$$u = \lim_{p \to 1} v = \mathrm{v}_0 + v_1 + v_2 + ... \quad \text{(A9)}$$

This is the basic idea of the HPM.

Appendix B. Approximate analytic solutions of Mathematical analysis of Tumor-Immune and Host cells interactions with Chemotherapy.

The solution for Eq.(10) with the initial is

$$D = \frac{u_o}{\gamma} + M_1 e^{-\gamma t} \qquad \text{where } M_1 = G - \frac{u_0}{\gamma} \quad \text{(B1)}$$

Substituting the value *D* in Eqs. (6) to (9) and using homotopy perturbation method, we construct a homotopy for the Eq. (6) to Eq. (9) as follows.

$$(1-p)\left(\frac{dT}{dt} - M_2 T\right) + p\left(\frac{dT}{dt} - M_2 T - \alpha_1 TH - \alpha_2 TN - \alpha_3 T M_1 e^{-\gamma t} + r_1 b_1 T^2\right) = 0 \quad \text{(B2)}$$

$$\text{where } M_2 = r_2 - \frac{\alpha_3 u_0}{\gamma}$$

$$(1-p)\left(\frac{dH}{dt} + M_3 H\right) + p\left(\frac{dH}{dt} + M_3 H - \beta RH + \sigma_2 TH + \alpha_3 M_1 e^{-\gamma t} H\right) = 0 \quad \text{(B3)}$$

$$\text{where } M_3 = \sigma_1 + \frac{\sigma_3 u_0}{\gamma}$$

$$(1-p)\left(\eta \frac{dR}{dt} + \eta M_4 R\right) + p\left(\eta \frac{dR}{dt} + \eta M_4 R_1 + T \frac{dR}{dt} - T r_2 R + (T+\eta) r_2 b_2 R^2 + (T+\eta)\rho_1 RH + (\rho_2 - \rho) TR + \rho_2 TR\left(\frac{u_0}{\gamma} + M_1 e^{-\gamma t}\right) + \rho_2 R \eta M_1 e^{-\gamma t}\right) = 0 \quad \text{(B4)}$$

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where $M_4 = \frac{\rho_2 u_0}{\gamma} - (r_2 - \rho_2)$

$$(1-p)(\frac{dN}{dt} + M_5 N) + p(\frac{dN}{dt} + M_5 N - r_3 T + r_3 b_3 TN + \delta_1 TN + \delta_3 N M_1 e^{-\gamma t}) = 0 \quad \text{(B5)}$$

where $M_5 = \delta_2 + \frac{\delta_3 u_0}{\gamma}$

The approximate solutions of Eq.(B2) to Eq.(B5) are as follows,

$$T = T_0 + pT_1 + p^2 T_2 + \ldots \quad \text{(B6)}$$

$$N = N_0 + pN_1 + p^2 N_2 + \ldots. \quad \text{(B7)}$$

$$H = H_0 + pH_1 + p^2 H_2 + \ldots. \quad \text{(B8)}$$

$$R = R_0 + pR_1 + p^2 R_2 + \ldots. \quad \text{(B9)}$$

Substituting Eq.(B1) and Eq.(B6) into Eq.(6) and arranging the coefficients of p powers, we have

$$p^0 : \frac{dT_0}{dt} - M_2 T_0 = 0 \quad (B10)$$

$$p^1 : \frac{dT_1}{dt} - M_2 T_1 - \alpha_1 T_0 H_0 - \alpha_2 T_0 N_0 - \alpha_3 T_0 M_1 e^{-\gamma t} + r_1 b_1 {T_0}^2 = 0 \quad (B11)$$

Substituting Eq.(B1) and Eq.(B7) into Eq.(7) and arranging the coefficients of p powers, we have

$$p^0 : \frac{dH_0}{dt} + M_3 H_0 = 0 \quad (B12)$$

$$p^1 : \frac{dH_1}{dt} + M_3 H_1 - \beta R_0 H_0 + \sigma_2 T_0 H_0 + \alpha_3 H_0 M_1 e^{-\gamma t} = 0 \quad (B13)$$

Substituting Eq.(B1) and Eq.(B8) into Eq.(8) and arranging the coefficients of p powers, we have

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$$p^0 : \frac{dR_0}{dt} + M_4 R_0 = 0 \qquad (B14)$$

$$p^1 : \eta \frac{dR_1}{dt} + \eta M_4 R_1 + T_0 \frac{dR}{dt} - T_0 r_2 R_0 + (T_0 + \eta) r_2 b_2 R_0{}^2 + (T_0 + \eta)\rho_1 R_0 H_0 + (\rho_2 - \rho) T_0 R_0 +$$

$$\rho_2 T_0 R_0 \left(\frac{u_0}{\gamma} + M_1 e^{-\gamma t} \right) + \rho_2 R_0 \eta M_1 e^{-\gamma t} = 0 \qquad (B15)$$

Substituting Eq.(B1) and Eq.(B9) into Eq.(9) and arranging the coefficients of p powers, we have

$$p^0 : \frac{dN_0}{dt} + M_5 N_0 = 0 \qquad (B16)$$

$$p^1 : \frac{dN_1}{dt} + M_5 N_1 - r_3 T_0 + r_3 b_3 T_0 N_0 + \delta_1 T_0 N_0 + \delta_3 N_0 M_1 e^{-\gamma t} = 0 \qquad (B17)$$

The initial approximations are as follows,

$$T_0(0) = 0, \qquad T_1(0) = 0 \qquad (B18)$$

$$H_0(0) = 0, \qquad H_1(0) = 0 \qquad (B19)$$

$$R_0(0) = 0, \qquad R_1(0) = 0 \qquad (B20)$$

$$N_0(0) = 0, \qquad N_1(0) = 0 \qquad (B21)$$

From Eq.(B10), Eq.(B12), Eq.(B14) and Eq.(B16), we get

$$T_0 = Ae^{M_2 t} \qquad (B23)$$

$$H_0 = Be^{-M_3 t} \qquad (B24)$$

$$R_0 = Ee^{-M_4 t} \qquad (B25)$$

$$N_0 = Fe^{-M_5 t} \qquad (B26)$$

Substituting Eq.(B23), Eq.(B24),and Eq.(B26) in Eq.(B11),we obtain the solution of Eq.(B11),

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$$T_1 = \frac{\alpha_1 AB}{M_3} e^{M_2 t}(1-e^{-M_3 t}) + \frac{\alpha_2 AF}{M_5} e^{M_2 t}(1-e^{-M_5 t}) + \frac{\alpha_3 AM_1}{\gamma} e^{M t}(1-e^{-\gamma t}) + \frac{r_1 b_1 A^2}{M_2} e^{M_2 t}(1-e^{M_2 t}) \quad (B27)$$

Substituting Eq.(B23), Eq.(B24),and Eq.(B25) in Eq.(B13),we obtain the solution of Eq.(B13),

$$H_1 = \frac{\beta BE}{M_4} e^{-M_3 t}(1-e^{-M_4 t}) + \frac{\sigma_2 AB}{M_2} e^{-M_3 t}(1-e^{M_2 t}) + \frac{\sigma_3 M_1 B}{\gamma} e^{-M_3 t}(e^{-\gamma t}-1) \quad (B28)$$

Substituting Eq.(B23), Eq.(B24),and Eq.(B25) in Eq.(B15),we obtain the solution of Eq.(B15),

$$R_1 = \frac{AE}{\eta M_2}(M_4 + r_2)e^{-M_4 t}(e^{M_2 t}-1) + \frac{AE^2 r_2 b_2}{\eta(M_2 - M_4)} e^{-M_4 t}(1-e^{(M_2-M_4)t}) + \frac{r_2 b_2 E^2}{M_4} e^{-M_4 t}(e^{-M_4 t}-1) + \frac{AEB\rho_1}{\eta(M_2 - M_3)} e^{-M_4 t}(1-e^{(M_2-M_3)t}) + \frac{\rho_1 BE}{M_3} e^{-M_4 t}(e^{-M_4 t}-1) \quad (B29)$$

Substituting Eq.(B23),and Eq.(B26) in Eq.(B17),we obtain the solution of Eq.(B17),

$$N_1 = \frac{r_3 A}{(M_2 + M_3)}(e^{M_2 t} - e^{-M_3 t}) + \frac{AF(r_3 b_3 + \delta_1)}{(M_2 + M_3 - M_5)}(e^{-M_3 t} - e^{(M_2 - M_5)t}) + \frac{\delta_3 FM_1}{(M_5 + \gamma - M_3)} (e^{-(M_5+\gamma)t} - e^{-M t}) \quad (B30)$$

Thus we get the the concentrations of tumor cells, hunting immune cells, resting immune cells, normal cells and drug concentration given by Eq.(16) to Eq.(20).

Appendix C. Nomenclature

Symbols	Description
T(t)	Concentration of tumor cells
H(t)	Concentration of hunting immune cells
R(t)	Concentration of resting immune cells
N(t)	Concentration of normal cells
D(t)	Concentration of drug concentration

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A	Initial concentration of tumor cells
B	Initial concentration of hunting immune cells
E	Initial concentration of resting immune cells
F	Initial concentration of normal cells
G	Initial concentration of drug concentration
r_1	Growth rate of tumor cells
r_2	Growth rate of resting immune cells
r_3	Growth rate of normal cells
k_1	Capacities of tumor cells
k_2	Capacities of resting immune cells
k_3	Capacities of normal cells
δ_1	Rate of death of health cell due to tumor cell
δ_2	Natural death rates of normal cells
δ_3	Rates of decrease in normal cells
α_1	Killing rate of tumor cell by hunting immune cells
α_2	Rate of death of tumor cell death due to normal cell competition
α_3	Rates of decrease in tumor cells
σ_1	Natural death rates of hunting immune cells
σ_2	Clearance of hunting immune cells
σ_3	Rates of decrease in hunting immune cells
ρ	Proliferation rate
ρ_1	Rate of conversion of resting immune cells due to hunting immune cells
ρ_2	Natural death rates of resting immune cells

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ρ_3	Rates of decrease in resting immune cells
u_0	Initial amount of drug injected
β	Rate of conversion of helper T cells to active CTL cells
η	Half saturation of the proliferation term

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MATHEMATICAL MODELING AND OPTIMIZATION OF IMMOBILIZED GLUCOSE ISOMERASE REACTORS USING HILL KINETICS

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Abstract

Glucose isomerization is a critical bioprocess for producing high-fructose corn syrup (HFCS), a widely used sweetener in the food industry. This process, catalyzed by immobilized glucose isomerase (GI), enhances enzyme reuse and reactor efficiency. However, optimizing the performance of these reactors remains challenging due to factors such as enzyme efficiency, mass transfer limitations, and complex reaction dynamics. In this study, a mathematical model is developed that integrates Hill kinetics with reaction-diffusion principles to simulate the transient behavior of glucose and fructose concentrations in immobilized enzyme reactors. The model accounts for enzyme cooperativity and the interplay between enzyme kinetics and mass transfer effects. Simulation results provide concentration profiles over time and along the reactor, and are used to optimize reactor parameters, such as enzyme concentration and flow rate, to maximize fructose yield. The findings offer valuable guidance for improving the efficiency of glucose isomerization in industrial applications.

Keywords: Glucose isomerization, Immobilized glucose isomerase, Hill kinetics, Mathematical modeling, Reaction-diffusion, Enzyme kinetics, Fructose yield optimization, Bioprocess optimization, Reactor design, Mass transfer.

1. Introduction

Glucose isomerization is a key step in the production of high-fructose corn syrup (HFCS), an important sweetener in the food industry. The reaction, catalyzed by glucose isomerase (GI), is typically carried out in immobilized enzyme reactors, which offer advantages such as enhanced enzyme stability and the ability to reuse enzymes. Despite these benefits, optimizing reactor

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performance remains a challenge, influenced by enzyme efficiency, mass transfer limitations, and environmental factors.

Mathematical modeling provides a valuable tool for optimizing such systems by simulating enzyme kinetics and mass transfer dynamics without the need for exhaustive experimental trials. This paper presents a mathematical model that incorporates Hill kinetics and reaction-diffusion principles to simulate glucose isomerization in immobilized enzyme reactors. The goal of the model is to optimize reactor parameters (e.g., enzyme concentration and flow rate) to maximize fructose yield, providing insights into improving industrial glucose isomerization processes.

2. Literature Review

Glucose isomerization is crucial for the production of HFCS. Immobilized glucose isomerase (GI) reactors are widely used due to their ability to reuse enzymes, enhancing reactor efficiency. However, optimizing these reactors remains challenging, as factors such as enzyme deactivation, substrate inhibition, and mass transfer limitations influence performance (Navajas et al., 2019).

Previous models have typically employed Michaelis-Menten kinetics to describe enzyme-substrate interactions. While effective in many cases, this approach does not account for enzyme cooperativity, which is significant when multiple substrate molecules interact with enzyme sites. The Hill equation, which models this non-linear enzyme behavior, offers a more accurate representation of the reaction kinetics (Hill, 1910).

Many reactor models focus on continuous stirred tank reactors (CSTR) or plug flow reactors (PFR), combining enzyme kinetics with reaction-diffusion equations to describe concentration profiles. However, these models often neglect mass transfer limitations or simplify enzyme dynamics, which can significantly affect reactor performance. This study aims to bridge these gaps by integrating Hill kinetics with reaction-diffusion principles, providing a comprehensive model for optimizing glucose isomerization in immobilized enzyme reactors.

3. Mathematical Model Development

3.1 System Description

The system under study involves glucose isomerization, where glucose is converted to fructose by immobilized glucose isomerase. The enzyme is assumed to be immobilized on a solid matrix, facilitating its reuse without significant loss of activity. The main components are:

- **Glucose (S)**: Substrate
- **Fructose (F)**: Product
- **Immobilized glucose isomerase (GI)**: Catalyst

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Assumptions:

- The system is modeled as either a Plug Flow Reactor (PFR) or Continuous Stirred Tank Reactor (CSTR), where flow regimes dictate the concentration gradients of glucose and fructose.
- Enzyme activity remains steady throughout the process.
- Enzyme concentration is uniform across the reactor.
- No enzyme deactivation occurs during the reaction.

3.2 Enzyme Kinetics (Hill Equation)

The Hill equation models the enzyme kinetics for glucose isomerization, accounting for enzyme cooperativity:

$$v = \frac{V_{\max} S^n}{K^n{}_m + S^n}$$

Where:

- V = Reaction rate (mol/time),
- S = substrate concentration (mol/L),
- Vmax = maximum reaction rate (mol/time),
- Km = Michaelis constant (mol/L), the substrate concentrationat which the reaction rate is half of V_{max}
- n = is the Hill coefficient, indicating enzyme cooperativity.
- n = 1 → No cooperativity (Michaelis-Menten behavior)
- n > 1 → Positive cooperativity (binding of one substrate increases affinity for others)
- n < 1 → Negative cooperativity (binding of one substrate decreases affinity for others)

This equation accounts for non-linear enzyme-substrate interactions, which are critical for accurately modeling enzyme efficiency at different substrate concentrations.

3.3 Mass Balance Equations

Mass balance equations describe the conservation of mass for glucose and fructose in the reactor. For glucose (S), the mass balance equation is:

$$\frac{\partial S}{\partial t} = \mathrm{D}_S \nabla^2 S - \frac{v}{\eta}$$

Where:

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- D^S is the diffusion coefficient of glucose (m²/s),
- $\nabla^2 S$ represents the spatial variation in glucose concentration,
- v is the reaction rate for glucose isomerization,
- η is the enzyme concentration in the reactor (mol/L).

For fructose (F), the mass balance equation is:

$$\frac{\partial F}{\partial t} = D_F \nabla^2 F + \frac{v}{\eta}$$

Where:

- D^F is the diffusion coefficient of fructose (m²/s),
- $\nabla^2 F$ represents the spatial variation in fructose concentration.
- v is the reaction rate.
- η is the enzyme concentration in the reactor (mol/L)

These equations describe how glucose and fructose concentrations change over time and space within the reactor.

3.4 Reaction-Diffusion Model

The reaction-diffusion model is expressed as a set of partial differential equations (PDEs):

$$\frac{\partial S}{\partial t} = D_S \nabla^2 S - \frac{v}{\eta}$$

$$\frac{\partial F}{\partial t} = D_F \nabla^2 F + \frac{v}{\eta}$$

Where x represents spatial position within the reactor. These equations describe glucose consumption and fructose production, along with the diffusion of both substances through the reactor.

3.5 Boundary Conditions and Assumptions

The following boundary conditions are applied to solve the reaction-diffusion equations:

- **At the inlet (x=0)**:

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- Glucose concentration is constant: S(0,t)=Sinlet
- Fructose concentration is zero: F(0,t)=0

- **At the outlet (x=L)**:
  - The flux of glucose and fructose is zero: $\frac{\partial S}{\partial x}(L,t)=0$, $\frac{\partial F}{\partial x}(L,t)=0$

Additional assumptions include no enzyme deactivation, perfect mixing at the reactor inlet (for CSTR), and uniform enzyme distribution in immobilized systems.

4. Model Solution and Numerical Simulation

4.1 Numerical Methods

The system of partial differential equations (PDEs) is solved using a combination of numerical methods for spatial and temporal discretization. The Finite Difference Method (FDM) is employed for spatial discretization, and the fourth-order Runge-Kutta method (RK4) is used for time integration.

4.2 Simulation Setup

The simulation parameters are as follows:

Plug Flow Reactor (PFR) Model

For a steady-state **PFR**, the mass balance equation is:

$$v\frac{ds}{dx}=-r(s)$$
$$v\frac{dF}{dx}=r(s)$$

where:

- S(x) and F(x) are the glucose and fructose concentrations along the reactor length x,
- v=1 m/s (flow velocity),
- r(S) is the reaction rate given by the **Hill equation**:

$$r(S)=V_{\max}\frac{S^n}{K^n{}_m+S^n}$$

- **Boundary conditions**:

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- At x=0 (inlet): S(0)=100 mM, F(0)=0
- At x=L m (outlet): No specific boundary conditions but solved numerically.

5. Results and Discussion

5.1 Interpretation of Results

- **Enzyme Concentration**: Increasing enzyme concentration enhances the reaction rate, improving conversion efficiency. However, diminishing returns are observed beyond a certain threshold.
- **Flow Rate**: Higher flow rates reduce residence time, decreasing conversion efficiency. Lower flow rates increase residence time but may cause operational inefficiencies.
- **Mass Transfer Limitations**: High glucose concentrations combined with slow diffusion rates reduce enzyme-substrate interaction, limiting reaction rate and efficiency.
- **Reactor Design Insights**: PFRs show higher conversion efficiency at the inlet, but careful management of flow rates is essential. CSTRs offer more uniform mixing but typically result in lower conversion efficiency.

5.2 Comparison with Literature

Our results align with previous studies. For instance, Deng et al. (2020) reported a maximum glucose conversion of approximately 80% in a CSTR, consistent with our simulation results. Our model also confirms the diffusion limitations identified by Singh et al. (2019), particularly at high glucose concentrations.

6. Conclusion

This study presents a comprehensive mathematical model for glucose isomerization in immobilized glucose isomerase reactors. Key findings emphasize the importance of enzyme concentration, flow rate, and mass transfer in optimizing reactor performance. The model offers valuable insights for industrial glucose isomerization processes, highlighting the need for balanced reactor conditions to maximize fructose yield. Future work will focus on experimental validation and exploring alternative enzyme kinetics models for further optimization.

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MATHEMATICAL ANALYSIS OF CHEMICALLY REACTIVE MHD FLOW WITH INDUCED MAGNETIC FIELD THROUGH CHANNEL

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Abstract

In this paper, a mathematical analysis is performed on the MHD flow across a channel made by two infinite vertical walls. A source of heat/sink is placed near one of the walls and first order chemical reaction is observed in the flow. Induced magnetic field arises due to applied magnetic field of strength is taken in the fluid motion equation to study its impact. The system of ordinary differential equations expressing the fluid model are solved analytically under the appropriate boundary conditions after going through non dimensional process. The solutions obtained for fluid velocity, Induced magnetic field, temperature distribution and concentration distribution in the system are plotted graphically w.r.t to various fluid parameters. In addition to this, the behavior of drag at the walls and induced current density are shown via tables. The significant impression of induced magnetic field and chemical reaction is seen on velocity and drag.

Key Words: MHD, Induced magnetic field, chemical reaction, Channel

1. Introduction

The study on MHD flows under application of magnetic field has been performed mathematically by numerous scholars. These studies predicted the significant impressions of magnetic field on the flows. The rate of heat and mass transfers are seen altered under applied magnetic field. The inputs of Hyat et al. [1] Hossain et al [2], Chamkha [3], Rahman and Sarkar [4], Malashetty and Umavathi [5], Makinde and Mohane [6] , Singh and Singh [7], Liu [8] has contributed towards various heat and mass transfer processes of MHD fluids under magnetic field applications. But in such kind of systems only applied magnetic field alone doesn't impact the flow, however the magnetic fluids generates its own magnetic field other than applied one. This is called induced magnetic field (IMF). Most of the researchers don't consider IMF as it complicate the system. But its existence cannot be ignored due to its contribution in converting mechanical energy to electrical energy. This finds applications in many electrical devices and power generating system like pumps, transformers etc. So various geometrical fluid flow systems has been analyzed under induced magnetic field with other one or

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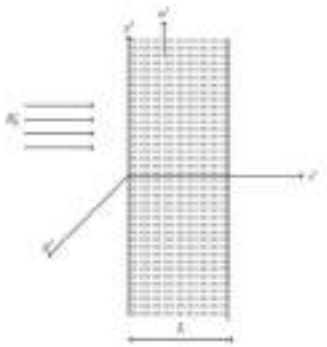
more components in the flow model by the researchers like Mazumdar et al [9], Singh et al [[10]-[11]], Jha et al [[12]-[15]], Kumar[16] and many more.

In lieu of above studies, my investigation is motivated. So in the present problem, I have taken MHD flow through the channel which is chemically reactive and induced magnetic field impacts on this flow model are observed mathematically on solving the fluid flow equations. The further part of the paper is divided into sections which involves 2. Mathematical presentation of the problem 3. Solution procedure 4. Graphical interpretation 5. Main findings

2. Mathematical Representation of the problem

The present flow model includes the electrical conducting fluid through the vertical channel placed along x' axis and z' axis is normal to walls. The fluid is incompressible and Newtonian in nature .The walls are distanced L apart. A magnetic field of strength B_0 acts perpendicular to channel. The physical model of the flow is shown below which describes the clear picture of flow.



Under the assumptions of heat source/sink, chemical reaction, IMF and Boussinesq approximation the model equations are:

$$\vartheta \frac{d^2u'}{dz'^2} + \frac{\mu_e B_0'}{\rho}\frac{dh'}{dz'} + g\beta(T' - T_L') + g\beta'(C' - C_L') = 0 \quad (1)$$

$$\frac{d^2h'}{dz'^2} + \sigma\mu_e B_0'\frac{du'}{dz'} = 0 \quad (2)$$

$$\frac{\kappa}{\rho C_p}\frac{d^2u'}{dz'^2} + \frac{Q_0}{\rho C_p}(T' - T_L') = 0 \quad (3)$$

$$\mathrm{D}\frac{d^2u'}{dz'^2} - k_1(C' - C_L') = 0 \quad (4)$$

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With boundary wall conditions as

$$u' = 0,\ h' = 0,\quad T' = T_0' \qquad C' = C_0' \qquad at\ z' = 0$$

$$u' = 0,\ h' = 0,\quad T' = T_L',\qquad C' = C_L' \qquad at\ z' = L \tag{5}$$

The terms involved here in the equations are explained in nomenclature

3. Solution Procedure

To obtain the solution of our flow model, we need to change the system into unit less form which is also known as non-dimension form. This is performed as below:

For non-dimension form of above system, we consider the parameters and variables as

$$u = \frac{u'}{U}, z = \frac{z'}{L}, B = \frac{h'}{\sigma \mu_e B_0' UL}, T = \frac{T' - T_L'}{T_0' - T_L'}, C = \frac{C' - C_L'}{C_0' - C_L'}, \mathrm{G} = \frac{\mathrm{g}\beta(T_0' - T_L')\mathrm{L}^2}{\nu \mathrm{U}},\ G' = \frac{\mathrm{g}\beta'(C_0' - C_L')\mathrm{L}^2}{\nu \mathrm{U}},\qquad Ha = \mu_e B_0' L\sqrt{\frac{\sigma}{\mu}}, S = -\frac{Q_0\,\mathrm{L}^2}{\kappa}, \lambda = \frac{\mathrm{k}_1\mathrm{L}^2}{\mathrm{D}}$$

Where B (Induced magnetic field parameter), G &G' (Thermal and mass Grashof numbers), Ha (Hartmann number), S (heat source/sink parameter), λ (chemical reaction parameter)

On substituting above the system of equation (1) - (5) takes the form as

$$\frac{d^2u}{dz^2} + Ha^2\frac{dB}{dz} + G\,T + G'C = 0 \tag{6}$$

$$\frac{d^2B}{dz^2} + \frac{du}{dz} = 0 \tag{7}$$

$$\frac{d^2T}{dz^2} - S\,T = 0 \tag{8}$$

$$\frac{d^2C}{dz^2} - \lambda\,C = 0 \tag{9}$$

$$u = 0,\ B = 0,\ T = 1,\ C = 1 \qquad at\ z = 0$$

$$\mathrm{u} = 0,\ B = 0,\ T = 0,\ C = 0 \qquad at\ z = 1 \tag{10}$$

As the above system, represents the simultaneous ODE system, so these equations (6)-(9) are solved analytically under boundary conditions (10). The solutions are given below:

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$$u = -Ha\, r_5 \exp(Ha\, z) + Ha\, r_6 \exp(-Ha\, z) - \frac{G}{(S-Ha^2)}\left(r_3 \exp\left(\sqrt{S}z\right) + r_4 \exp\left(-\sqrt{S}z\right)\right) - \frac{G'}{(\lambda-Ha^2)}\left(r_1 \exp\left(\sqrt{\lambda}z\right) + r_2 \exp\left(-\sqrt{\lambda}z\right)\right) + r_8 \quad (11)$$

$$B = r_7 + r_5 \exp(Ha\, z) + r_6 \exp(-Ha\, z) + \frac{G}{\sqrt{S}\,(S-Ha^2)}\left(r_3 \exp\left(\sqrt{S}z\right) - r_4 \exp\left(-\sqrt{S}z\right)\right) + \frac{G'}{\sqrt{\lambda}\,(\lambda-Ha^2)}\left(r_1 \exp\left(\sqrt{\lambda}z\right) - r_2 \exp\left(-\sqrt{\lambda}z\right)\right) \quad (12)$$

$$T = r_3 \exp\left(\sqrt{S}z\right) + r_4 \exp\left(-\sqrt{S}z\right) \quad (13)$$

$$C = r_1 \exp\left(\sqrt{\lambda}z\right) + r_2 \exp\left(-\sqrt{\lambda}z\right) \quad (14)$$

The other variables ICD (J) and skin friction (τ) are given by

$$J = -\frac{dB}{dz} = -r_5 Ha \exp(Ha\, z) + r_6 Ha \exp(-Ha\, z) - \frac{G}{\sqrt{S}\,(S-Ha^2)}\left(r_3\sqrt{S} \exp\left(\sqrt{S}z\right) + r_4\sqrt{S}\exp\left(-\sqrt{S}z\right)\right) - \frac{G'}{\sqrt{\lambda}\,(\lambda-Ha^2)}\left(r_1\sqrt{\lambda} \exp\left(\sqrt{\lambda}z\right) + r_2\sqrt{\lambda} \exp\left(-\sqrt{\lambda}z\right)\right) \quad (15)$$

$$(\tau)_{z=0} = \left(\frac{\mathrm{du}}{\mathrm{dz}}\right)_{z=0} = -Ha^2\, r_5 - Ha^2\, r_6 - \frac{G\sqrt{S}}{(S-Ha^2)}(r_3 - r_4) - \frac{G'\sqrt{\lambda}}{(\lambda-Ha^2)}(r_1 - r_2) \quad (16)$$

$$(\tau)_{z=1} = -\left(\frac{\mathrm{du}}{\mathrm{dz}}\right)_{z=1} = Ha^2\, r_5 \exp(Ha\,) + Ha^2\, r_6 \exp(-Ha\,) + \frac{G}{(S-Ha^2)}\left(r_3\sqrt{S} \exp\left(\sqrt{S}\right) - r_4\sqrt{S} \exp\left(-\sqrt{S}\right)\right) + \frac{G'}{(\lambda-Ha^2)}\left(r_1\sqrt{\lambda} \exp\left(\sqrt{\lambda}\right) - r_2\sqrt{\lambda} \exp\left(-\sqrt{\lambda}\right)\right) \quad (17)$$

The Sherwood number (Sh) (rate of mass transfer) is given as

$$(Sh)_{z=0} = -\left(\frac{dC}{dz}\right)_{z=0} = -r_1\sqrt{\lambda} + r_2\sqrt{\lambda} \quad (18)$$

$$(Sh)_{z=1} = -\left(\frac{dC}{dz}\right)_{z=1} = -r_1\sqrt{\lambda} \exp\left(\sqrt{\lambda}\right) + r_2\sqrt{\lambda} \exp\left(-\sqrt{\lambda}\right) \quad (19)$$

The terms $r_1, r_2, r_3, r_4, r_5, , r_6, r_7, r_8$ are shown in the Appendix

4. Graphical Interpretation

The solutions attained in the above section are studied graphical to visualize the effects of main parameters viz Hartmann number (Ha), heat source parameter (S) and chemical Reaction parameter (λ). While plotting

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velocity, temperature and concentration graphs other parameters are kept constant. In the discussion we will be using the symbols of parameters for simplification.

The behavior of velocity w.r.t to above mentioned parameters can be viewed from the figures 1-3The velocities can be seen declined with increase in Ha and S values. This depicts the dominance of electromagnetic forces and heat source/sink. On the other side, the velocities pattern rises on increment in λ values which reflects that chemical reaction speed up the velocity by suppressing the viscous forces.

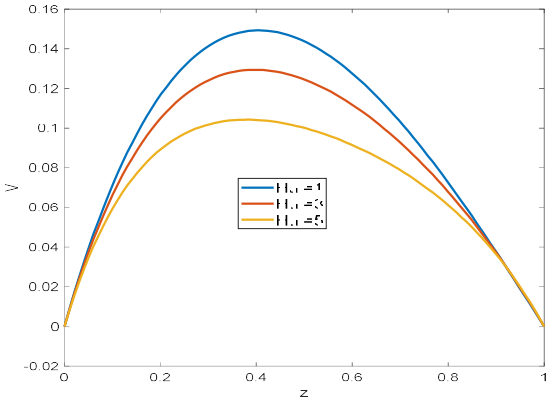

Fig 1. Velocity correspond to varying Ha

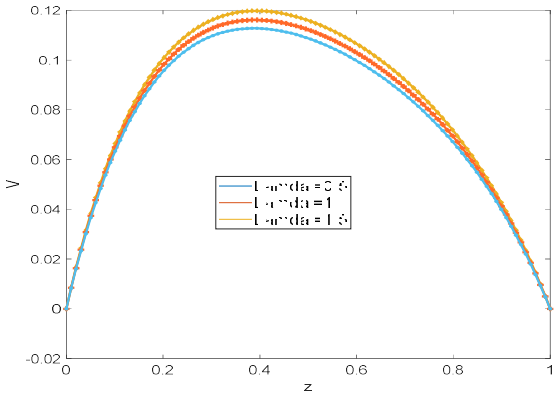

Fig 2. Velocity correspond to varying λ

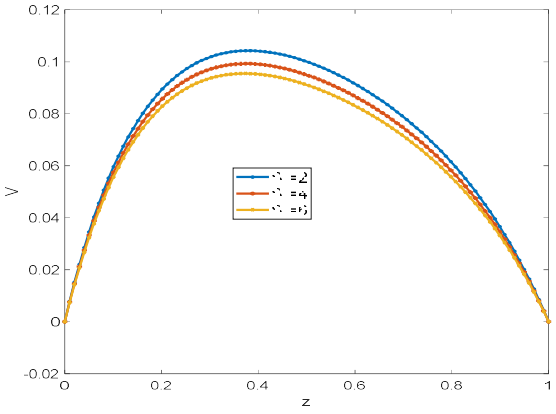

Fig 3. Velocity correspond to varying S

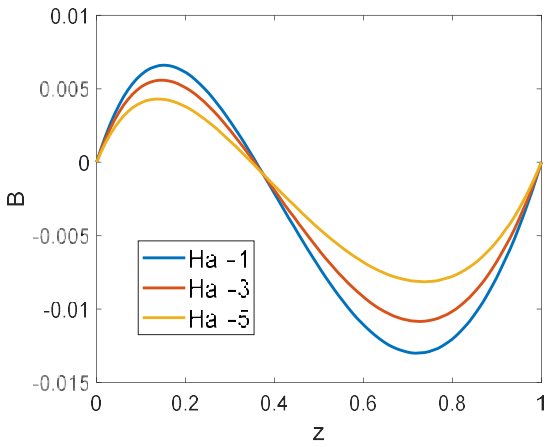

Fig 4. IMF correspond to varying Ha

The induced magnetic field (IMF) action corresponding to various parameters has been displayed via figures 4-6. The IMF graphs on increasing Ha and λ values shows declined nature near the left wall but acts adversely

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near the right wall. This indicates that the electromagnetic forces are more dominant near left wall as compared to right wall. But it shows decreasing pattern throughout the channel with increasing S values.

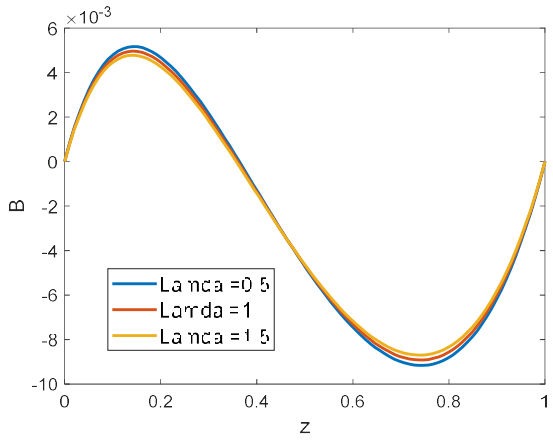

Fig 5. IMF correspond to varying λ

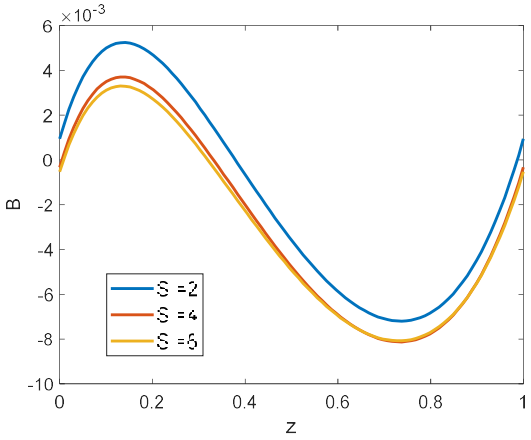

Fig 6. IMF correspond to varying S

The induced current density (J) significance w.r.t to parameters is shown through the figures 7-9. ICD curves behaves differently across the channel. With increasing in Ha values, ICD behaves decreasing near left and right walls but increases in the middle of channel. However ICD plots shows reverse behavior with rise in S and λ values

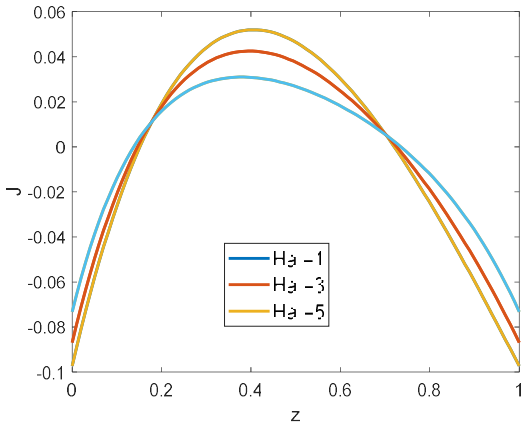

Fig 7. ICD correspond to varying Ha

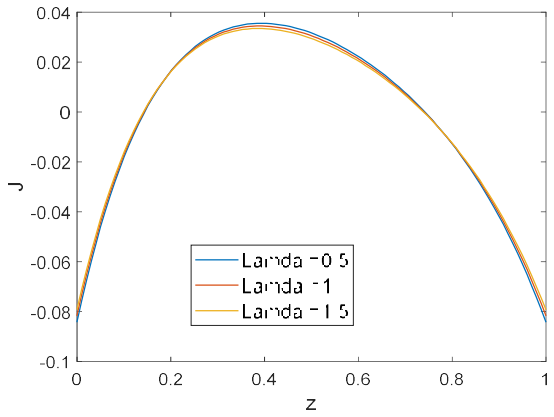

Fig 8. ICD correspond to varying λ

Figure 10 depicts the decline in concentration on rise in λ values which is due to deceleration in the mass diffusion. The drag at the walls of the channel are enumerated for different values of Ha and λ as diplayed in Table 1. The increment in Ha values imparts the decrease in drag at left wall but increases the drag at the right

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wall. This signifies the effectiveness of Lorentz forces at the right wall as compared to left one. However the drag the both walls is decreased with increment in λ values.

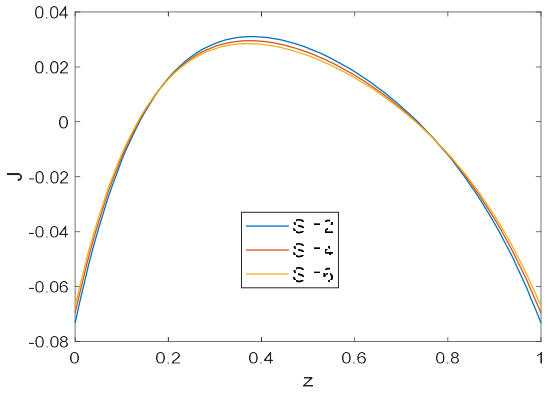

Fig 9. ICD correspond to varying Ha

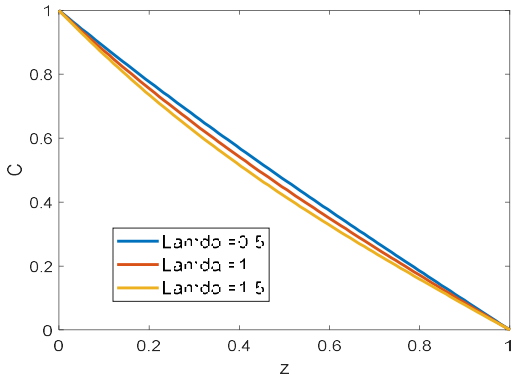

Fig.10 Concentration with varying λ

Table 1. Skin friction at the wall z = 0 and z = 1

Ha	τ_0	τ_1	λ	τ_0	τ_1
1	0.8548	0.3833	0.5	0.8769	0.5139
3	0.8295	0.4086	1	0.8581	0.4967
5	0.7965	0.4416	3	0.7965	0.4416

Table 2 is shown for Sherwood number variation. The mass transfer rate (Sh) is speeded up at the left wall with the increase in λ values but slower down close to right wall of channel

Table 2. Sherwood number at walls z = 0 and z = 1

λ	Sh_0	Sh_1
0.5	1.1614	-2.3554
1	1.3130	-3.5692
3	1.8440	-10.4226

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5. Main Findings

The present work analysis is concluded as

- IMF and chemical reaction has prominent impact on velocities and drag near the walls of the channel.
- IMF reduces the velocities and chemical reaction work oppositely.
- IMF reduces drag at the left wall but increase the drag at the right wall. However the chemical reaction minimizes the drag at the both walls.
- The chemical reaction in the flow enhanced the mass transfer rate at the left wall in comparison to right wall.

The study reflects that IMF and chemical reaction in the flow system has remarkable effect. Such kind of analysis helps in advancing electrical devices and many other medical instruments

Appendix

$$r_1 = \frac{-e^{-2\sqrt{\lambda}}}{1-e^{-2\sqrt{\lambda}}},\ \ r_2 = \frac{1}{1-e^{-2\sqrt{\lambda}}},\qquad r_3 = \frac{-e^{-2\sqrt{S}}}{1-e^{-2\sqrt{S}}},\ \ r_4 = \frac{1}{1-e^{-2\sqrt{S}}},$$

$$r_5 = \frac{-1}{2(1-e^{Ha})}\left(\frac{G}{(S-Ha^2)}\left(r_3\left(1-e^{\sqrt{S}}\right)\left(\frac{1}{\sqrt{S}}+\frac{1}{Ha}\right)-r_4\left(1-e^{-\sqrt{S}}\right)\left(\frac{1}{\sqrt{S}}-\frac{1}{Ha}\right)\right)\right.$$
$$\left.+\frac{G'}{(\lambda-Ha^2)}\left(r_1\left(1-e^{\sqrt{\lambda}}\right)\left(\frac{1}{\sqrt{\lambda}}+\frac{1}{Ha}\right)-r_2\left(1-e^{-\sqrt{\lambda}}\right)\left(\frac{1}{\sqrt{\lambda}}-\frac{1}{Ha}\right)\right)\right)$$

$$r_6 = \frac{-1}{2(1-e^{-Ha})}\left(\frac{G}{(S-Ha^2)}\left(r_3\left(1-e^{\sqrt{S}}\right)\left(\frac{1}{\sqrt{S}}-\frac{1}{Ha}\right)-r_4\left(1-e^{-\sqrt{S}}\right)\left(\frac{1}{\sqrt{S}}+\frac{1}{Ha}\right)\right)\right.$$
$$\left.+\frac{G'}{(\lambda-Ha^2)}\left(r_1\left(1-e^{\sqrt{\lambda}}\right)\left(\frac{1}{\sqrt{\lambda}}-\frac{1}{Ha}\right)-r_2\left(1-e^{-\sqrt{\lambda}}\right)\left(\frac{1}{\sqrt{\lambda}}+\frac{1}{Ha}\right)\right)\right)$$

$$r_7 = -\frac{G(r_3-r_4)}{\sqrt{S}\,(S-Ha^2)}-\frac{G'\,(r_1-r_2)}{\sqrt{\lambda}\,(\lambda-Ha^2)}-r_6-r_5$$

$$r_8 = \frac{G(r_3+r_4)}{(S-Ha^2)}+\frac{G'\,(r_1+r_2)}{(\lambda-Ha^2)}-Ha\,r_6+Ha\,r_5$$

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Nomenclarure

ν Coefficient of Kinematic viscosity, m^2s^{-1}

β Volumetric coefficient of thermal expansion, K^{-1}

β' Volumetric coefficient of expansion of concentration, K^{-1}

μ Coefficient of viscosity, $kgm^{-1}s^{-1}$

κ Thermal conductivity, $Wm^{-1}K^{-1}$

ρ Fluid density, kgm^{-3}

σ Electrical conductivity, $\Omega^{-1}m^{-1}$

λ Chemical reaction parameter

μ_e Magnetic permeability

C' Dimensional species concentration, Kgm^{-3}

C'_0 Species concentration at the left wall, Kgm^{-3}

C'_L Species concentration at the right wall, Kgm^{-3}

D Chemical molecular diffusivity, m^2s^{-1}

Q_0 Heat source/sink coefficient

B_0 Strength of applied magnetic field, Wbm^{-2}

T Dimensionless fluid temperature

T_0' Temperature at left wall, K

T' Dimensional fluid temperature,

T'_L Fluid temperature at right wall, K

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On Neutrosophic p*gp-Closed Sets in NeutrosophicTopological Spaces

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Abstract

Aim of these **c**oncepts, we introduced a new class of neutrosophic p*gp-closed sets on neutrosophic topological space X, known as neutrosophicp*gp-closed (briefly Np*gp-closed) sets. Also, inspect this new set is relate with other existing sets. Further, we discuss about their characterizations of neutrosophic p*gp-closed sets.

Keywords: Neutrosophic Closed Sets, Neutrosophic Open Sets, Neutrosophic Pre Closed Set

AMS Subject Classification: 18B30, 03E72, 11B05, 26A21

1. Introduction

In 1986, intuitionistic fuzzy set as a generalization of fuzzy set extended by Atanassov K [2]. Recently, Salama A.A and Alblowi S.A [6] discussed the concept of neutrosophictopological spaces. WadeiAl-Omeri and Saeid Jafari [8] discovered the generalized closed sets and generalized pre-closed sets in neutrosophictopological space which belong to the important class of neutrosophicsets. Salama, Alblowi [7] described about the concept of generalized neutrosophicset and generalized neutrosophictopological spaces. neutrosophictopological spaces are very realistic generalizations of fuzzy topological spaces, allow for the study of neutrosophicopen sets, closed sets, interior and closure etc.

This conceptis arranged as follows: Section 2, some basic definitions and results to given. Section 3, neutrosophicp*gp-closed set is introduced and study the basic properties of neutrosophicp*g-closed sets. Section 4, discussed the characterizations of neutrosophicp*gp-closed sets. Section 5, conclusion of neutrosophicp*gp-closed sets on neutrosophictopological space.

2. Preliminaries

In this section, we recall some relevant basic preliminaries about neutrosophicsets and its properties.

Throughout this paper (X,τ_N) represents a neutrosophictopological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl(A) and int(A) and C(A) denote the closure of A, the interior of A and the complement of A respectively. (X,τ_N)will be replaced by X if there are no changes of confusion. Recollect the following definitions and results.

Definition 2.1. A neutrosophictopology (briefly NT) **[6]** is a non-empty set X is a family τ_N of neutrosophicsubsets in X satisfying the following axioms:

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(i).0_N, $1_N \in \tau_N$.

(ii)$G_1 \cap G_2 \in \tau_N$for any $G_1, G_2 \in \tau_N$.

(iii)$\cup G_i \in \tau_N$for every $\{G_i; i \in I\} \subseteq \tau_N$.

Throughout this article, the pair of (X,τ_N) is called a neutrosophictopological space.

Definition 2.2. A neutrosophicset A in a neutrosophictopological space (X,τ_N) is called neutrosophicpreopen set **[5]** (briefly NPOS) if A $\subseteq Nint(Ncl(A))$

Definition 2.3. Let (X,τ_N) be a neutrosophictopological space. A subset A of (X,τ_N) is called neutrosophicgeneralized closed set (neutrosophic-g-closed) **[5]** if Ncl(A) $\subseteq$ G whenever A $\subseteq$ G and G is neutrosophicopen set (NOS). Complement of neutrosophic-g-closed set is called the neutrosophic-g-open set.

Definition 2.4. Let (X,τ_N) be a neutrosophictopological space. A subset A of (X,τ_N) is called neutrosophicgeneralized pre regular closed set Ngpr-closed) **[6]** if Ncl(A) $\subseteq$ G whenever A $\subseteq$ G and G is neutrosophicregular open set (NROS). Complement of neutrosophic-gpr-closed set is called the neutrosophic-gpr-open set.

3.NeutrosophicPre*generalized Pre-Closed Sets

In this sequel, we introduce the concept of neutrosophicp*gp-closed sets and study their basic properties in neutrosophictopological spaces. In addition, their relationship among the various types of neutrosophicgeneralized closed sets is discussed.

Definition 3.1. A subset A of a topological space (X, τ_N) is called **Neutrosophicpre*generalized pre closed** (briefly**Np*gp-closed**) if $Npcl(A) \subseteq G$whenever$A \subseteq G$and $G$is neutrosophicpre*open in (X,τ_N) .The collection of all neutrosophicp*gp-closed sets in(X,τ_N) is denoted by$Np^* gp\text{-}C(X, \tau)$or simply by $Np^* gp\text{-}C(X)$.

Example 3.2. Let X = {a, b} with τ_N={0_N, 1_N, A, B} where A = $\{\langle 0.5, 0.6, 0.7\rangle, \langle 0.4, 03, 0.5\rangle\}$ and B = $\{\langle 0.4, 0.5, 0.6\rangle, \langle 0.4, 0.4, 0.6\rangle\}$. Then (X, τ_N) is a neutrosophic topological space. The closed sets of (X, τ_N) are A'=$\{\langle 0.7, 0.4, 0.5\rangle, \langle 0.5, 0.7, 0.4\rangle\}$ &B' = $\{\langle 0.6, 0.5, 0.4\rangle, \langle 0.6, 0.6, 0.4\rangle\}$. Consider the neutrosophic set C = $\{\langle 0.5, 0.5, 0.6\rangle, \langle 0.5, 0.6, 0.7\rangle\}$ in (X, τ_N). Here C is a neutrosophic p*gp-closed set in (X, τ_N).

Theorem 3.3. In any neutrosophictopological space, every neutrosophicp*gp-closed set isneutrosophicgpr-closed.

Proof. Let $A \subseteq G$and G be neutrosopic regular open in (X,τ_N).Thus, N$pcl(A) \subseteq G$ whenever $A \subseteq$G and G is neutrosopic regular open. Therefore, A is neutrosophicgpr-closed. The complement of a neutrosophicgpr-closed set is called a neutrosophicgpr-open set.

Remark 3.4. The converse of the above theorem need not be true as seen in the following example.

Example 3.5. Let X = {a, b} with τ_N={0_N, 1_N, A, B} where A = $\{\langle 0.4, 0.6, 0.8\rangle, \langle 0.7, 0.5, 0.5\rangle\}$ and B = $\{\langle 0.6, 0.5, 0.6\rangle, \langle 0.5, 0.5, 0.7\rangle\}$. Then (X, τ_N) is a neutrosophic topological space. The closed sets of (X, τ_N) are

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A'={⟨0.8, 0.4, 0.4⟩, ⟨0.5, 0.5, 0.7⟩} &B' = {⟨0.6, 0.5,0.6⟩, ⟨0.7,0.5, 0.5⟩}. Consider the neutrosophicgpr closed set C = {⟨0.5, 0.4, 0.6⟩, ⟨0.5,0.6, 0.7⟩} in (X, τ_N). Here C is a not neutrosophic p*gp-closed set in (X, τ_N).

Theorem 3.6. In any neutrosophictopological space, every neutrosophic closed set is neutrosophicp*gp-closed.

Proof. Let $A \subseteq G$ and G be neutrosophicpre*open. $\mathrm{N}cl(A) \subseteq$ G.Thus, $Npcl(A) \subseteq G$ whenever $A \subseteq$ G and G is neutrosophicpre*open. Therefore, G is neutrosophicp*gp-closed. The complement of aneutrosophicp*gp-closed set is called a neutrosophicp*gp-open set.

Remark 3.7. The converse of the above theorem need not be true as seen in the following example.

Example 3.8. Let X = {a, b} with τ_N={0_N, 1_N, A, B} where A = {⟨0.4, 0.6, 0.6⟩, ⟨0.5, 03, 0.5⟩} and B = {⟨0.4, 0.5, 0.6⟩, ⟨0.4,0.6, 0.6⟩}. Then (X, τ_N) is a neutrosophic topological space. The closed sets of (X, τ_N) are A'= {⟨0.6, 0.4, 0.4⟩, ⟨0.5, 0.7, 0.5⟩} &B' = {⟨0.6, 0.5, 0.4⟩, ⟨0.6,0.4, 0.4⟩}. Consider the neutrosophic p*gp-closed set C = {⟨0.5, 0.5, 0.6⟩, ⟨0.5,0.6, 0.7⟩} in (X, τ_N). Here C is a not neutrosophic closed set in (X, τ_N).

4. Characterizations of neutrosophicp*gp-closed sets

Theorem 4.1. The arbitrary of union of the neutrosophicp*gp-closed sets is neutrosophicp*gp-closed.

Proof. Let $A$and B be neutrosophicp*gp-closed sets in (X,τ_N). Let G be a neutrosophicpre*open in (X,τ_N) such that $A \cup B \subseteq G$. Then A$\subseteq G$ and $B \subseteq G$. Since A and B are neutrosophicp*gp-closed sets,$Npcl(A) \subseteq G$and$Npcl(B) \subseteq G$. Then$Npcl(A \cup B) = Npcl(A) \cup Npcl(A) \subseteq G$. Hence $A \cup B$ is neutrosophicp*gp-closed sets.

Theorem 4.2. The finite of intersection of two neutrosophicp*gp-closed sets is neutrosophicp*gp-closed.

Proof. The proof is obvious.

Theorem 4.3. The intersection of a neutrosophicp*gp-closed set and a neutrosophicclosed set is a neutrosophicpre*closed.

Proof. Let A be a neutrosophicp*gp-closed set and B is a neutrosophicclosed set. If G is a neutrosophicopen set with A∩ $B \subseteq G$, then G is neutrosophicpre*open and A ⊆G∪ (1_N-B). So, Npcl(A) ⊆G∪ (1_N-B).Then, Ncl(A∩ B) = Ncl(A)∩Ncl(B) ⊆ Npcl(A)∩Ncl(B) = Npcl(A)∩B⊆ G. Hence, A∩ B is a neutrosophicpre*closed set.

Theorem 4.4. Let A⊆ $B \subseteq Npcl(A)$ and A is a Np*gp-closed subset of (X,τ_N), then B is also a Np*gp-closed subset of (X,τ_N).

Proof. Given A is a Np*gp-closed subset of (X,τ_N), then Npcl(A) ⊆ G, whenever A ⊆ G, G is anneutrosophicprre*open subset of (X,τ_N). Let A ⊆ B, $Npcl(B) \subseteq$ pcl(A). Let, if possible, there exist a neutrosophicpre*open subset Hof X,τ_N), such that B ⊆ H. So, A ⊆ H and B being Np*gp-closed subset of (X,τ_N), Npcl(A) ⊆ H, then Npcl(B) ⊆ H. Hence B is also a Np*gp-closed subset of (X,τ_N).

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5. Conclusion

In this chapter, initiallyneutrosophicp*gp-closed sets is defined. Then, the properties of closed sets, gpr-closed sets are studied in relation with newly introduced neutrosophicp*gp-closed sets. Some new theorems are derived and their relationships with existing sets are found out.

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SHADOW SOFT SEPARATION AXIOMS: A THEORETICAL FRAMEWORK VIA SHADOW SOFT POINTS

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Abstract

Shadow soft set improves the accuracy of modelling complex systems characterized by partial or ambiguous information. In this work, we introduce the concept of shadow soft point, which serves as the fundamental element for developing shadow soft topology, accompanied by illustrative examples. Furthermore, explore neighborhood system of shadow soft point and its properties. Additionally, we formulate shadow soft T_i-spaces (i =0,1,2)in terms of shadow soft points and provide a detailed analysis of relationships among the spaces.

Keywords: Shadow Soft Set, Shadow Soft Topological Space, Shadow Soft Point, Shadow Soft Separation Axioms.

2020 Mathematics Subject Classifications:54A40, 03E72, 94D05

Introduction:

Uncertainty is an unavoidable aspect of life, influencing every choice we make and shaping how we interpret the world.To capture the unpredictability present in real world scenarios,Pedrycz [1] introduced shadowed set theory in 1998, which incorporate a shadow region around the fuzzy set to represent elements that are ambiguous or uncertain.It focuses on representing the impact of vagueness in specific regions around the fuzzy set, rather than simply classifying objects as belonging or not belonging to a set.Studies on this concept from various perspectives and applications of shadowed sets in various fields have been studied by several researchers includes Ibrahim, A. M. et al [2], Deng and Yao [3], Cattaneo and Ciucci [4] and Zhang and Yao [5].Shadow soft set theory, introduced by Alkhazaleh [6], builds upon the concept of shadowed sets associated with a suitable set of parameters.The shadow soft set can be induced from fuzzy soft set by determining the shadow parameter set. In shadow soft sets, each element of soft set belongs to any of the three distinct

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regions namely, core region, shadow region and exclusion regions using corresponding pair of shadow threshold value (α,β) from [0,1] based on the membership grades of its elements.

In this work, we defined shadow soft point and analyzed how shadow soft point belongs to shadow soft set with examples. Also, neighborhood system of shadow soft point is introduced by the collection of shadow soft sets that describe the local structure around the shadow soft point. Furthermore, the concept of shadow soft T_i-spaces (i=0,1,2) are defined using shadow soft points and investigated its characterizations.

Preliminaries:

Definition 2.1 [6]

Let U = $\{u_1, u_2, \dots, u_n\}$ be the universal set, E= $\{e_1, e_2, \dots, e_m\}$ be the set of parameters then (F,E) be a fuzzy soft set overU. Let $shdw$= $\{(\alpha_1, \beta_1), (\alpha_2, \beta_2) \dots, (\alpha_m, \beta_m)\}$ be the shadow parameter set relevant to E. Let$shdw$(U)be the set of all shadowed subsets on U. A pair

$(F, E)_{shdw}$is called shadow soft setoverU, where $F_{(\alpha_i,\beta_i)}$ is a mapping given by

$F_{(\alpha_i,\beta_i)}$: E→$shdw$(U)

for all$i$= 1,2 …$m$andwe can write it as $F_{(\alpha_i,\beta_i)}(e_i)$=$\left\{\frac{x_j}{f_i(x_j)}\right\}$,∀$j$= 1,2 …$n$ and$i$= 1,2 …m, where

$$f_{\mathrm{i}}(x_{\mathrm{j}}) = \begin{cases} 0, & \mu_i(u_j) \le \alpha_i \\ 1, & \mu_i(u_j) \ge \beta_i \\ [0,1], & \alpha_i < \mu_i(u_j) < \beta_i \end{cases}$$

Definition 2.2 [6]

A null shadow soft set $(\Phi, E)_{shdw}$over a universe U is a shadow soft set with $\phi_{(\alpha_i,\beta_i)}(e_i) = 0$, ∀ e_i ∈E.

Definition 2.3 [6]

Anabsolute shadow soft set $(\Psi, E)_{shdw}$over a universe U is a shadow soft set with$\Psi_{(\alpha_i,\beta_i)}(e_i)$ = 1, ∀ e_i ∈E.

Definition 2.4 [7]

A Shadow soft topology on(U, E)is a collectionτ ofshadow soft set over Uif

i) $(\Phi, E)_{shdw}$, $(\Psi, E)_{shdw}$∈τ
ii) If$(A, E)_{shdw}$, $(B, E)_{shdw}$∈τ then $(A, E)_{shdw} \cap (B, E)_{shdw}$∈τ
iii) If $(A_i, E)_{shdw}$∈τ for each $i \in$ I then $\cup_{i\in I}$ $(A_i, E)_{shdw}$∈τ

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Then (U, τ, E) is called shadow soft topological space over U. Also, members of τ are called shadow soft open set in (U, τ, E). A shadow soft set$(A,E)_{shdw}$over U is called shadow soft closed set in (U, τ, E) if and only if its complement$(A,E)^{C}_{shdw}$is shadow soft open set in (U, τ, E).

Definition 2.5 [7]

An indiscrete shadow soft topological space (U, τ, E) contains only $(\Phi,E)_{shdw}$and $(\Psi,E)_{shdw}$and discrete shadow soft topology τ is the collection of all shadow soft sets over (U, E).

Definition 2.6 [7]

Let (U, τ, E) be ashadow soft topological space. Let V⊆U and$(V,E)_{shdw}$be an absolute shadow soft seton V, then

τ_V = {$(V,E)_{shdw}\cap(A,E)_{shdw}$: $(A,E)_{shdw}\in\tau$} is known asthe shadow soft subspace topology over V and (V,τ_V,E)is shadow soft subspace of (U, τ, E).

Shadow soft separation axioms

Definition 3.1

A shadow soft point of a shadow soft set $(A,E)_{shdw}$ over (U, E) is denoted by ${e_i}^{u_j}(A^E_{shdw})$ such that for the element e_i∈E and u_j∈U, $A_{(\alpha_i,\beta_i)}({e_i}^{u_j})=[0,1]$ or 1.

Example 3.2

Let U = {u_1,u_2, u_3}be the universal set of elements and E = {e_1,e_2,e_3}be the set of parameters.Consider the shadow soft set$(A,E)_{shdw}$ over (U, E),

$(A,E)_{shdw}$= {$(e_1,\left\{\frac{u_1}{1},\frac{u_2}{0},\frac{u_3}{[0,1]}\right\})$,$(e_2,\left\{\frac{u_1}{[0,1]},\frac{u_2}{1},\frac{u_3}{1}\right\})$,$(e_3,\left\{\frac{u_1}{[0,1]},\frac{u_2}{[0,1]},\frac{u_3}{0}\right\})$}

Then the shadow soft points of $(A,E)_{shdw}$are${e_1}^{u_1}(A^E_{shdw})$ = $(e_1,\left\{\frac{u_1}{1}\right\})$, ${e_1}^{u_3}(A^E_{shdw})$ = $(e_1,\left\{\frac{u_3}{[0,1]}\right\})$,${e_2}^{u_1}(A^E_{shdw})$ = $(e_2,\left\{\frac{u_1}{[0,1]}\right\})$, ${e_2}^{u_2}(A^E_{shdw})$ = $(e_2,\left\{\frac{u_2}{1}\right\})$, ${e_2}^{u_3}(A^E_{shdw})$ = $(e_1,\left\{\frac{u_1}{1}\right\})$, ${e_3}^{u_1}(A^E_{shdw})$ = $(e_3,\left\{\frac{u_1}{[0,1]}\right\})$, ${e_3}^{u_2}(A^E_{shdw})$ = $(e_3,\left\{\frac{u_2}{[0,1]}\right\})$.

Definition 3.3

A shadow soft set consist of a single shadow soft point is known as shadow soft singletonset.

Remark3.4

Shadow soft union of all the shadow soft points of a shadow soft set is equal to the shadow soft set.

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Definition 3.5

The complement of a shadow soft point ${e_i}^{u_j}(A^E_{shdw})$ is a shadow soft point denoted by $({e_i}^{u_j}(A^E_{shdw}))^c$ such that $A_{(\alpha_i,\beta_i)}({e_i}^{u_j}) = c(A_{(\alpha_i,\beta_i)}({e_i}^{u_j}))$ where c is a shadow compliment.

Remark3.6

The shadow soft complement of a shadow soft point over (U, E) need not to be a shadow soft point.

Definition 3.7

A shadow soft point ${e_i}^{u_j}(A^E_{shdw})$ is said to belong to a shadow soft set $(G,E)_{shdw}$, if for the element $e_i \in$E and $u_j \in$U, $A_{(\alpha_i,\beta_i)}({e_i}^{u_j}) \leq G_{(\alpha_i,\beta_i)}({e_i}^{u_j})$and denoted by ${e_i}^{u_j}(A^E_{shdw}) \in (G,E)_{shdw}$.

Example 3.8

Let U = $\{u_1, u_2, u_3\}$ and E = $\{e_1, e_2\}$

Consider a shadow soft point ${e_2}^{u_1}(A^E_{shdw}) = (e_2, \left\{\frac{u_1}{[0,1]}\right\})$ from example 3.2 and a shadow soft set$(G,E)_{shdw}$ over (U, E) is defined as,

$(G,E)_{shdw}$= $\{(e_1, \left\{\frac{u_1}{1}, \frac{u_2}{0}, \frac{u_3}{[0,1]}\right\}), (e_2, \left\{\frac{u_1}{1}, \frac{u_2}{0}, \frac{u_3}{1}\right\})\}$

Here, for$e_2 \in$ E,$u_1 \in$U, $A_{(\alpha_2,\beta_2)}({e_2}^{u_1}) \leq G_{(\alpha_2,\beta_2)}({e_2}^{u_1})$. Thus, ${e_2}^{u_1}(A^E_{shdw}) \in (G,E)_{shd}$.

Definition 3.9

A shadow soft set $(L,E)_{shdw}$in a shadow soft topological space(U, τ, E) is said to be a shadow soft neighborhood of a shadow soft point ${e_i}^{u_j}(A^E_{shdw})$ if there exists a shadow soft open set $(M,E)_{shdw}$such that ${e_i}^{u_j}(A^E_{shdw}) \in (M,E)_{shdw} \subseteq (L,E)_{shdw}$, for$e_i \in$E and $u_j \in$U.

Note:

The family of all shadow soft neighborhoods of ${e_i}^{u_j}(A^E_{shdw})$ is called its shadow soft neighborhood system and is denoted by $\mathcal{N}_\tau({e_i}^{u_j}(A^E_{shdw}))$.

Proposition 3.10

An arbitrary shadow soft union of shadow soft neighbourhoods of a shadow softpointis again a shadow softneighbourhood.

Proof

Let $\{(A_\lambda,E)_{shdw}\}$ be an arbitrary collection of shadow soft neighbourhoods of ${e_i}^{u_j}(A^E_{shdw})$.

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For each λ ∈ I, there exists a shadow soft open set$(H_\lambda, E)_{shdw}$such that$e_i^{u_j}(A_{shdw}^E) \in (H_\lambda, E)_{shdw} \subseteq (A_\lambda, E)_{shdw} \subseteq \cup_{\lambda \in I}\ (A_\lambda, E)_{shdw}$.

That is $e_i^{u_j}(A_{shdw}^E) \in (H_\lambda, E)_{shd} \subseteq \cup_{\lambda \in I}\ (A_\lambda, E)_{shdw}$. Therefore, $\cup_{\lambda \in I}\ (A_\lambda, E)_{shdw}$ is a shadow soft neighbourhoodof$e_i^{u_j}(A_{shdw}^E)$. That is, arbitrary shadow soft union of shadow soft neighbourhoods of $e_i^{u_j}(A_{shdw}^E)$ is again a shadow softneighbourhood of $e_i^{u_j}(A_{shdw}^E)$.

Remark 3.11

Theshadow soft intersection of shadow soft neighbourhoods of a shadow softpoint need not to be a shadow softneighbourhood of that point.

Theorem 3.12

A shadow soft set in a shadow soft topological space (U, τ, E) is shadow softopen if and only if it is a shadow soft neighborhood ofeach of its shadow soft points.

Proof

Let (U, τ, E) be a shadow soft topological space and $e_i^{u_j}(A_{shdw}^E)$be a shadow soft point in a shadow soft open set $(A, E)_{shdw}$. Then by definition $(A, E)_{shdw}$ is a shadow soft neighborhood of $e_i^{u_j}(A_{shdw}^E)$.

Conversely, suppose$(A, E)_{shdw}$is a shadow soft set such that it is shadow soft neighborhoodof each of its shadow soft points. Then foreach$e_i^{u_j}(A_{shdw}^E) \in (A, E)_{shdw}$, there exists a shadow soft open set $(H_\lambda, E)_{shdw}$, such that $e_i^{u_j}(A_{shdw}^E) \in (H_\lambda, E)_{shdw} \subseteq (A, E)_{shdw}$.

Also, $(A, E)_{shdw}$ is the shadow soft union of each of its shadow soft points which implies $(A, E)_{shdw} = \cup_{\lambda \in I}\ (H_\lambda, E)_{shdw}$.

Since the arbitrary union of family of shadow soft openset is open, $(A, E)_{shdw}$ is shadow soft open.

Theorem3.13

The shadow soft neighborhood system of a shadow soft point $e_i^{u_j}(A_{shdw}^E)$in a shadow soft topological space (U, τ, E) satisfies the followingproperties:

i) If $(A, E)_{shdw} \in \tilde{\mathcal{N}}_\tau(e_i^{u_j}(A_{shdw}^E))$ then $e_i^{u_j}(A_{shdw}^E) \in (A, E)_{shdw}$
ii) A shadow soft superset of a shadow soft neighborhoodof a shadow soft point is also a shadow soft neighborhood of the point.
iii) Intersection of two shadow soft neighborhoods ofa shadow soft point is again a shadow softneighborhood.

Proof

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i) If $(A,E)_{shdw} \in \tilde{\mathcal{N}}_{\tau}(e_i^{u_j}(A_{shdw}^E))$ then there exist a shadow soft open set $(H,E)_{shdw} \in \tau$ such that $e_i^{u_j}(A_{shdw}^E) \in (H,E)_{shdw} \subseteq (A,E)_{shdw} \Longrightarrow e_i^{u_j}(A_{shdw}^E) \in (A,E)_{shdw}$.

ii) Let $(A,E)_{shdw} \in \tilde{\mathcal{N}}_{\tau}(e_i^{u_j}(A_{shdw}^E))$ and $(A,E)_{shdw} \subseteq (K,E)_{shdw}$
Then there exist a shadow soft open set $(H,E)_{shdw}$ such that $e_i^{u_j}(A_{shdw}^E) \in (H,E)_{shdw} \subseteq (A,E)_{shdw} \Longrightarrow$
$e_i^{u_j}(A_{shdw}^E) \in (H,E)_{shdw} \subseteq (K,E)_{shdw} \Longrightarrow (K,E)_{shdw} \in \tilde{\mathcal{N}}_{\tau}(e_i^{u_j}(A_{shdw}^E))$

iii) Let $(A,E)_{shdw}, (B,E)_{shdw} \in \tilde{\mathcal{N}}_{\tau}(e_i^{u_j}(A_{shdw}^E))$ then there exists a shadow soft open sets $(H,E)_{shdw}$ and $(S,E)_{shdw}$ such that $e_i^{u_j}(A_{shdw}^E) \in (H,E)_{shdw} \subseteq (A,E)_{shdw}$ and $e_i^{u_j}(A_{shdw}^E) \in (S,E)_{shdw} \subseteq (B,E)_{shdw} \Longrightarrow e_i^{u_j}(A_{shdw}^E) \in (H,E)_{shdw} \cap (S,E)_{shdw} \subseteq (A,E)_{shdw} \cap (B,E)_{shdw}$. Now, $(H,E)_{shdw} \cap (S,E)_{shdw}$ is shadow soft open and hence $(A,E)_{shdw} \cap (B,E)_{shdw} \in \tilde{\mathcal{N}}_{\tau}(e_i^{u_j}(A_{shdw}^E))$.

Definition 3.14

A shadow soft topological space (U, τ, E) is said to be shadow soft T_0 – space if for every pair of distinct shadow soft points $e^u(A_{shdw}^E), e^u(B_{shdw}^E)$ over (U, E), there exists a shadow soft open set containing one but not the other.

Example 3.15

Let U = $\{u_1, u_2\}$, E = $\{e_1, e_2\}$ and τ be the discrete shadow softtopology on U.

Here, for every distinct shadow soft point in U, there exists a shadow soft open set containing one of the points but not contains the other which is distinct.

Hence,(U, τ, E) is shadow soft T_0 – space.

Theorem 3.16

A shadow soft subspace of a shadow soft T_0 – space is shadow soft T_0 – space. That is, the property of being a T_0 – space of a shadow soft topological space is hereditary.

Proof

Let (V, τ_V, E)be a shadow soft subspace of a shadow soft T_0 –space (U, τ, E).

Let $e^u(A_{shdw}^E), e^u(B_{shdw}^E)$be two distinct shadow soft pointsin (V, E). Then these shadow soft points are also in (U, E).

Thus, there exist a shadow soft open set $(G,E)_{shdw} \in \tau$containing one shadow soft point say, $e^u(A_{shdw}^E)$ but not the other $e^u(B_{shdw}^E)$.

That implies $(V,E)_{shdw} \cap (G,E)_{shdw}$ is a shadow soft open set in (V, τ_V, E)containing $e^u(A_{shdw}^E)$ but not $e^u(B_{shdw}^E)$.

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Hence (V,τ_V,E)is a shadow soft T_0 – space.

Definition 3.17

A shadow soft topological space (U, τ, E) is said to be shadow soft T_1 – space if for every pair of distinct shadow soft points $e^u(A^E_{shdw})$,$e^u(B^E_{shdw})$ over (U, E), there exists a shadow soft open sets $(G,E)_{shdw}$,$(H,E)_{shdw}$such that $e^u(A^E_{shdw})\in(G,E)_{shdw}$, $e^u(B^E_{shdw})\notin(G,E)_{shdw}ande^u(B^E_{shdw})\in(H,E)_{shdw}$, $e^u(A^E_{shdw})\notin(H,E)_{shdw}$.

Example 3.18

Let U = $\{u_1,u_2\}$, E = $\{e_1,e_2\}$ and τ be the discrete shadow softtopology on U.

Here, for every arbitrary distinct pair of shadow soft points there exist a pair of shadow soft open sets which satisfies the condition of shadow soft T_1 – space.

Thus,(U, τ, E) is shadow soft T_1 – space.

Theorem 3.19

A shadow soft subspaceof a shadow soft T_1 – space (U, τ, E) is shadow soft T_1 – space.

Proof

Let (V,τ_V,E)be a shadow soft subspace of a shadow soft T_1 –space (U, τ, E).

Let $e^u(A^E_{shdw})$,$e^u(B^E_{shdw})$be two distinct shadow soft pointsin (V, E). Then these shadow soft points are also in (U, E). Thus there exist a shadow soft open sets $(G,E)_{shdw}$ and $(H,E)_{shdw}\in\tau$such that $e^u(A^E_{shdw})\in(G,E)_{shdw}$, $e^u(B^E_{shdw})\notin(G,E)_{shdw}$and $e^u(B^E_{shdw})\in(H,E)_{shdw}$, $e^u(A^E_{shdw})\notin(H,E)_{shdw}$.

That implies $(V,E)_{shdw}\cap(G,E)_{shdw}$, $(V,E)_{shdw}\cap(H,E)_{shdw}$ are shadow soft open sets in (V,τ_V,E)such that $e^u(A^E_{shdw})\in(V,E)_{shdw}\cap(G,E)_{shdw}$, $e^u(B^E_{shdw})\notin(V,E)_{shdw}\cap(G,E)_{shdw}$and $e^u(B^E_{shdw})\in(V,E)_{shdw}\cap(H,E)_{shdw}$, $e^u(A^E_{shdw})\notin(V,E)_{shdw}\cap(H,E)_{shdw}$.

Hence (V,τ_V,E)is a shadow soft T_1 – space.

Definition 3.20

A shadow soft topological space (U, τ, E) is said to be a shadow soft T_2 – spaceif for every pair of distinctshadow soft points $e^u(A^E_{shdw})$, $e^u(B^E_{shdw})$ over (U, E), there exists disjoint shadow soft open sets $(G,E)_{shdw}$,$(H,E)_{shdw}$such that $e^u(A^E_{shdw})\in(G,E)_{shdw}$and $e^u(B^E_{shdw})\in(H,E)_{shdw}$.

Example 3.21

Let U = $\{u_1,u_2\}$, E = $\{e_1,e_2\}$ and τ be the discrete shadow softtopology on U.

Then, for every pair of distinctshadow soft points there exists disjoint shadow soft open setsin τ containing them.

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Thus,(U, τ, E) is shadow soft T_2 – space.

Theorem 3.22

A shadow soft subspace of a shadow soft T_2 – space (U, τ, E) is shadow soft T_2 – space.

Proof

Let (V,τ_V,E)be a shadow soft subspace of a shadow soft T_2 –space (U, τ, E).

Let $e^u(A^E_{shdw})$,$e^u(B^E_{shdw})$be two distinct shadow soft points in (V, E). Then these shadow soft points are also in (U, E).

Hence there exists disjoint shadow soft open sets $(G,E)_{shdw}$ and $(H,E)_{shdw}$inτsuch that $e^u(A^E_{shdw})\in (G,E)_{shdw}$ and $e^u(B^E_{shdw})\in(H,E)_{shdw}$.

Thus, $(V,E)_{shdw}\cap(G,E)_{shdw}$ and $(V,E)_{shdw}\cap(H,E)_{shd}$ are disjoint shadow soft open sets in τ_V such that $e^u(A^E_{shdw})\in (V,E)_{shdw}\cap(G,E)_{shdw}$ and $e^u(B^E_{shdw})\in (V,E)_{shdw}\cap(H,E)_{shdw}$.

Hence, (V,τ_V,E)is a shadow soft T_2 – space.

Proposition 3.23

For a shadow soft topological space (U, τ, E) we have:

Shadow soft T_2 – space $\Longrightarrow$ Shadow soft T_1 – space $\Longrightarrow$Shadow soft T_0 – space.

Proof

(1) Let (U, τ, E) be a shadow soft T_2 – space and let $e^u(A^E_{shdw})$, $e^u(B^E_{shdw})$be two distinct shadow soft points. Then there exist disjoint shadow soft open sets $(G,E)_{shdw}$and $(H,E)_{shdw}$inτsuch that $e^u(A^E_{shdw})\in (G,E)_{shdw}$ and $e^u(B^E_{shdw})\in(B,E)_{shdw}$. Since $(G,E)_{shdw}\cap(H,E)_{shdw} = (\Phi,E)_{shdw}$, $e^u(A^E_{shdw})\notin (H,E)_{shdw}$and $e^u(B^E_{shdw})\notin (G,E)_{shdw}$. Thus there exists shadow soft open sets$(G,E)_{shdw}$ and $(H,E)_{shdw}$ such that $e^u(A^E_{shdw})\in (G,E)_{shdw}$,$e^u(B^E_{shdw})\notin (G,E)_{shdw}$and $e^u(B^E_{shdw})\in (H,E)_{shdw}$, $e^u(A^E_{shdw})\notin (H,E)_{shdw}$. Hence,(U, τ, E) is a shadow soft T_1 – space.

(2) Let (U, τ, E) be a shadow soft T_1 – space and let $e^u(A^E_{shdw})$,$e^u(B^E_{shdw})$be two distinct shadow soft points.Then there exist disjoint shadow soft open sets $(G,E)_{shdw}$ and $(H,E)_{shdw}$inτsuch that$e^u(A^E_{shdw})\in (G,E)_{shdw}$,$e^u(B^E_{shdw})\notin (G,E)_{shdw}$and $e^u(B^E_{shdw})\in (H,E)_{shdw}$, $e^u(A^E_{shdw})\notin (H,E)_{shdw}$.Thus,we obtain a shadow soft set containing one of the shadow soft points but not the other. Hence,(U, τ, E) is a shadow soft T_0 – space.

Note:

The inverse of the above result implications need not be true.

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Conclusion:

In this work, we defined shadow soft points in terms of the exclusion region and the fundamental properties of shadow soft points are studied with examples. Results about neighborhood of shadow soft point are investigated. Additionally, we have introduced shadow soft T_i-spaces (i =0,1,2) by using shadow soft points and analyze the relations between eachT_i-spaces. In future we will investigates the concept of shadow soft regular space and shadow soft normal space, it will be very useful for further research work in shadow soft topological spaces.

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THEORETICAL INSIGHTS INTO TYPES OF TURIYAM SOFT RELATIONS

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Abstract

In this paper we study the Types of Turiyam Soft Relations. We establish the Equivalence Class of Turiyam Soft Relations. We also introduce the concepts of Turiyam Soft Partition and Turiyam Soft Block and the theorems related to this topic has also been discussed.

Keywords: Symmetric Turiyam Soft Relation, Transitive Turiyam Soft Relation, Reflexive Turiyam Soft Relation, Equivalence Class, Partition of Turiyam Soft Set.

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1. Introduction:

An intriguing theoretical idea called "Soft Set Theory" was developed in 1999 by Molodtsov [1], aids in coping with life's uncertainties and directs us toward making wise decisions.In 2005, SmarandacheintroducedNeutrosophic Set [2] as the generalization of Intuitionistic Fuzzy Set involving Indeterminacy values along with the Truth and Falsity membership values. P. K. Maji studied this concept of Neutrosophic Set and defined Neutrosophic Soft Set in 2011 [3].

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The concept of Neutrosophic Soft Relations, their Composition, Inverse and its Types was defined by Deli and Broumiin 2015 [4]. In 2021, Orhan Dalkilic [5] defined the concept of Relations on Neutrosophic Soft Set in the context of the theory of Neutrosophic Soft Set proposed by Deli and Broumi. As a generalization of Neutrosophic Set, Prem Kumar Singh introduced and studied the concept of Turiyam Set [6-9]. Turiyam Set involves the Liberalization value as the Fourth dimension in addition to the Truth, Indeterminacy and Falsity membership values. Gamachu Adugna Ganati, V. N. Srinivasa Rao Repalle and Mamo Abebe Ashebo defined Relations on Turiyam sets in 2023 [10].

In this paper, we define Turiyam Soft Relation and study their types, the Equivalence Class of Turiyam Soft Relation and the Partition of Turiyam Soft Sets together with the theorems related to Types of Turiyam Soft Relation.

2.1.Preliminaries:

Definition – 2.1.1 [1]

Let U be an initial universe and E be the set of parameters. Let $P(U)$ denotes the power set of U. Consider a nonempty set A, $A \subset E$. A pair (F, A) is called a Soft Set over U, where F is a mapping given by $F: A \to P(U)$.

Definition – 2.1.2 [6]

A Turiyam set A on U has the form

$$A = \{< u, t_A(u), i_A(u), f_A(u), l_A(u) >: u \in U\}$$

where $t_A(u): U \to [0,1], i_A(u): U \to [0,1], f_A(u): U \to [0,1]$ and$l_A(u): U \to [0,1]$ denote the truth value, theindeterminacy value, the falsity value and the Turiyamstate (or liberal) value for each $u \in U$, correspondingly bywhich $t_A(u), i_A(u), f_A(u)$ and $l_A(u)$ satisfies the condition$0 \leq t_A(u) + i_A(u) + f_A(u) + l_A(u) \leq 4, \quad \forall u \in U$.

Definition - 2.1.3 [11]

A Turiyam Soft Set $(\mathrm{F}, \mathrm{A})_{\mathrm{L}}$ on the universe U over the fixed parameter E is defined as,

$$(\mathrm{F}, \mathrm{A})_{\mathrm{L}} = \{\big(e, F(e)\big): e \in A, F(e) \in T(U)\}$$

Where, $\mathrm{T}(\mathrm{U})$ denotes the set of all Turiyam Sets of U and F is a mapping from $\mathrm{F}: \mathrm{A} \to \mathrm{T}(\mathrm{U}); \mathrm{A} \subset \mathrm{E}$.

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3. Relation on Turiyam Soft Set:

3.1.Turiyam Soft Relation:

In this section, we define the Turiyam Soft Relation with an example.

Definition – 3.1.1:

Let $(\mathrm{F},\mathrm{A})_{\mathrm{L}}$ and $(\mathrm{G},\mathrm{B})_{\mathrm{L}}$ be two Turiyam Soft Sets over a fixed universe U, then a Turiyam Soft Relation from $(\mathrm{F},\mathrm{A})_{\mathrm{L}}$ to $(\mathrm{G},\mathrm{B})_{\mathrm{L}}$ is a Turiyam Soft Subset of $(F,A)_L \times (G,B)_L$ and is of the form $(\mathcal{R},\mathrm{C}\times\mathrm{D})_L$, where $\mathrm{C}\times\mathrm{D} \subseteq \mathrm{A}\times\mathrm{B}$ and $\mathcal{R}(\mathrm{e},\mathrm{f}) \subseteq (F,A)_L \times (G,B)_L, \forall (e,f) \in C \times D$ and is defined as,

$$(\mathcal{R},\mathrm{C}\times\mathrm{D})_L = \{((e,f),\mathcal{R}(\mathrm{e},\mathrm{f})) : (e,f) \in C \times D \subseteq A \times B\}$$

Example – 3.1.2:

Let $U = \{u_1,u_2,u_3\}, E = \{e_1,e_2,e_3,e_4,e_5\}$ and $\mathrm{A},\mathrm{B} \subset \mathrm{E}$, where $A = \{e_1,e_2\}$, $B = \{e_1,e_2,e_5\}$

Consider the Turiyam Soft Sets $(\mathrm{F},\mathrm{A})_{\mathrm{L}}$ and $(\mathrm{G},\mathrm{B})_L$ as follows:

$$(\mathrm{F},\mathrm{A})_{\mathrm{L}} = \left\{\begin{array}{l}\left(e_1,\left\{\frac{\mathrm{u}_1}{0.3,0.4,0.6,0.5},\frac{\mathrm{u}_2}{0.2,0.4,0.5,0.3},\frac{\mathrm{u}_3}{0.7,0.8,0.4,0.6}\right\}\right),\\ \left(e_2,\left\{\frac{\mathrm{u}_1}{0.6,0.4,0.5,0.6},\frac{\mathrm{u}_2}{0.4,0.5,0.3,0.1},\frac{\mathrm{u}_3}{0.1,0.3,0.5,0.7}\right\}\right)\end{array}\right\}$$

$$(\mathrm{G},\mathrm{B})_L = \left\{\begin{array}{l}\left(e_1,\left\{\frac{\mathrm{u}_1}{0.4,0.3,0.5,0.7},\frac{\mathrm{u}_2}{0.3,0.5,0.6,0.8},\frac{\mathrm{u}_3}{0.6,0.5,0.7,0.8}\right\}\right),\\ \left(e_2,\left\{\frac{\mathrm{u}_1}{0.2,0.4,0.3,0.8},\frac{\mathrm{u}_2}{0.5,0.2,0.2,0.6},\frac{\mathrm{u}_3}{0.4,0.4,0.9,0.6}\right\}\right),\\ \left(e_5,\left\{\frac{\mathrm{u}_1}{0.5,0.6,0.8,0.3},\frac{\mathrm{u}_2}{0.4,0.3,0.7,0.3},\frac{\mathrm{u}_3}{0.5,0.7,0.8,0.3}\right\}\right)\end{array}\right\}$$

$(\mathrm{F},\mathrm{A})_{\mathrm{L}}$ and $(\mathrm{G},\mathrm{B})_L$ are Turiyam Soft Sets. Then the Turiyam Soft Relation from $(\mathrm{F},\mathrm{A})_{\mathrm{L}}$ to $(\mathrm{G},\mathrm{B})_{\mathrm{L}}$ is,

$$(\mathcal{R},\mathrm{C}\times\mathrm{D})_L = \left\{\begin{array}{l}\left((e_1,e_2),\left\{\frac{\mathrm{u}_1}{0.2,0.4,0.6,0.5},\frac{\mathrm{u}_2}{0.2,0.4,0.5,0.3},\frac{\mathrm{u}_3}{0.4,0.8,0.9,0.6}\right\}\right),\\ \left((e_1,e_5),\left\{\frac{\mathrm{u}_1}{0.3,0.6,0.8,0.3},\frac{\mathrm{u}_2}{0.2,0.4,0.7,0.3},\frac{\mathrm{u}_3}{0.5,0.8,0.8,0.3}\right\}\right),\\ \left((e_2,e_2),\left\{\frac{\mathrm{u}_1}{0.2,0.4,0.5,0.6},\frac{\mathrm{u}_2}{0.4,0.5,0.3,0.1},\frac{\mathrm{u}_3}{0.1,0.4,0.9,0.6}\right\}\right)\end{array}\right\}$$

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3.2.Types of Turiyam Soft Relation:

Definition – 3.2.1:

Let $(\mathcal{R}, C \times D)_L$ be a Turiyam Soft Relations on $(\mathrm{F}, \mathrm{A})_{\mathrm{L}}$.

i) $(\mathcal{R}, C \times D)_L$ is Reflexive Turiyam Soft Relation if $\mathcal{R}(\mathrm{e}, \mathrm{e}) \in (\mathcal{R}, C \times D)_L, \forall\, e \in A$, $(e, e) \in C \times D \subseteq A \times A$.

ii) $(\mathcal{R}, C \times D)_L$ is Symmetric Turiyam Soft Relation if $\mathcal{R}(\mathrm{e}, \mathrm{f}) \in (\mathcal{R}, C \times D)_L \Rightarrow \mathcal{R}(\mathrm{f}, \mathrm{e}) \in (\mathcal{R}, C \times D)_L, \forall\, e, f \in A,\ (e, f) \in C \times D \subseteq A \times A$.

iii) $(\mathcal{R}, C \times D)_L$ is Transitive Turiyam Soft Relation if $\mathcal{R}(\mathrm{e}, \mathrm{f}) \in (\mathcal{R}, C \times D)_L$ *and* $\mathcal{R}(\mathrm{f}, \mathrm{g}) \in (\mathcal{R}, C \times D)_L \Rightarrow \mathcal{R}(\mathrm{e}, \mathrm{g}) \in (\mathcal{R}, C \times D)_L,\ \forall\, e, f, g \in A,\ (e, f), (f, g), (e, g) \in C \times D \subseteq A \times A$.

iv) $(\mathcal{R}, C \times D)_L$ is Turiyam Soft Equivalence Relation if it is Reflexive, Symmetric and Transitive.

Definition – 3.2.2:

Let $(\mathcal{R}, C \times D)_L$ be a Turiyam Soft Relation from $(\mathrm{F}, \mathrm{A})_{\mathrm{L}}$to $(\mathrm{F}, \mathrm{A})_{\mathrm{L}}$. Then, the Equivalence class of $(\mathrm{e}, \mathrm{F}(\mathrm{e}))$ denoted by $\left[(\mathrm{e}, \mathrm{F}(\mathrm{e}))\right]_{\mathcal{R}}$ is defined as,

$$\left[(\mathrm{e}, \mathrm{F}(\mathrm{e}))\right]_{\mathcal{R}} = \left\{(f, F(f)) : \mathcal{R}(e, f) \in (\mathcal{R}, C \times D)_L\right\}$$

Example – 3.2.3:

From the Relation,

$$(\mathcal{R}, \mathrm{C} \times \mathrm{D})_L = \left\{\begin{array}{l}\left((e_1, e_2), \left\{\frac{\mathrm{u}_1}{0.2, 0.4, 0.6, 0.5}, \frac{\mathrm{u}_2}{0.2, 0.4, 0.5, 0.3}, \frac{\mathrm{u}_3}{0.4, 0.8, 0.9, 0.6}\right\}\right), \\ \left((e_1, e_5), \left\{\frac{\mathrm{u}_1}{0.3, 0.6, 0.8, 0.3}, \frac{\mathrm{u}_2}{0.2, 0.4, 0.7, 0.3}, \frac{\mathrm{u}_3}{0.5, 0.8, 0.8, 0.3}\right\}\right), \\ \left((e_2, e_2), \left\{\frac{\mathrm{u}_1}{0.2, 0.4, 0.5, 0.6}, \frac{\mathrm{u}_2}{0.4, 0.5, 0.3, 0.1}, \frac{\mathrm{u}_3}{0.1, 0.4, 0.9, 0.6}\right\}\right)\end{array}\right\}$$

the Equivalence class is,

$$\left[(\mathrm{e}_1, \mathrm{F}(\mathrm{e}_1))\right]_{\mathcal{R}} = \left\{\left(\left(e_2, \left\{\frac{\mathrm{u}_1}{0.2, 0.4, 0.3, 0.8}, \frac{\mathrm{u}_2}{0.5, 0.2, 0.2, 0.6}, \frac{\mathrm{u}_3}{0.4, 0.4, 0.9, 0.6}\right\}\right)\right)\right\}$$

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Lemma – 3.2.4:

Let $(\mathcal{R}, C \times D)_L$ be an Equivalence Turiyam Soft Relation on $(\mathrm{F}, \mathrm{A})_\mathrm{L}$. For any $\big(\mathrm{e}, \mathrm{F}(e)\big), \big(\mathrm{f}, \mathrm{F}(f)\big) \in (\mathrm{F}, \mathrm{A})_\mathrm{L}$, $\mathcal{R}(e, f) \in (\mathcal{R}, C \times D)_L$ if and only if $\big[\big(\mathrm{e}, \mathrm{F}(\mathrm{e})\big)\big]_{\mathcal{R}} = \big[\big(\mathrm{f}, \mathrm{F}(\mathrm{f})\big)\big]_{\mathcal{R}}$

Proof:

Assume that, $\big(\mathrm{e}_1, \mathrm{F}(e_1)\big) \in \big[\big(\mathrm{e}, \mathrm{F}(e)\big)\big]_{\mathcal{R}}$. Then, $\mathcal{R}(e_1, e) \in (\mathcal{R}, C \times D)_L$. Since $(\mathcal{R}, C \times D)_L$ be an Equivalence Turiyam Soft Relation, $\big(\mathrm{e}_1, \mathrm{F}(e_1)\big) \in \big[\big(\mathrm{f}, \mathrm{F}(f)\big)\big]_{\mathcal{R}}$ (by Transitive Turiyam Soft Relation). Hence, $\big[\big(\mathrm{e}, \mathrm{F}(e)\big)\big]_{\mathcal{R}} \subseteq \big[\big(\mathrm{f}, \mathrm{F}(f)\big)\big]_{\mathcal{R}}$.

Assume$\big(\mathrm{f}_1, \mathrm{F}(f_1)\big) \in \big[\big(\mathrm{f}, \mathrm{F}(f)\big)\big]_{\mathcal{R}}$. Then, $\mathcal{R}(f_1, f) \in (\mathcal{R}, C \times D)_L$. Since $(\mathcal{R}, C \times D)_L$ be an Equivalence Turiyam Soft Relation, $\big(\mathrm{f}_1, \mathrm{F}(f_1)\big) \in \big[\big(\mathrm{e}, \mathrm{F}(e)\big)\big]_{\mathcal{R}}$ (by Transitive Turiyam Soft Relation). Hence, $\big[\big(\mathrm{f}, \mathrm{F}(f)\big)\big]_{\mathcal{R}} \subseteq \big[\big(\mathrm{e}, \mathrm{F}(e)\big)\big]_{\mathcal{R}}$.

Thus, $\big[\big(\mathrm{e}, \mathrm{F}(e)\big)\big]_{\mathcal{R}} = \big[\big(\mathrm{f}, \mathrm{F}(f)\big)\big]_{\mathcal{R}}$.

Conversely,

Assume, $\big[\big(\mathrm{e}, \mathrm{F}(e)\big)\big]_{\mathcal{R}} = \big[\big(\mathrm{f}, \mathrm{F}(f)\big)\big]_{\mathcal{R}}$. Since $(\mathcal{R}, C \times D)_L$ be an Equivalence Turiyam Soft Relation, $\mathcal{R}(f, f) \in (\mathcal{R}, C \times D)_L$(by Reflexive Turiyam Soft Relation). Hence, $\big(\mathrm{f}, \mathrm{F}(f)\big) \in \big[\big(\mathrm{f}, \mathrm{F}(f)\big)\big]_{\mathcal{R}} = \big[\big(\mathrm{e}, \mathrm{F}(e)\big)\big]_{\mathcal{R}} \Rightarrow \mathcal{R}(e, f) \in (\mathcal{R}, C \times D)_L$.

Definition – 3.2.5:

A collection of non-empty Turiyam Soft Subsets $\rho = \{(F_i, A_i)_L : i \in I\}$ of a Turiyam Soft Set $(F, A)_L$ is called a Turiyam Soft Partition of $(F, A)_L$ if

i) $(F, A)_L = \cup_i (F_i, A_i)_L$

ii) $A_i \cap A_j = \phi$, whenever $i \neq j$.

Example – 3.2.6:

Let $\mathrm{U} = \{\mathrm{u}_1, \mathrm{u}_2, \mathrm{u}_3\}$ be the fixed universe and $\mathrm{A} = \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_5\}$ be the set of parameters, then,

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$$(\mathrm{F},\mathrm{A})_L = \left\{ \begin{array}{l} \left(e_1, \left\{ \frac{\mathrm{u}_1}{0.4, 0.3, 0.5, 0.7}, \frac{\mathrm{u}_2}{0.3, 0.5, 0.6, 0.8}, \frac{\mathrm{u}_3}{0.6, 0.5, 0.7, 0.8} \right\} \right), \\ \left(e_2, \left\{ \frac{\mathrm{u}_1}{0.2, 0.4, 0.3, 0.8}, \frac{\mathrm{u}_2}{0.5, 0.2, 0.2, 0.6}, \frac{\mathrm{u}_3}{0.4, 0.4, 0.9, 0.6} \right\} \right), \\ \left(e_5, \left\{ \frac{\mathrm{u}_1}{0.5, 0.6, 0.8, 0.3}, \frac{\mathrm{u}_2}{0.4, 0.3, 0.7, 0.3}, \frac{\mathrm{u}_3}{0.5, 0.7, 0.8, 0.3} \right\} \right) \end{array} \right\}$$

be the Turiyam Soft Set over U. Suppose, $\mathrm{A}_1 = \{e_2, e_5\}, A_2 = \{e_1\}$, where $(F_1, A_1)_L = \{F_1(e_2, e_5)\}$ and $(F_2, A_2)_L = \{F_2(e_1)\}$ are Turiyam Soft Subsets of $(F, A)_L$ such that, $F_i = F$ for $i = 1,2$. $\{(F_1, A_1)_L, (F_2, A_2)_L\}$is a Turiyam Soft Partition of $(F, A)_L$, since, $(F, A)_L = (F_1, A_1)_L \cup (F_2, A_2)_L$ and $A_1 \cap A_2 = \phi$.

Definition – 3.2.7:

Elements of the Turiyam Soft Partition are called a Turiyam Soft Block of $(F, A)_L$.

Theorem – 3.2.8:

Let $\{(F_i, A_i)_L : i \in I\}$ be a Turiyam Soft Partition of the Turiyam Soft Set $(F, A)_L$. Then, the Turiyam Soft Relation defined on $(F, A)_L$ as $\mathrm{F(e)}\ \mathcal{R}\ \mathrm{F(f)}$ is a Turiyam Soft Equivalence Relation if and only if $\mathrm{F(e)}$ and $\mathrm{F(f)}$ are elements of the same block.

Proof:

i) Let $(\mathrm{e}, \mathrm{F(e)})$ be any element of the Turiyam Soft Set $(F, A)_L$. Obviously, $(\mathrm{e}, \mathrm{F(e)})$ is in the same block. Hence, $\mathcal{R}(e, e) \in (\mathcal{R}, C \times D)_L$.

$\Rightarrow (\mathcal{R}, C \times D)_L$ is a Reflexive Turiyam Soft Relation.

ii) If $\mathcal{R}(e, f) \in (\mathcal{R}, C \times D)_L$, then $(\mathrm{e}, \mathrm{F(e)})$ and $(\mathrm{f}, \mathrm{F(f)})$ are in the same block. Thus, $\mathcal{R}(f, e) \in (\mathcal{R}, C \times D)_L$.

$\Rightarrow (\mathcal{R}, C \times D)_L$ is a Symmetric Turiyam Soft Relation.

iii) If $\mathcal{R}(e, f) \in (\mathcal{R}, C \times D)_L$ and $\mathcal{R}(f, g) \in (\mathcal{R}, C \times D)_L$ then $(\mathrm{e}, \mathrm{F(e)})$, $(\mathrm{f}, \mathrm{F(f)})$ and $(\mathrm{g}, \mathrm{F(g)})$ are in the same block. Thus, $\mathcal{R}(e, g) \in (\mathcal{R}, C \times D)_L$.

$\Rightarrow (\mathcal{R}, C \times D)_L$ is a Transitive Turiyam Soft Relation.

Hence, $(\mathcal{R}, C \times D)_L$is a Turiyam Soft Equivalence Relation.

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Remark – 3.2.9:

The Turiyam Soft Relation given in Theorem –3.2.8 is a Turiyam Soft Equivalence Relation determined by the Turiyam Soft Partition ρ. Consider the Turiyam Soft Partition, $\rho = \{(F_1, A_1)_L, (F_2, A_2)_L\}$ from Example - 3.2.6. The Turiyam Soft Equivalence Relation determined by the partition ρ is,

$$\mathcal{R} = \{F(e_1) \times F(e_1), F(e_2) \times F(e_2), F(e_5) \times F(e_5), F(e_2) \times F(e_5), F(e_5) \times F(e_2)\}$$

Theorem – 3.2.10:

Corresponding to every Turiyam Soft Equivalence Relation defined on a Turiyam Soft Set $(F, A)_L$ there exists a Turiyam Soft Partition on $(F, A)_L$ and this Turiyam Soft Partition precisely consists of the Turiyam Soft Equivalence Classes of $(\mathcal{R}, C \times D)_L$.

Proof:

Let $\left[\left(\mathrm{e}, \mathrm{F}(\mathrm{e})\right)\right]$be the Turiyam Soft Equivalence Relation with respect to the Turiyam Soft Relation $(\mathcal{R}, C \times D)_L$on $(\mathrm{F}, \mathrm{A})_\mathrm{L}$.Let $A_e$be the elements in A corresponding to $\left[\left(\mathrm{e}, \mathrm{F}(\mathrm{e})\right)\right]$. Thus, $A_e = \{f \in A : \mathcal{R}(f, e) \in (\mathcal{R}, C \times D)_L\}$. Let us denote $\left[\left(\mathrm{e}, \mathrm{F}(\mathrm{e})\right)\right]$ as $(F_i, A_i)_L$ on A_e. We have to prove that the collection $\left[\left(\mathrm{e}, \mathrm{F_x}(\mathrm{e})\right)\right]$ of such distinct sets forms a partition ρ of $(\mathrm{F}, \mathrm{A})_\mathrm{L}$.

To prove this, we need to show that,

i) $(F, A)_L = \cup_i (F_i, A_i)_L$

ii) $A_e \cap A_f = \phi$, whenever $e \neq f$.

Since, $(\mathcal{R}, C \times D)_L$is a Turiyam Soft Equivalence Relation, $\mathcal{R}(e, e) \in (\mathcal{R}, C \times D)_L$, $\forall\, e \in A$ (by Reflexive Turiyam Soft Relation). This proves i).

Let $m \in A_e \cap A_f$. Since, $\mathrm{F}(\mathrm{m}) \in (\mathrm{F}, \mathrm{A_e})$ and $F(m) \in \left(F, A_f\right)$, then, $\mathcal{R}(m, e) \in (\mathcal{R}, C \times D)_L$and $\mathcal{R}(m, f) \in (\mathcal{R}, C \times D)_L \Rightarrow \mathcal{R}(e, f) \in (\mathcal{R}, C \times D)_L$ (by Transitive Turiyam Soft Relation). By Lemma – 3.2.4, $\left[\left(\mathrm{e}, \mathrm{F}(\mathrm{e})\right)\right]_\mathcal{R} = \left[\left(\mathrm{f}, \mathrm{F}(\mathrm{f})\right)\right]_\mathcal{R} \Rightarrow A_e = A_f$. Which is a contradiction. Thus, $A_e \neq A_f$, hence $A_e \cap A_f = \phi$. This completes the theorem.

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Remark – 3.2.11:

The Turiyam Soft Equivalence Class of $(\mathcal{R}, C \times D)_L$constructed from the Turiyam Soft Partition in Theorem – 3.2.10 is called the Quotient Turiyam Soft Set of $(F, A)_L$ and is denoted by $(F, A)_L \setminus (\mathcal{R}, C \times D)_L$.

4. Conclusion:

Turiyam Soft Relations extend the fundamental principles of the Soft Set Theory by incorporating the Turiyam framework, allowing for more refined representation of uncertain, indeterminate, inconsistent degrees. By introducing Truth, Indeterminacy, Falsity and Liberalization Values, these relations offer a powerful tool for modelling complex interdependencies in various domains.

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DISTANCE BASED SIMILARITY MEASURES OF INTUITIONISTIC FUZZY BINARY SOFT SETS AND ITS APPLICATIONS

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Abstract

Distance based similarity measures of Intuitionistic fuzzy binary soft sets is defined and their basic properties are discussed. The distance-based similarity measure is calculated for different Intuitionistic Fuzzy Binary Soft Sets. Additionally, we defined an algorithm to compute the similarities of Intuitionistic Fuzzy Binary Soft Set (IFBSS), and use the proposed algorithm for decision making problem in the real-life.

Keywords: Soft set, Intuitionistic, Fuzzy, Binary, Similarity measure, Distance based similarity.

AMS Subject Classification:03D45, 03F55, 03E72

1.INTRODUCTION

Intuitionistic fuzzy set [3,7] incorporate a distinctive hesitancy function, while soft set theory provides powerful parameterization techniques to address vagueness effectively. Renowned for its flexibility, soft set theory [1] enables approximate descriptions without imposing strict constraints, making it widely applicable in fields such as Complex systems, medical science, Engineering and Clustering. Over the years, this theory has evolved with the introduction of new concepts and applications.

Fuzzy logic and fuzzy set theory, pioneered by Zadeh [8], have significantly advanced computing but face certain limitations, including ambiguity in assigning membership values and a lack of parameterization techniques. In contrast, intuitionistic fuzzy soft sets [9] are gaining traction across

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numerous disciplines, with their rapid growth highlighting their increasing importance and utility in diverse applications. Acikgoz and N.Tas [2] has introduced the concept of Binary soft sets in 2016. In 2020, J. Subhashini and P. Gino Metilda introduced the concept of Fuzzy Binary Soft set [4]. Similarity measures for IFBSS [11] have already been established in our previous work. In this paper, our aim is to introduce a novel approach by defining distance-based similarity measures providing an alternative perspective to evaluate the similarity between IFBSS.

In this paper, first part serves as the preliminary section, where the essential definitions required for the study are provided. In second part, we introduce the distance-based similarity measures of IFBSS and prove their key properties. Finally, we define an algorithm and use this for the decision-making problem in the real life.

2. DEFINITIONS AND PRELIMINARY RESULTS

Definition 2.1 [5]

Let U and V be the two initial universal sets and E be the set of parameters. Let F(𝕌) and F(V) represents the set of all Intuitionistic fuzzy sets of U and V respectively. A pair (F_{IFBSS}, E) is an ***Intuitionistic fuzzy binary soft set*** over U and V where $F_{IFBSS}$is a mapping defined by, $F_{IFBSS}: E \to F(U) \times F(V)$.

Intuitionistic fuzzy binary soft sets can be written as,

$$(F_{IFBSS}, E) = \{(x, \gamma_{IFSS}(x), \delta_{IFSS}(x)) : x \in E\}$$

where $\gamma_{IFSS}(x)$ is an Intuitionistic fuzzy soft set called x-element of F_{IFBSS} with respect to the universe U. That is for all $x \in E$ is characterized as follows,

$$\gamma_{IFSS}(x) = \{(x, \gamma_E(x) : x \in E)\}$$

Here, $\gamma_E(x) = \left\{\frac{u}{\mu_{\gamma_E}(x)(u), \nu_{\gamma_E}(x)(u)} : u \in U\right\}$,

the functions μ_{γ_E}andν_{γ_E} are the membership and non -membership functions of the mapping $\gamma_{IFSS}$with the condition $0 \leq \mu_{\gamma_E}(x) + \nu_{\gamma_E}(x) \leq 1$.

Similarly, $\delta_{IFSS}(x)$ is an Intuitionistic fuzzy soft set called x-element of F_{IFBSS} with respect to the universe V. That is for all $x \in$Eis characterized as follows,

$$\delta_{IFSS}(x) = \{(x, \delta_E(x) : x \in E)\}$$

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Here, $\delta_E(x) = \left\{\frac{v}{\mu_{\delta_E}(x)(v),\nu_{\delta_E}(x)(v)} : v \in V\right\}$,

the functions μ_{δ_E}andν_{δ_E} are the membership and non -membership functions of the mapping $\delta_{IFSS}$with the condition $0 \leq \mu_{\delta_E}(x) + \nu_{\delta_E}(x) \leq 1$.

Definition 2.2 [6]

Let Uand V be a universal set of elements, E = $\{e_1, e_2, \ldots, e_k\}$ be a fixed set of parameters. Let (F_{IFBSS}, E) and (G_{IFBSS}, E) be two IFBSS for all $e \in E$ and d is the distance measure between two IFBSS.

For all $e_k \in$E,u_i, $\in$ U and $v_j \in$ V where k = 1 ton, i = 1 to l and j = 1 to m, some distance measures are defined as follows:

i)Hamming Distance:

$$d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_H = \frac{1}{2n}\sum_{k=1}^{n}\left[\sum_{i=1}^{l}\sum_{j=1}^{m}\left(\left|\mu_{\gamma_{A(e_k)}}(u_i) - \mu_{\gamma_{B(e_k)}}(u_i)\right| + \left|\vartheta_{\gamma_{A(e_k)}}(u_i) - \vartheta_{\gamma_{B(e_k)}}(u_i)\right| + \left|\mu_{\delta_{A(e_k)}}(v_j) - \mu_{\delta_{B(e_k)}}(v_j)\right| + \left|\vartheta_{\delta_{A(e_k)}}(v_j) - \vartheta_{\delta_{B(e_k)}}(v_j)\right|\right)\right]$$

ii)Euclidean Distance:

$$d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_E = \sqrt{\frac{1}{2n}\sum_{k=1}^{n}\left[\sum_{i=1}^{l}\sum_{j=1}^{m}\left(\begin{array}{c}\left|\mu_{\gamma_{A(e_k)}}(u_i) - \mu_{\gamma_{B(e_k)}}(u_i)\right|^2 + \left|\vartheta_{\gamma_{A(e_k)}}(u_i) - \vartheta_{\gamma_{B(e_k)}}(u_i)\right|^2 \\ + \left|\mu_{\delta_{A(e_k)}}(v_j) - \mu_{\delta_{B(e_k)}}(v_j)\right|^2 + \left|\vartheta_{\delta_{A(e_k)}}(v_j) - \vartheta_{\delta_{B(e_k)}}(v_j)\right|^2\end{array}\right)\right]}$$

iii)Normalized Hamming Distance:

$$d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_{nH} = \frac{1}{2lmn}\sum_{k=1}^{n}\left[\sum_{i=1}^{l}\sum_{j=1}^{m}\left(\left|\mu_{\gamma_{A(e_k)}}(u_i) - \mu_{\gamma_{B(e_k)}}(u_i)\right| + \left|\vartheta_{\gamma_{A(e_k)}}(u_i) - \vartheta_{\gamma_{B(e_k)}}(u_i)\right| + \left|\mu_{\delta_{A(e_k)}}(v_j) - \mu_{\delta_{B(e_k)}}(v_j)\right| + \left|\vartheta_{\delta_{A(e_k)}}(v_j) - \vartheta_{\delta_{B(e_k)}}(v_j)\right|\right)\right]$$

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iv)Normalized Euclidean Distance:

$$d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_{nE} = \sqrt{\frac{1}{2lmn}\sum_{k=1}^{n}\left[\sum_{i=1}^{l}\sum_{j=1}^{m}\begin{pmatrix}\left|\mu_{\gamma_{A(e_k)}}(u_i)-\mu_{\gamma_{B(e_k)}}(u_i)\right|^2+\left|\vartheta_{\gamma_{A(e_k)}}(u_i)-\vartheta_{\gamma_{B(e_k)}}(u_i)\right|^2\\+\left|\mu_{\delta_{A(e_k)}}(v_j)-\mu_{\delta_{B(e_k)}}(v_j)\right|^2+\left|\vartheta_{\delta_{A(e_k)}}(v_j)-\vartheta_{\delta_{B(e_k)}}(v_j)\right|^2\end{pmatrix}\right]}$$

3. DISTANCE BASED SIMILARITY MEASURES ON INTUITIONISTIC FUZZY

BINARY SOFT SETS

3.1 SIMILARITY MEASURES BASED ON DISTANCE MEASURES

We have introduced a distance measure for pair of IFBSS. Building on these distance measures, we can define the similarity measures for Intuitionistic Fuzzy Binary Soft sets. In this paper, (F_{IFBSS}, E) and (G_{IFBSS}, E) are two IFBSS defined on the fixed parameter over U and V.

Definition 3.1.1

Let (F_{IFBSS}, E) and (G_{IFBSS}, E) be two Intuitionistic Fuzzy Binary Soft Sets over Uand V. LetE be the set of parameters.

i) Hamming distance-based similarity:

Based on Hamming distance the similarity measures between (F_{IFBSS}, E) and (G_{IFBSS}, E) is defined as,

$$\boldsymbol{S}^{'}(\mathbf{F}_{\mathrm{IFBSS}}, \mathbf{G}_{\mathrm{IFBSS}})_{\mathrm{H}} = \frac{1}{1+d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_H}$$

where, $d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_H$ is the Hamming distance between the Intuitionistic Fuzzy Binary Soft Sets(F_{IFBSS}, E) and (G_{IFBSS}, E).

ii) Normalized hamming distance-based similarity:

Based on Normalized Hamming distance, the similarity measures between (F_{IFBSS}, E) and (G_{IFBSS}, E) is defined as,

$$\boldsymbol{S}^{'}(\mathbf{F}_{\mathrm{IFBSS}}, \mathbf{G}_{\mathrm{IFBSS}})_{\mathrm{nH}} = \frac{1}{1+d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_{nH}}$$

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where, $d_{IFBSS}(F_{IFBSS},\ G_{IFBSS})_{nH}$ is the Normalized Hamming distance between the Intuitionistic Fuzzy Binary Soft Sets(F_{IFBSS}, E) and (G_{IFBSS}, E).

iii) Euclidean distance-based similarity:

Based on Euclidean distance the similarity measures between (F_{IFBSS}, E) and (G_{IFBSS}, E) is defined as,

$$\boldsymbol{S}'(\mathbf{F}_{\mathrm{IFBSS}},\mathbf{G}_{\mathrm{IFBSS}})_{\mathrm{E}}=\frac{1}{1+d_{IFBSS}(F_{IFBSS},G_{IFBSS})_{E}}$$

where, $d_{IFBSS}(F_{IFBSS},\ G_{IFBSS})_{E}$ is the Euclidean distance between the Intuitionistic Fuzzy Binary Soft Sets (F_{IFBSS}, E) and (G_{IFBSS}, E).

iv) Normalized Euclidean based similarity:

Based on Normalized Euclidean distance the similarity measures between (F_{IFBSS}, E) and (G_{IFBSS}, E) is defined as,

$$\boldsymbol{S}'(\mathbf{F}_{\mathrm{IFBSS}},\mathbf{G}_{\mathrm{IFBSS}})_{\mathrm{nE}}=\frac{1}{1+d_{IFBSS}(F_{IFBSS},G_{IFBSS})_{nE}}$$

where, $d_{IFBSS}(F_{IFBSS},\ G_{IFBSS})_{nE}$ is the Normalized Euclidean distance between the Intuitionistic Fuzzy Binary Soft Sets(F_{IFBSS}, E) and (G_{IFBSS}, E).

Example 3.1.2

A mother and her daughter visit a clothing store to purchase a salwar based on their fabric preferences. The universal set U represents the set of salwar available in the store defined as U = {S_1 - salwar 1, S_2 – salwar 2, S_3 – salwar 3}. The universal set V represents the set of fabric type available for salwar defined as V = {F_1 – cotton, F_2 – silk, F_3 – polyster} under the choice of parameters, E = {p_1 - Expensive, p_2 – Modern, p_3 - Quality}. Let (F_{IFBSS}, E) is the choice of a mother and (G_{IFBSS}, E) is the choice of daughter who is going to purchase salwar based on their fabric preferences. Let (F_{IFBSS}, E) and (G_{IFBSS}, E) are represented as follows.

$$(\mathrm{F}_{\mathrm{IFBSS}},\mathrm{E})=\left\{\begin{array}{l}\left(p_1,\left(\left\{\frac{S_1}{0.4,0.6},\frac{S_2}{0.2,0.7},\frac{S_3}{0.7,0.2}\right\},\left\{\frac{F_1}{0.2,0.8},\frac{F_2}{0.5,0.5},\frac{F_3}{0.4,0.6}\right\}\right)\right),\\ \left(p_2,\left(\left\{\frac{S_1}{0.7,0.3},\frac{S_2}{0.3,0.6},\frac{S_3}{0.2,0.8}\right\},\left\{\frac{F_1}{0.3,0.7},\frac{F_2}{0.8,0.1},\frac{F_3}{0.6,0.3}\right\}\right)\right),\\ \left(p_3,\left(\left\{\frac{S_1}{0.4,0.3},\frac{S_2}{0.7,0.2},\frac{S_3}{0.8,0.1}\right\},\left\{\frac{F_1}{0.5,0.4},\frac{F_2}{0.7,0.2},\frac{F_3}{0.9,0.1}\right\}\right)\right)\end{array}\right\}$$

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$$(G_{IFBSS}, E) = \left\{ \begin{array}{l} \left(p_1, \left(\left\{\frac{S_1}{0.3,0.7}, \frac{S_2}{0.6,0.4}, \frac{S_3}{0.4,0.6}\right\}, \left\{\frac{F_1}{0.4,0.6}, \frac{F_2}{0.3,0.7}, \frac{F_3}{0.5,0.5}\right\}\right)\right), \\ \left(p_2, \left(\left\{\frac{S_1}{0.4,0.5}, \frac{S_2}{0.3,0.7}, \frac{S_3}{0.6,0.4}\right\}, \left\{\frac{F_1}{0.5,0.5}, \frac{F_2}{0.6,0.4}, \frac{F_3}{0.4,0.5}\right\}\right)\right), \\ \left(p_3, \left(\left\{\frac{S_1}{0.5,0.5}, \frac{S_2}{0.7,0.3}, \frac{S_3}{0.5,0.5}\right\}, \left\{\frac{F_1}{0.8,0.2}, \frac{F_2}{0.5,0.5}, \frac{F_3}{0.2,0.8}\right\}\right)\right) \end{array} \right\}$$

i) Hamming Distance:

$d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_H$= 1.7666

ii) Euclidean Distance:

$d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_E$ = 0.7078

iii) Normalized Hamming Distance:

$d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_{nH}$ = 0.1962

iv) Normalized Euclidean Distance:

$d_{IFBSS}(F_{IFBSS}, G_{IFBSS})_{nE}$ = 0.23603

Now, to find the distance-based similarity measures,

i) Hamming distance

$S'(F_{IFBSS}, G_{IFBSS})_H = \frac{1}{1+1.7666}$

= 0.3614

ii) Normalized Hamming distance

$S'(F_{IFBSS}, G_{IFBSS})_{nH} = \frac{1}{1+0.1962}$

= 0.83598

iii) Euclidean distance

$S'(F_{IFBSS}, G_{IFBSS})_E = \frac{1}{1+0.7078}$

= 0.5855

iv) Normalized Euclidean distance

$S'(F_{IFBSS}, G_{IFBSS})_{nE} = \frac{1}{1+0.23603}$

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= 0.80904

Note 3.1.3

In distance-based similarity measures, the similarity between an IFBSS and its complement is never zero.

Lemma 3.1.4

For any two IFBSS (F_{IFBSS}, E) and (G_{IFBSS}, E) the norms listed below are satisfied.

i) $S'(F_{IFBSS}, G_{IFBSS}) = S'(G_{IFBSS}, F_{IFBSS})$

ii) $0 \leq S'(F_{IFBSS}, G_{IFBSS}) \leq 1$

iii) $S'(F_{IFBSS}, G_{IFBSS}) = 1$ iff $F_{IFBSS} = G_{IFBSS}$

Proof:

From the definition it is obvious.

3.2 APPLICATION OF SIMILARITY MEASURES BASED ON DISTANCE MEASURES

Distance based similarity measures play a significant role in this framework by quantifying the closeness between elements or alternative. These measures use mathematical formulations to calculate the "distance" between the characteristics of different options, ensuring a reliable comparison even in uncertain conditions. To address the complexities of decision-making, we have developed a new algorithm based on distance-based similarity measures. This algorithm systematically evaluates alternatives by considering their attributes and the associated uncertainty, enabling precise and objective decision-making.

3.2.1 Algorithm

Step 1: Implement the IFBSS related to the problem.

Step 2: Identify the standard model that can be constructed with the assistance of area of expertise.

Step 3: Calculate the distance between the standard model and other IFBSS.

Step 4: Calculate the distance-based similarity measures between the standard model and other IFBSS.

Step 5: A ***maximal***distance-based similarity is a great choice.

3.2.2 Illustration

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In a competitive image processing challenge, two photographers are tasked with categorizing and analyzing images under predefined criteria. The universal set U = $\{I_{S_1}, I_{S_2}, I_{S_3}\}$ represents the images and the universal set V represents the image features, defined as V = $\{F_{S_1} - \text{Color}, F_{S_2} - \text{Texture}, F_{S_3} - \text{Shape}\}$. The competition uses a carefully chosen parameter E = {p_1 – Color contrast, p_2 – Edge density, p_3 – Brightness uniformity, p_4 – Color saturation} to evaluate the relationship between image categories and features.

Each photographer provides their preferences under their choice of parameter E. Additionally, judge evaluates the photographer's submissions expressing his own judgement criteria under the choice of parameter. The main aim is to identify the photographer who demonstrates the best understanding of image processing and image feature category correlations.

Now, we use the algorithm 3.2.1 to solve the above decision-making problem.

Step 1

The data represented above can be written as IFBSS (F_{IFBSS}, E) and (G_{IFBSS}, E) which is generated by two photographers.

$$(F_{IFBSS}, E) = \left\{ \begin{array}{l} \left(p_1, \left(\left\{\frac{I_{S_1}}{0.7,0.3}, \frac{I_{S_2}}{0.5,0.5}, \frac{I_{S_3}}{0.6,0.4}\right\}, \left\{\frac{F_{S_1}}{0.5,0.4}, \frac{F_{S_2}}{0.3,0.4}, \frac{F_{S_3}}{0.2,0.5}\right\}\right)\right) \\ \left(p_2, \left(\left\{\frac{I_{S_1}}{0.8,0.1}, \frac{I_{S_2}}{0.7,0.2}, \frac{I_{S_3}}{0.4,0.4}\right\}, \left\{\frac{F_{S_1}}{0.5,0.3}, \frac{F_{S_2}}{0.6,0.2}, \frac{F_{S_3}}{0.5,0.1}\right\}\right)\right) \\ \left(p_3, \left(\left\{\frac{I_{S_1}}{0.4,0.4}, \frac{I_{S_2}}{0.3,0.2}, \frac{I_{S_3}}{0.1,0.5}\right\}, \left\{\frac{F_{S_1}}{0.3,0.4}, \frac{F_{S_2}}{0.7,0.1}, \frac{F_{S_3}}{0.6,0.4}\right\}\right)\right) \\ \left(p_4, \left(\left\{\frac{I_{S_1}}{0.25,0.5}, \frac{I_{S_2}}{0.7,0.2}, \frac{I_{S_3}}{0.5,0.4}\right\}, \left\{\frac{F_{S_1}}{0.7,0.2}, \frac{F_{S_2}}{0.45,0.45}, \frac{F_{S_3}}{0.3,0.1}\right\}\right)\right) \end{array} \right\}$$

$$(G_{IFBSS}, E) = \left\{ \begin{array}{l} \left(p_1, \left(\left\{\frac{I_{S_1}}{0.4,0.4}, \frac{I_{S_2}}{0.5,0.5}, \frac{I_{S_3}}{0.3,0.2}\right\}, \left\{\frac{F_{S_1}}{0.7,0.2}, \frac{F_{S_2}}{0.6,0.4}, \frac{F_{S_3}}{0.2,0.6}\right\}\right)\right) \\ \left(p_2, \left(\left\{\frac{I_{S_1}}{0.3,0.4}, \frac{I_{S_2}}{0.5,0.2}, \frac{I_{S_3}}{0.7,0.1}\right\}, \left\{\frac{F_{S_1}}{0.5,0.5}, \frac{F_{S_2}}{0.7,0.1}, \frac{F_{S_3}}{0.3,0.4}\right\}\right)\right) \\ \left(p_3, \left(\left\{\frac{I_{S_1}}{0.9,0.1}, \frac{I_{S_2}}{0.75,0.3}, \frac{I_{S_3}}{0.4,0.5}\right\}, \left\{\frac{F_{S_1}}{0.6,0.3}, \frac{F_{S_2}}{0.5,0.4}, \frac{F_{S_3}}{0.8,0.25}\right\}\right)\right) \\ \left(p_4, \left(\left\{\frac{I_{S_1}}{0.4,0.5}, \frac{I_{S_2}}{0.9,0.1}, \frac{I_{S_3}}{0.8,0.2}\right\}, \left\{\frac{F_{S_1}}{0.7,0.4}, \frac{F_{S_2}}{0.2,0.2}, \frac{F_{S_3}}{0.4,0.2}\right\}\right)\right) \end{array} \right\}$$

Step 2

The standard model that can be constructed with the assistance of area of expertise under the choice of parameters. We construct his IFBSS as (H_{IFBSS}, E).

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$$(\mathrm{H}_{\mathrm{IFBSS}}, \mathrm{E}) = \left\{ \begin{array}{l} \left(p_1, \left(\left\{\frac{I_{S_1}}{0.9,0.2}, \frac{I_{S_2}}{0.5,0.4}, \frac{I_{S_3}}{0.3,0.4}\right\}, \left\{\frac{F_{S_1}}{0.6,0.4}, \frac{F_{S_2}}{0.3,0.7}, \frac{F_{S_3}}{0.5,0.4}\right\}\right)\right) \\ \left(p_2, \left(\left\{\frac{I_{S_1}}{0.7,0.1}, \frac{I_{S_2}}{0.5,0.2}, \frac{I_{S_3}}{0.6,0.3}\right\}, \left\{\frac{F_{S_1}}{0.7,0.2}, \frac{F_{S_2}}{0.8,0.2}, \frac{F_{S_3}}{0.4,0.1}\right\}\right)\right) \\ \left(p_3, \left(\left\{\frac{I_{S_1}}{0.5,0.3}, \frac{I_{S_2}}{0.4,0.2}, \frac{I_{S_3}}{0.6,0.3}\right\}, \left\{\frac{F_{S_1}}{0.4,0.2}, \frac{F_{S_2}}{0.5,0.4}, \frac{F_{S_3}}{0.9,0.1}\right\}\right)\right) \\ \left(p_4, \left(\left\{\frac{I_{S_1}}{0.4,0.3}, \frac{I_{S_2}}{0.6,0.4}, \frac{I_{S_3}}{0.7,0.3}\right\}, \left\{\frac{F_{S_1}}{0.5,0.5}, \frac{F_{S_2}}{0.7,0.2}, \frac{F_{S_3}}{0.4,0.4}\right\}\right)\right) \end{array} \right\}$$

Step 3

Calculate the distance between the standard model and other IFBSS. Here we are using Hamming distance-based similarity measures.

$$d_{IFBSS}(\mathrm{F}_{\mathrm{IFBSSS}}, \mathrm{H}_{\mathrm{IFBSS}})_H = \frac{1}{2 \times 3}[7.35]$$

= 1.225

$$\boldsymbol{d_{IFBSS}(\mathrm{F}_{\mathrm{IFBSSS}}, \mathrm{H}_{\mathrm{IFBSS}})_H = 1.225}$$

$$d_{IFBSS}(\mathrm{G}_{\mathrm{IFBSSS}}, \mathrm{H}_{\mathrm{IFBSS}})_H = \frac{1}{2 \times 3}[8.5]$$

= 1.41667

$$\boldsymbol{d_{IFBSS}(\mathrm{G}_{\mathrm{IFBSSS}}, \mathrm{H}_{\mathrm{IFBSS}})_H = 1.41667}$$

Step 4

Now, calculate the distance-based similarity measures.

$S'(\mathrm{F}_{\mathrm{IFBSSS}}, \mathrm{H}_{\mathrm{IFBSS}})_{\mathrm{H}} = \frac{1}{1+1.225}$

= 0.44943

$\boldsymbol{S'(\mathrm{F}_{\mathrm{IFBSSS}}, \mathrm{H}_{\mathrm{IFBSS}})_{\mathrm{H}} = 0.44943}$

$S'(\mathrm{G}_{\mathrm{IFBSSS}}, \mathrm{H}_{\mathrm{IFBSS}})_{\mathrm{H}} = \frac{1}{1+1.41667}$

= 0.41379

$\boldsymbol{S'(\mathrm{G}_{\mathrm{IFBSSS}}, \mathrm{H}_{\mathrm{IFBSS}})_{\mathrm{H}} = 0.41379}$

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Conclusion of the problem

By analyzing, the distance-based similarity measures is obtained, it is evident that photographer (F_{IFBSSS}, E) choice is greater than photographer (G_{IFBSSS}, E) choice. This indicate that photographer (F_{IFBSSS}, E)'s has the best understanding of image processing and image feature correlation.

4. CONCLUSION

Distance based similarity measures for IFBSS play a crucial role in addressing complex decision-making problems in various domains. By integrating the concept of distance into similarity computation, these measures enable a robust evaluation of relationships between objects under uncertainty. Distance based similarity measures for IFBSSs are introduced, along with a discussion of their fundamental properties. Additionally, a novel algorithm is defined for calculating the distance-based similarity of IFBSSs, and use this for the decision-making problem in the real-life problem.

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ON NEUTROSOPHIC Υ −REGULAR AND Υ −NORMAL SPACES

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Abstract

The focus of this paper is to explore the concepts of regular and normal spaces in neutrosophic topology by means neutrosophicΥ −open sets. We further establish their stronger forms and discuss their attributes.

Keywords:neutrosophicΥ −open, neutrosophic Υ −closed, neutrosophic Υ −regular, neutrosophic Υ −normal

AMS Subject Classification:18B30, 03B52, 11B05,

I. Introduction

The theory of neutrosophic sets initiated by Smarandache[10]entrenched as a significant mathematical tool in dealing with uncertainties. This led to the origination of neutrosophic topological spaces and various other topological concepts. Ahu Acikgoz and F. Esenbel[1] implemented the idea of quasi coincidence to study the concept of separation axioms in neutrosophic topological spaces. Further, regular and normal spaces in neutrosophic theory have been discussed in [2,4,12]. Recently, the authors of this paper[7] presented some weaker separation axioms by means of neutrosophicΥ −open sets. This paper

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focuses on the concepts of neutrosophic Υ −regular and normal spaces. The stronger forms are also discussed. Further, their characterizations and inter-relations are studied.

II. Preliminaries

Definition 2.1:[9]Let U be a non-empty fixed set. A **neutrosophic set**L is an object having the form $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ where $\mu_L(u), \sigma_L(u)$and $\gamma_L(u)$represent the membership, indeterminacy and non-membership functions respectively of each element $u \in U$. A neutrosophic set $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ can be identified to an ordered triple $< \mu_L, \sigma_L, \gamma_L >$ in $]^-0,1^+[$ on U.

Definition 2.2:[9]Let U be a non-empty set and $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}, M = \{< u, \mu_M(u), \sigma_M(u), \gamma_M(u) >: u \in U\}$ be neutrosophic sets in U. Then

(i) $L \subseteq M$ if $\mu_L(u) \leq \mu_M(u), \sigma_L(u) \leq \sigma_M(u)$ and $\gamma_L(u) \geq \gamma_M(u)$ for all $u \in U$.

(ii)$L \cup M = \{< u,$max$\{\mu_L(u), \mu_M(u)\},$ max$\{\sigma_L(u), \sigma_M(u)\},$ min$\{\gamma_L(u), \gamma_M(u)\}>: u \in U\}$

(iii) $L \cap M = \{< u,$min$\{\mu_L(u), \mu_M(u)\},$ min$\{\sigma_L(u), \sigma_M(u)\},$ max$\{\gamma_L(u), \gamma_M(u)\}>: u \in U\}$

(iv) $L^c = \{< u, \gamma_L(u), 1 - \sigma_L(u), \mu_L(u) >: u \in U\}$

(v) $0_{N_{tr}} = \{< u, 0,0,1 >: u \in U\}$ and $1_{N_{tr}} = \{< u, 1,1,0 >: u \in U\}$

Definition 2.3:[9]A **neutrosophic topology** on a non-empty set U is a family $\tau_{N_{tr}}$ of neutrosophic sets in U satisfying the following axioms:

(i) $0_{N_{tr}}, 1_{N_{tr}} \in \tau_{N_{tr}}$

(ii)$\cup L_i \in \tau_{N_{tr}} \forall \{L_i : i \in I\} \subseteq \tau_{N_{tr}}$

(iii) $L_1 \cap L_2 \in \tau_{N_{tr}}$ for any $L_1, L_2 \in \tau_{N_{tr}}$

The pair $(U, \tau_{N_{tr}})$ is called a neutrosophic topological space. The members of $\tau_{N_{tr}}$ are called neutrosophic open$(N_{tr}O)$ and its complements are called neutrosophic closed$(N_{tr}C)$.

Definition 2.4:[1]A neutrosophic set $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ is called a **neutrosophic point**$(N_{tr}P)$ if for any element $v \in U, \mu_L(v) = a, \sigma_L(v) = b, \gamma_L(v) = c$ for $u = v$ and $\mu_L(v) = 0, \sigma_L(v) =$

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$0, \gamma_L(v) = 1$ for $u \neq v$, where a, b, c are real standard or non-standard subsets of $]^-0,1^+[$. A neutrosophic point is denoted by $u_{a,b,c}$. For the neutrosophic point $u_{a,b,c}$, u will be called its support.

Definition 2.5:[1]A neutrosophic point $u_{a,b,c}$ is said to be **neutrosophic quasi – coincident** with a neutrosophic set L, denoted by $u_{a,b,c}qL$ if $u_{a,b,c} \notin L^c$. If $u_{a,b,c}$ is not neutrosophic quasi – coincident with L, we denote it by $u_{a,b,c}\hat{q}L$.

Definition 2.6:[5]A neutrosophic set L of a neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be **neutrosophicΥ −open**$(N_{tr}\Upsilon O)$ if for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}, L \subseteq N_{tr}cl\big(N_{tr}int(L \cup F)\big)$. The complement of neutrosophicΥ −open set is neutrosophicΥ −closed. The class of neutrosophic Υ −open sets is denoted by $N_{tr}\Upsilon O\big(U, \tau_{N_{tr}}\big)$.

Theorem 2.7:[5]Every N_{tr}open set is $N_{tr}\Upsilon$ −open.

Definition 2.8:[6]A function $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is said to be **stronglyneutrosophicΥ −continuous** if $f_{N_{tr}}^{-1}(M)$ is N_{tr}open in $(U, \tau_{N_{tr}})$ for every $N_{tr}\Upsilon$ −open set M in $\big(V, \rho_{N_{tr}}\big)$.

Definition 2.9:[7]A function $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is said to be a **neutrosophicΥ −open** if $f_{N_{tr}}(L)$ is $N_{tr}\Upsilon$ −open in $(V, \rho_{N_{tr}})$ for every N_{tr}open set L in $(U, \tau_{N_{tr}})$.

Definition 2.10:[12]A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a **neutrosophichausdorff space** if for every pair of neutrosophic points $u_{a,b,c}$and $v_{a',b',c'}, u \neq v$, there exists N_{tr}open sets L and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in L, v_{a',b',c'} \in M$ and $L \subseteq M^c$.

Definition 2.11:[8]A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to **neutrosophicΥ −compact** if every $N_{tr}\Upsilon$ −open cover of $(U, \tau_{N_{tr}})$ has a finite subcover.

Definition 2.12:[1]A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a **neutrosophicregular space** if for every pair consisting of a $N_{tr}Pu_{a,b,c}$ and a N_{tr}closed set $H, u_{a,b,c}\hat{q}H$, there exist N_{tr}open sets L and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in L, H \subseteq M$ and $L\hat{q}M$.

Definition 2.13:[1]A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a **neutrosophicnormal space** if for every pair of N_{tr}closed sets G and $H, G\hat{q}H$, there exist N_{tr}open sets L and M in $(U, \tau_{N_{tr}})$ such that $G \subseteq L, H \subseteq M$ and $L\hat{q}M$.

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Lemma 2.14:[7]Let $(S, \tau^*_{N_{tr}})$ be a neutrosophic subspace of $(U, \tau_{N_{tr}})$. Then a neutrosophic set L in S is$N_{tr}\Upsilon$ −closed in $(U, \tau_{N_{tr}})$if and only if L is $N_{tr}\Upsilon$ −closed in $(S, \tau^*_{N_{tr}})$.

Theorem 2.15:[8]Let L be a $N_{tr}\Upsilon$ −closed subset of a $N_{tr}\Upsilon$ −compact space $(U, \tau_{N_{tr}})$ such that $L \cap L^c = 0_{N_{tr}}$. Then L is $N_{tr}\Upsilon$ −compact relative to U.

III. NeutrosophicΥ −regular Spaces

Definition 3.1: A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a $\boldsymbol{N_{tr}\Upsilon}$ **−regular space** if for every pair consisting of a $N_{tr}Pu_{a,b,c}$and a $N_{tr}\Upsilon$ −closed set $H, u_{a,b,c}\hat{q}H$, there exist $N_{tr}\Upsilon$ −open sets L and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in L, H \subseteq M$ and $L\hat{q}M$.

Theorem 3.2: Let $(U, \tau_{N_{tr}})$ be a neutrosophic topological space. Then, the following are equivalent:

(i) $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon$ −regular.

(ii) For every $N_{tr}\Upsilon - nbhdN$ of $u_{a,b,c}$, there exists a $N_{tr}\Upsilon$ −open set L containing $u_{a,b,c}$ such that $N_{tr}\Upsilon cl(L) \subseteq N$.

(iii) For each $N_{tr}Pu_{a,b,c}$in U and for each $N_{tr}\Upsilon$ −closed set L such that $u_{a,b,c}\hat{q}L$, there exists a $N_{tr}\Upsilon$ −open set M such that $N_{tr}\Upsilon cl(M)\hat{q}L$.

Proof:

(i)⟹(ii) Let N be a $N_{tr}\Upsilon - nbhd$ of $u_{a,b,c}$. Then, there exists a $N_{tr}\Upsilon$ −open set G in U such that $u_{a,b,c} \in G \subseteq N$. Now, $u_{a,b,c} \in G \Longrightarrow u_{a,b,c}\hat{q}G^c$ and G^c is $N_{tr}\Upsilon$ −closed. Since U is $N_{tr}\Upsilon$ −regular, there exist $N_{tr}\Upsilon$ −open sets L and M such that $u_{a,b,c} \in L, G^c \subseteq M$ and $L\hat{q}M$. Now, $L\hat{q}M \Longrightarrow L \subseteq M^c \Longrightarrow N_{tr}\Upsilon cl(L) \subseteq N_{tr}\Upsilon cl(M^c) = M^c \subseteq G \subseteq N$.Hence$N_{tr}\Upsilon cl(L) \subseteq N$.

(ii)⟹(iii) Let $u_{a,b,c}$ be a $N_{tr}P$ in U and L be a $N_{tr}\Upsilon$ −closed set such that $u_{a,b,c}\hat{q}L$. Then, L^c is $N_{tr}\Upsilon$ −open and $u_{a,b,c} \in L^c$. Since L^c is a $N_{tr}\Upsilon - nbhd$ of $u_{a,b,c}$, by (ii), there exists a $N_{tr}\Upsilon$ −open set M such that $N_{tr}\Upsilon cl(M) \subseteq L^c \Longrightarrow N_{tr}\Upsilon cl(M)\hat{q}L$.

(iii)⟹(i) Let $u_{a,b,c}$ be a $N_{tr}P$ in U and L be a $N_{tr}\Upsilon$ −closed set such that $u_{a,b,c}\hat{q}L$. Now, L^c is $N_{tr}\Upsilon$ −open and $u_{a,b,c}\hat{q}L \Longrightarrow u_{a,b,c} \in L^c$. By (iii), there exists a $N_{tr}\Upsilon$ −open set M such that $N_{tr}\Upsilon cl(M)\hat{q}L \Longrightarrow N_{tr}\Upsilon cl(M) \subseteq L^c \Longrightarrow L \subseteq (N_{tr}\Upsilon cl(M))^c$. Since $u_{a,b,c}$ and L are arbitrary, for every pair of $N_{tr}Pu_{a,b,c}$and a N_{tr}closed set L

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such that $u_{a,b,c}\hat{q}L$, there exist $N_{tr}\Upsilon$ −open sets L^c and $(N_{tr}\Upsilon cl(M))^c$ such that $u_{a,b,c} \in L^c$ and $L \subseteq (N_{tr}\Upsilon cl(M))^c$. Therefore, U is $N_{tr}\Upsilon$ −regular.

Theorem 3.3: A neutrosophic topological space $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon$ −regular if and only if for each $N_{tr}\Upsilon$ −closed set H and each $u_{a,b,c}\hat{q}H$, there exist $N_{tr}\Upsilon$ −open sets L and M such that $u_{a,b,c} \in L, H \subseteq M$ and $N_{tr}\Upsilon cl(L)\hat{q}N_{tr}\Upsilon cl(M)$.

Proof: Let U be $N_{tr}\Upsilon$ −regular, $u_{a,b,c}$ be $N_{tr}P$ in U and H be $N_{tr}\Upsilon$ −closed set such that $u_{a,b,c}\hat{q}H$. Then, there exist $N_{tr}\Upsilon$ −open sets L and M such that $u_{a,b,c} \in L, H \subseteq M$ and $L\hat{q}M \Longrightarrow L\hat{q}N_{tr}\Upsilon cl(M)$. Hence $u_{a,b,c}\hat{q}N_{tr}\Upsilon cl(M)$ and $N_{tr}\Upsilon cl(M)$ is $N_{tr}\Upsilon$ −closed. Since U is $N_{tr}\Upsilon$ −regular, there exist $N_{tr}\Upsilon$ −open sets P and Q such that $u_{a,b,c} \in P$ and $N_{tr}\Upsilon cl(M) \subseteq Q$ and $P\hat{q}Q$. Now, $M \subseteq N_{tr}\Upsilon cl(M) \subseteq Q \Longrightarrow Q^c \subseteq M^c$. Further, $P\hat{q}Q \Longrightarrow P \subseteq Q^c \subseteq M^c \Longrightarrow P\hat{q}M$. Since $N_{tr}\Upsilon cl(P)$is the smallest $N_{tr}\Upsilon$ −closed set containing P, $N_{tr}\Upsilon cl(P) \subseteq M^c$. Take $L = P$. Then $N_{tr}\Upsilon cl(L) = N_{tr}\Upsilon cl(P) \subseteq Q^c \subseteq (N_{tr}\Upsilon cl(M))^c$. Hence $N_{tr}\Upsilon cl(L)\hat{q}N_{tr}\Upsilon cl(M)$. Conversely, let $u_{a,b,c}$be a $N_{tr}P$ in U and H be a $N_{tr}\Upsilon$ −closed set such that $u_{a,b,c}\hat{q}H$. Then, by assumption, there exist $N_{tr}\Upsilon$ −open sets L and M in U such that $u_{a,b,c} \in L, H \subseteq M$ and $N_{tr}\Upsilon cl(L)\hat{q}N_{tr}\Upsilon cl(M)$. Now, $N_{tr}\Upsilon cl(L)\hat{q}N_{tr}\Upsilon cl(M) \Longrightarrow N_{tr}\Upsilon cl(L) \subseteq \left(N_{tr}\Upsilon cl(M)\right)^c \Longrightarrow L \subseteq \left(N_{tr}\Upsilon cl(M)\right)^c = N_{tr}\Upsilon int(M^c) \subseteq M^c \Longrightarrow L\hat{q}M$. Therefore U is $N_{tr}\Upsilon$ −regular.

Theorem 3.4: Let $(U, \tau_{N_{tr}})$ be a $N_{tr}\Upsilon$ −regular space. Then

(i) Every $N_{tr}\Upsilon$ −open set in $(U, \tau_{N_{tr}})$ is the union of $N_{tr}\Upsilon$ −closed sets.

(ii) Every $N_{tr}\Upsilon$ −closed set in $(U, \tau_{N_{tr}})$ is the intersection of $N_{tr}\Upsilon$ −open sets.

Proof: (i) Let U be $N_{tr}\Upsilon$ −regular, L be a $N_{tr}\Upsilon$ −open set in U and $u_{a,b,c} \in L$. Then L^c is $N_{tr}\Upsilon$ −closed and $u_{a,b,c}\hat{q}L^c$. Since U is $N_{tr}\Upsilon$ −regular, there exist $N_{tr}\Upsilon$ −open sets M and N such that $u_{a,b,c} \in M, L^c \subseteq N$ and $M\hat{q}N$. Now, $M\hat{q}N \Longrightarrow M \subseteq N^c \subseteq L$. Let $H = N_{tr}\Upsilon cl(M)$. N^c is $N_{tr}\Upsilon$ −closed set containing M but $N_{tr}\Upsilon cl(M)$ is the smallest $N_{tr}\Upsilon$ −closed set containing M. Hence$H = N_{tr}\Upsilon cl(M) \subseteq N^c \Longrightarrow u_{a,b,c} \in H \subseteq N^c \subseteq L$. Therefore $L = \cup\{H : H \in N_{tr}\Upsilon C(U, \tau_{N_{tr}})\}$.

(ii) can be proved similarly.

Theorem 3.5: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a strongly $N_{tr}\Upsilon$ −continuous and $N_{tr}\Upsilon$ −open bijective function. If $(U, \tau_{N_{tr}})$ is N_{tr}regular, then $(V, \rho_{N_{tr}})$ is $N_{tr}\Upsilon$ −regular.

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Proof: Let $u_{a,b,c}$ be a $N_{tr}P$ and H be a $N_{tr}\Upsilon$ −closed set in V such that $u_{a,b,c}\hat{q}H$. Then $f_{N_{tr}}^{-1}(u_{a,b,c})\hat{q}f_{N_{tr}}^{-1}(H)$. Moreover, since $f_{N_{tr}}$ is strongly $N_{tr}\Upsilon$ −continuous, $f_{N_{tr}}^{-1}(H)$ is N_{tr}closed in U. Hence, by hypothesis, there exists N_{tr}open sets L and M in U such that $f_{N_{tr}}^{-1}(u_{a,b,c}) \in L$, $f_{N_{tr}}^{-1}(H) \subseteq M$ and $L\hat{q}M$. Then, $u_{a,b,c} \in f_{N_{tr}}(L), H \subseteq f_{N_{tr}}(M)$ and $f_{N_{tr}}(L)\hat{q}f_{N_{tr}}(M)$. Now, since $f_{N_{tr}}$ is $N_{tr}\Upsilon$ −open, $f_{N_{tr}}(L)$ and $f_{N_{tr}}(M)$ are $N_{tr}\Upsilon$ −open in V and therefore V is $N_{tr}\Upsilon$ −regular.

Theorem 3.6: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a strongly $N_{tr}\Upsilon$ −continuous and $N_{tr}\Upsilon$ −closed bijective function. If $(V, \rho_{N_{tr}})$ is $N_{tr}\Upsilon$ −regular, then $(U, \tau_{N_{tr}})$ is N_{tr}regular.

Proof: Let $u_{a,b,c}$ be a $N_{tr}P$ and H be a $N_{tr}\Upsilon$ −closed set in U such that $u_{a,b,c}\hat{q}H$. Then $f_{N_{tr}}(u_{a,b,c})\hat{q}f_{N_{tr}}(H)$. Moreover, since $f_{N_{tr}}$ is $N_{tr}\Upsilon$ −closed, $f_{N_{tr}}(H)$ is $N_{tr}\Upsilon$ −closed in V. By hypothesis, there exists $N_{tr}\Upsilon$ −open sets L and M in U such that $f_{N_{tr}}(u_{a,b,c}) \in L$, $f_{N_{tr}}(H) \subseteq M$ and $L\hat{q}M$. Then, $u_{a,b,c} \in f_{N_{tr}}^{-1}(L), H \subseteq f_{N_{tr}}^{-1}(M)$ and $f_{N_{tr}}^{-1}(L)\hat{q}f_{N_{tr}}^{-1}(M)$. Now, since $f_{N_{tr}}$ is strongly $N_{tr}\Upsilon$ −continuous, $f_{N_{tr}}^{-1}(L)$ and $f_{N_{tr}}^{-1}(M)$ are N_{tr}open in V and hence U is N_{tr}regular.

Theorem 3.7: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a $N_{tr}\Upsilon - i$ homeomorphism. If $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon$ −regular, then $(V, \rho_{N_{tr}})$ is also $N_{tr}\Upsilon$ −regular.

Proof: Let $u_{a,b,c}$ be a $N_{tr}P$ and H be a $N_{tr}\Upsilon$ −closed set in V such that $u_{a,b,c}\hat{q}H$. Then $f_{N_{tr}}^{-1}(u_{a,b,c})\hat{q}f_{N_{tr}}^{-1}(H)$ and $f_{N_{tr}}^{-1}(H)$ is $N_{tr}\Upsilon$ −closed in U as $f_{N_{tr}}$ is $N_{tr}\Upsilon - i$homeomorphism. Since $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon$ −regular, there exists $N_{tr}\Upsilon$ −open sets L and M in U such that $f_{N_{tr}}^{-1}(u_{a,b,c}) \in L$, $f_{N_{tr}}^{-1}(H) \subseteq M$ and $L\hat{q}M$. Then, $u_{a,b,c} \in f_{N_{tr}}(L), H \subseteq f_{N_{tr}}(M)$ and $f_{N_{tr}}(L)\hat{q}f_{N_{tr}}(M)$. Again, since $f_{N_{tr}}$ is a $N_{tr}\Upsilon - i$homeomorphism, $f_{N_{tr}}(L)$ and $f_{N_{tr}}(M)$ are $N_{tr}\Upsilon$ −open in V. Hence V is $N_{tr}\Upsilon$ −regular.

Theorem 3.8: Let $(U, \tau_{N_{tr}})$ be a $N_{tr}\Upsilon$ −compact and N_{tr}Hausdorff space in which $H \cap H^c = 0_{N_{tr}}$ for every $N_{tr}\Upsilon$ −closed set H in $(U, \tau_{N_{tr}})$. Then $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon$ −regular.

Proof: Let $u_{a,b,c}$be a $N_{tr}P$ and H be a $N_{tr}\Upsilon$ −closed set in U such that $u_{a,b,c}\hat{q}H$. Then $u_{a,b,c} \in H^c$. Since U is N_{tr}hausdorff, for every $v_{i_{a',b',c'}} \in H$, there exist N_{tr}open sets L_i and M_i such that $u_{a,b,c} \in L_i, v_{i_{a',b',c'}} \in M_i, u_{a,b,c} \notin M_i, v_{i_{a',b',c'}} \notin L_i$and $L_i \subseteq {M_i}^c$ for all i. Now, $\mathcal{M} = \{M_i : i \in \Delta\}$ forms a N_{tr}open cover for H and by theorem 2.7, $\mathcal{M}$ forms a $N_{tr}\Upsilon$ −open cover for H. Moreover, by theorem 2.15, H is $N_{tr}\Upsilon$ −compact. Then, there exists a finite subcover $\{M_{i_k} : k = 1,2, \dots, n\}$ for H. Now, let $L = \bigcap_{k=1}^{n} L_{i_k}$ and $M = \bigcup_{k=1}^{n} M_{i_k}$ where

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$L_{i_k}$are the N_{tr}open sets corresponding to M_{i_k}. Obviously, L is N_{tr}open and hence $N_{tr}\Upsilon$ −open. Since $L_{i_k} \subseteq M_{i_k}^c$ for all k, $L = \bigcap_{k=1}^{n} L_{i_k} \subseteq \bigcap_{k=1}^{n} M_{i_k}^c = \bigcup_{k=1}^{n} M_{i_k}^c = M^c \Longrightarrow L \subseteq M^c$. Hence there exists $N_{tr}\Upsilon$ −open sets L and M such that $u_{a,b,c} \in L, H \subseteq M$ and $L\hat{q}M$. Therefore U is $N_{tr}\Upsilon -$ regular.

Theorem 3.9: Let $(U, \tau_{N_{tr}})$ be a $N_{tr}\Upsilon$ −regular space in which the class of all $N_{tr}\Upsilon$ −open sets is closed under finite intersection. Then every neutrosophic open subspace of $(U, \tau_{N_{tr}})$ is also $N_{tr}\Upsilon$ −regular.

Proof: Let $(S, \tau_{N_{tr}}^{S})$ be a neutrosophic open subspace of $\left(U, \tau_{N_{tr}}\right), u_{a,b,c}$ be a $N_{tr}P$ and H be a $N_{tr}\Upsilon$ −closed set in S such that $u_{a,b,c}\hat{q}H$. By lemma 2.14, H is $N_{tr}\Upsilon$ −closed in U. Since U is $N_{tr}\Upsilon$ −regular, there exist $N_{tr}\Upsilon$ −open sets L and M in U such that $u_{a,b,c} \in L, H \subseteq M, L\hat{q}M$.Now, $u_{a,b,c} \in L \cap 1_{N_{tr}}^{S}, H \subseteq M \cap 1_{N_{tr}}^{S}$ and $\left(L \cap 1_{N_{tr}}^{S}\right)\hat{q}\left(M \cap 1_{N_{tr}}^{S}\right)$. By theorem 2.7, $1_{N_{tr}}^{S}$is $N_{tr}\Upsilon$ −open in U and by hypothesis, $L \cap 1_{N_{tr}}^{S}$ and $M \cap 1_{N_{tr}}^{S}$ are $N_{tr}\Upsilon$ −open in U. By lemma 2.14, both $L \cap 1_{N_{tr}}^{S}$ and $M \cap 1_{N_{tr}}^{S}$are $N_{tr}\Upsilon$ −open in S. Therefore $(S, \tau_{N_{tr}}^{S})$ is $N_{tr}\Upsilon$ −regular.

IV. NeutrosophicΥ −normal Spaces

Definition 4.1: A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a $\boldsymbol{N_{tr}\Upsilon}$ **−normal space** if for every pair of $N_{tr}\Upsilon$ −closed sets G and $H, G\hat{q}H$, there exist $N_{tr}\Upsilon$ −open sets L and M in $(U, \tau_{N_{tr}})$ such that $G \subseteq L, H \subseteq M$ and $L\hat{q}M$.

Theorem 4.2: Let $(U, \tau_{N_{tr}})$ be a neutrosophic topological space. Then the following are equivalent:

(i) $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon$ −normal.

(ii) For every $N_{tr}\Upsilon$ −closed set H and every $N_{tr}\Upsilon$ −open set L containing H, there exists a $N_{tr}\Upsilon$ −open set M containing H such that $N_{tr}\Upsilon cl(M) \subseteq L$.

(iii) For every pair of $N_{tr}\Upsilon$ −closed sets G and H in $(U, \tau_{N_{tr}})$ such that $G\hat{q}H$, there exists a $N_{tr}\Upsilon$ −open set L containing G such that $N_{tr}\Upsilon cl(L)\hat{q}H$.

(iv)For every pair of $N_{tr}\Upsilon$ −closed sets G and H in $(U, \tau_{N_{tr}})$ such that $G\hat{q}H$, there exist $N_{tr}\Upsilon$ −open sets L and M containing G and H respectively such that $N_{tr}\Upsilon cl(L)\hat{q}N_{tr}\Upsilon cl(M)$.

Proof:

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(i)⟹(ii) Let L be a $N_{tr}\Upsilon$ −open set containing the $N_{tr}\Upsilon$ −closed set H. Then $H\hat{q}L^c$. Since U is $N_{tr}\Upsilon$ −normal, there exists $N_{tr}\Upsilon$ −open sets M and N such that $H \subseteq M, L^c \subseteq N$ and $M\hat{q}N$. Hence $M \subseteq N^c \Longrightarrow N_{tr}\Upsilon cl(M) \subseteq N^c \subseteq L$.

(ii)⟹(iii) Let G and H be $N_{tr}\Upsilon$ −closed sets such that $G\hat{q}H$. Then H^c is $N_{tr}\Upsilon$ −open such that $G \subseteq H^c$. By assumption, there exists a $N_{tr}\Upsilon$ −open set L such that $G \subseteq L$ and $N_{tr}\Upsilon cl(L) \subseteq H^c \Longrightarrow N_{tr}\Upsilon cl(L)\hat{q}H$.

(iii)⟹(iv) Let G and H be $N_{tr}\Upsilon$ −closed sets such that $G\hat{q}H$. By assumption, there exists a $N_{tr}\Upsilon$ −open set L containing G such that $N_{tr}\Upsilon cl(L)\hat{q}H$. Now, since $N_{tr}\Upsilon cl(L)$ and H are a pair of $N_{tr}\Upsilon$ −closed sets which do not quasi-coincide, there exists a $N_{tr}\Upsilon$ −open set M such that $H \subseteq M$ and $N_{tr}\Upsilon cl(M)\hat{q}N_{tr}\Upsilon cl(L)$.

(iv)⟹(i) Let G and H be $N_{tr}\Upsilon$ −closed sets such that $G\hat{q}H$. Then, by (iv), there exist $N_{tr}\Upsilon$ −open sets L and M containing G and H respectively and $N_{tr}\Upsilon cl(L)\hat{q}N_{tr}\Upsilon cl(M)$. Now, $N_{tr}\Upsilon cl(L)\hat{q}N_{tr}\Upsilon cl(M) \Longrightarrow N_{tr}\Upsilon cl(L) \subseteq (N_{tr}\Upsilon cl(M))^c = N_{tr}\Upsilon int(M^c) \subseteq M^c$. Hence $L \subseteq N_{tr}\Upsilon cl(L) \subseteq M^c \Longrightarrow L\hat{q}M$. Therefore U is $N_{tr}\Upsilon$ −normal.

Theorem 4.3: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a strongly $N_{tr}\Upsilon$ −continuous and $N_{tr}\Upsilon$ −open bijective function. If $(U, \tau_{N_{tr}})$ is N_{tr}normal, then $(V, \rho_{N_{tr}})$ is $N_{tr}\Upsilon$ −normal.

Proof: Let $G$and H be $N_{tr}\Upsilon$ −closed sets in V such that $G\hat{q}H$. Then $f^{-1}_{N_{tr}}(G)\hat{q}f^{-1}_{N_{tr}}(H)$. Moreover, since $f_{N_{tr}}$ is strongly $N_{tr}\Upsilon$ −continuous,$f^{-1}_{N_{tr}}(G)$and$f^{-1}_{N_{tr}}(H)$are N_{tr}closed in U. Hence, by hypothesis, there exists N_{tr}open sets L and M in U such that $f^{-1}_{N_{tr}}(G) \subseteq L$, $f^{-1}_{N_{tr}}(H) \subseteq M$ and $L\hat{q}M$. Then, $G \subseteq f_{N_{tr}}(L), H \subseteq f_{N_{tr}}(M)$ and $f_{N_{tr}}(L)\hat{q}f_{N_{tr}}(M)$. Now, since $f_{N_{tr}}$ is $N_{tr}\Upsilon$ −open, $f_{N_{tr}}(L)$ and $f_{N_{tr}}(M)$ are $N_{tr}\Upsilon$ −open in V and therefore V is $N_{tr}\Upsilon$ −normal.

Theorem 4.4: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a $N_{tr}\Upsilon - i$ homeomorphism. If $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon$ −normal, then $(V, \rho_{N_{tr}})$ is also $N_{tr}\Upsilon$ −normal.

Proof: Let $G$and H be a $N_{tr}\Upsilon$ −closed sets in V such that $G\hat{q}H$. Then $f^{-1}_{N_{tr}}(G)$ and $f^{-1}_{N_{tr}}(H)$ are $N_{tr}\Upsilon$ −closed in U as $f_{N_{tr}}$ is a $N_{tr}\Upsilon - i$homeomorphism. Moreover, $f^{-1}_{N_{tr}}(G)\hat{q}f^{-1}_{N_{tr}}(H)$. Since U is $N_{tr}\Upsilon$ −normal, there exist $N_{tr}\Upsilon$ −open sets L and M in U such that $f^{-1}_{N_{tr}}(G) \subseteq L$, $f^{-1}_{N_{tr}}(H) \subseteq M$ and $L\hat{q}M$. Then, $G \subseteq f_{N_{tr}}(L), H \subseteq f_{N_{tr}}(M)$ and $f_{N_{tr}}(L)\hat{q}f_{N_{tr}}(M)$. Again, since $f_{N_{tr}}$ is a $N_{tr}\Upsilon - i$homeomorphism, $f_{N_{tr}}(L)$ and $f_{N_{tr}}(M)$ are $N_{tr}\Upsilon$ −open in V. Hence V is $N_{tr}\Upsilon$ −normal.

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Theorem 4.5: Let $(U, \tau_{N_{tr}})$ be a $N_{tr}\Upsilon$ −compact and N_{tr}Hausdorff space in which $H \cap H^c = 0_{N_{tr}}$ for every $N_{tr}\Upsilon$ −closed set H in $\left(U, \tau_{N_{tr}}\right)$. Then $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon$ −normal.

Proof: Let G and H be $N_{tr}\Upsilon$ −closed sets in U such that $G\hat{q}H$. Then $G \subseteq H^c$. Let $u_{a,b,c} \in G, v_{a',b',c'} \in H$ and by theorem 2.15, G and H are $N_{tr}\Upsilon$ −compact. Now, let us fix $u_{a,b,c}$. Since U is N_{tr}Hausdorff, for every point $v_{i_{a',b',c'}}(v_i \neq u)$, there exist N_{tr}open sets L_i and M_i such that $u_{a,b,c} \in L_i, u_{a,b,c} \notin M_i, v_{i_{a',b',c'}} \in M_i, v_{i_{a',b',c'}} \notin L_i$ and $L_i \subseteq M_i^c$ for all i. Consider the collection $\mathcal{M} = \left\{M_i : v_{i_{a',b',c'}} \in H\right\}$. This forms a N_{tr}open cover for H and by theorem 2.7, $\mathcal{M}$ forms a $N_{tr}\Upsilon$ −open cover for H. Since, H is $N_{tr}\Upsilon$ −compact, $\mathcal{M}$ has a finite subcover $\mathcal{M}' = \left\{M_{i_k} : k = 1,2,3, \dots, n\right\}$. Now, let $M_{u_{abc}} = \bigcup_{M_{i_k} \in \mathcal{M}'} M_{i_k}$and $L_{u_{a,b,c}} = \bigcap L_{i_k}$ where L_{i_k} are N_{tr}sets corresponding to each $M_{i_k} \in \mathcal{M}'$. Both $L_{u_{a,b,c}}$ and $M_{u_{a,b,c}}$ are N_{tr}open and hence $N_{tr}\Upsilon$ −open. Moreover, since $L_{i_k} \subseteq M_{i_k}^c$ for all $k, L_{u_{a,b,c}} = \bigcap L_{i_k} \subseteq \bigcap M_{i_k}^c = (\bigcup M_{i_k})^c = M_{u_{a,b,c}}^c \Longrightarrow L_{u_{a,b,c}} \subseteq M_{u_{a,b,c}}^c$. We observe that the collection $\mathcal{L} = \{L_{u_{a,b,c}} : u_{a,b,c} \in G\}$ forms a $N_{tr}\Upsilon$ −open cover for G. Since G is $N_{tr}\Upsilon$ −compact, $\mathcal{L}$ has a finite subcover $\mathcal{L}'$. Now, let $L = \bigcup_{L_u \in L'} L_{u_{a,b,c}}$ and $M = \bigcap\left\{M_{u_{a,b,c}} : L_{u_{a,b,c}} \in L'\right\}$. Then there exist $N_{tr}\Upsilon$ −open sets L and M such that $G \subseteq L, H \subseteq M$ and $L \subseteq M^c \Longrightarrow L\hat{q}M$. Hence $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon$ −normal.

Theorem 4.6: Let $(U, \tau_{N_{tr}})$ be a $N_{tr}\Upsilon$ −normal space in which the class of all $N_{tr}\Upsilon$ −open sets is closed under finite intersection. Then every neutrosophic open subspace of $(U, \tau_{N_{tr}})$ is also $N_{tr}\Upsilon$ −normal.

Proof: Let $(S, \tau_{N_{tr}}^S)$ be a neutrosophic open subspace of $\left(U, \tau_{N_{tr}}\right), G$ and H be $N_{tr}\Upsilon$ −closed sets in S such that $G\hat{q}H$. By lemma 2.14, G and H are $N_{tr}\Upsilon$ −closed in U. Since U is $N_{tr}\Upsilon$ −normal, there exist $N_{tr}\Upsilon$ −open sets L and M in U such that $G \subseteq L, H \subseteq M$ and $L\hat{q}M$. Now, $G \subseteq L \cap 1_{N_{tr}}^S, H \subseteq M \cap 1_{N_{tr}}^S$ and $\left(L \cap 1_{N_{tr}}^S\right)\hat{q}\left(M \cap 1_{N_{tr}}^S\right)$. By theorem 2.7, $1_{N_{tr}}^S$is $N_{tr}\Upsilon$ −open in U and by hypothesis, $L \cap 1_{N_{tr}}^S$ and $M \cap 1_{N_{tr}}^S$ are $N_{tr}\Upsilon$ −open in U. Again, by lemma 2.14, both $L \cap 1_{N_{tr}}^S$ and $M \cap 1_{N_{tr}}^S$are $N_{tr}\Upsilon$ −open in S. Therefore $(S, \tau_{N_{tr}}^S)$ is $N_{tr}\Upsilon$ −normal.

V. Stronger Forms of $N_{tr}\Upsilon$ −regular and $N_{tr}\Upsilon$ −normal Spaces

Definition 5.1: A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a **strongly$N_{tr}\Upsilon$ − regular space** if for every pair consisting of a $N_{tr}Pu_{a,b,c}$and a $N_{tr}\Upsilon$ −closed set $H, u_{a,b,c}\hat{q}H$, there exist N_{tr}open sets L and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in L, H \subseteq M$ and $L\hat{q}M$.

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Theorem 5.2: Every strongly $N_{tr}\Upsilon$ −regular space is $N_{tr}\Upsilon$ −regular space.

Proof: Let $(U, \tau_{N_{tr}})$ be a strongly $N_{tr}\Upsilon$ −regular space, H be a $N_{tr}\Upsilon$ −closed set and $u_{a,b,c}$ be a $N_{tr}P$ such that $u_{a,b,c}\hat{q}H$. Then, there exist N_{tr}open sets L and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in L, H \subseteq M$ and $L\hat{q}M$. By theorem 2.7, L and M are $N_{tr}\Upsilon$ −open. Hence U is $N_{tr}\Upsilon$ −regular.

Theorem 5.3: A neutrosophic topological space $\left(U, \tau_{N_{tr}}\right)$ is strongly $N_{tr}\Upsilon$ −regular if and only if for each $N_{tr}\Upsilon$ −closed set H and each $u_{a,b,c}\hat{q}H$, there exist N_{tr}open sets L and M such that $u_{a,b,c} \in L, H \subseteq M$ and $N_{tr}cl(L)\hat{q}N_{tr}cl(M)$.

Proof: Let U be a strongly $N_{tr}\Upsilon$ −regular space, $u_{a,b,c}$ be a $N_{tr}P$ in U and H be $N_{tr}\Upsilon$ −closed such that $u_{a,b,c}\hat{q}H$. Then, there exist N_{tr}open sets L and M such that $u_{a,b,c} \in L, H \subseteq M$ and $L\hat{q}M \Longrightarrow L\hat{q}N_{tr}cl(M)$. Now, $N_{tr}cl(M)$ is N_{tr}closed and $u_{a,b,c}\hat{q}N_{tr}cl(M)$. Hence there exist N_{tr}open sets P and Q such that $u_{a,b,c} \in P$ and $N_{tr}cl(M) \subseteq Q$ and $P\hat{q}Q$. Now, $M \subseteq N_{tr}cl(M) \subseteq Q \Longrightarrow Q^c \subseteq M^c$. Further, $P\hat{q}Q \Longrightarrow P \subseteq Q^c \subseteq M^c \Longrightarrow P\hat{q}M$. Since $N_{tr}cl(P)$is the smallest N_{tr}closed set containing $P, N_{tr}cl(P) \subseteq M^c$. Now, take $L = P$. Then $N_{tr}cl(L) = N_{tr}cl(P) \subseteq Q^c \subseteq (N_{tr}cl(M))^c$. Hence $N_{tr}cl(L)\hat{q}N_{tr}cl(M)$. Conversely, suppose $u_{a,b,c}$is a $N_{tr}P$ in U and H is a $N_{tr}\Upsilon$ −closed set such that $u_{a,b,c}\hat{q}H$. Then, by assumption, there exist N_{tr}open sets L and M in U such that $u_{a,b,c} \in L, H \subseteq M$ and $N_{tr}cl(L)\hat{q}N_{tr}cl(M)$.Now,$N_{tr}cl(L)\hat{q}N_{tr}cl(M) \Longrightarrow N_{tr}cl(L) \subseteq (N_{tr}cl(M))^c \Longrightarrow L \subseteq (N_{tr}cl(M))^c = (N_{tr}int(M))^c \subseteq M^c \Longrightarrow L\hat{q}M$. Therefore U is strongly $N_{tr}\Upsilon$ −regular.

Theorem 5.4: Let $\left(U, \tau_{N_{tr}}\right)$ be a strongly $N_{tr}\Upsilon$ −regular space. Then

(i) Every $N_{tr}\Upsilon$ −open set in $\left(U, \tau_{N_{tr}}\right)$ is the union of N_{tr}closed sets.

(ii) Every $N_{tr}\Upsilon$ −closed set in $\left(U, \tau_{N_{tr}}\right)$ is the intersection of N_{tr}open sets.

Proof: (i) Let L be a $N_{tr}\Upsilon$ −open set in U and $u_{a,b,c} \in L$. Then L^c is $N_{tr}\Upsilon$ −closed and $u_{a,b,c}\hat{q}L^c$. Since U is strongly $N_{tr}\Upsilon$ −regular, there exist N_{tr}open sets M and N such that $u_{a,b,c} \in M, L^c \subseteq N$ and $M\hat{q}N$. Now, $M\hat{q}N \Longrightarrow M \subseteq N^c \subseteq L$ and let $H = N_{tr}cl(M)$.Then, $H \subseteq N^c \Longrightarrow u_{a,b,c} \in H \subseteq N^c \subseteq L$. Therefore $L = \cup\{H : H \in \tau^c_{N_{tr}}\}$.

(ii) can be proved similarly.

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Theorem 5.5: A neutrosophic topological space $\left(U, \tau_{N_{tr}}\right)$ is strongly $N_{tr}\Upsilon$ −regular if and only if for every pair consisting of a $N_{tr}\Upsilon$ −compact set G and $N_{tr}\Upsilon$ −closed set H such that $G\hat{q}H$, there exist N_{tr}open sets L and M such that $G \subseteq L, H \subseteq M$ and $L\hat{q}M$.

Proof: Let U be a strongly $N_{tr}\Upsilon$ −regular space, G be a $N_{tr}\Upsilon$ −compact set and H be $N_{tr}\Upsilon$ −closed in U such that $G\hat{q}H$. Then, for each $u_{i_{a,b,c}} \in G$, there exist N_{tr}open sets L_i and M_i such that $u_{i_{a,b,c}} \in L_i$, $H \subseteq M_i$ and $L_i\hat{q}M_i$ for all i. Now, the collection $\{L_i : u_{i_{a,b,c}} \in G\}$ forms a N_{tr}open cover for G and by theorem 2.7, it forms a $N_{tr}\Upsilon$ −open cover for G. Since G is $N_{tr}\Upsilon$ −compact, there exists a finite subcollection $\{L_{i_k} : k = 1,2, \dots, n\}$ such that $G \subseteq \bigcup_{k=1}^{n} L_{i_k}$. Now, corresponding to each L_{i_k}, there exists someM_{i_k}such that $H \subseteq \bigcap_{k=1}^{n} M_{i_k}$. Take $L = \bigcup_{k=1}^{n} L_{i_k}$ and $M = \bigcap_{k=1}^{n} M_{i_k}$. Hence, there exist N_{tr}open sets L and M such that $G \subseteq L$, $H \subseteq M$ and $L\hat{q}M$. Conversely, let H be a $N_{tr}\Upsilon$ −closed set and $u_{a,b,c}\hat{q}H$. Now, $u_{a,b,c}$ is a $N_{tr}\Upsilon$ −compact set and by assumption, there exist N_{tr}open sets L and M such that $u_{a,b,c} \subseteq L, H \subseteq M$ and $L\hat{q}M$. Therefore U is strongly $N_{tr}\Upsilon$ −regular.

Theorem 5.6: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a $N_{tr}\Upsilon$ −irresolute, N_{tr}open bijective function. If $(U, \tau_{N_{tr}})$ is strongly $N_{tr}\Upsilon$ −regular, then $(V, \rho_{N_{tr}})$ is also strongly $N_{tr}\Upsilon$ −regular.

Proof: Let $u_{a,b,c}$ be a $N_{tr}P$ and H be a $N_{tr}\Upsilon$ −closed set in V such that $u_{a,b,c}\hat{q}H$. Then $f_{N_{tr}}^{-1}(u_{a,b,c})\hat{q}f_{N_{tr}}^{-1}(H)$ and since $f_{N_{tr}}$ is $N_{tr}\Upsilon$ −irresolute, $f_{N_{tr}}^{-1}(H)$ is $N_{tr}\Upsilon$ −closed in U. Since U is strongly $N_{tr}\Upsilon$ −regular, there exists N_{tr}open sets L and M in U such that $f_{N_{tr}}^{-1}(u_{a,b,c}) \in L$, $f_{N_{tr}}^{-1}(H) \subseteq M$ and $L\hat{q}M$. Then, $u_{a,b,c} \in f_{N_{tr}}(L), H \subseteq f_{N_{tr}}(M)$ and $f_{N_{tr}}(L)\hat{q}f_{N_{tr}}(M)$. Again, since $f_{N_{tr}}$ is N_{tr}open, $f_{N_{tr}}(L)$ and $f_{N_{tr}}(M)$ are N_{tr}open in V. Hence V is strongly $N_{tr}\Upsilon$ −regular.

Definition 5.7: A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a strongly $\boldsymbol{N_{tr}\Upsilon}$ **−normal space** if for every pair of $N_{tr}\Upsilon$ −closed sets G and $H, G\hat{q}H$, there exist N_{tr}open sets L and M in $(U, \tau_{N_{tr}})$ such that $G \subseteq L, H \subseteq M$ and $L\hat{q}M$.

Theorem 5.8: Every strongly $N_{tr}\Upsilon$ −normal space is $N_{tr}\Upsilon$ −normal space.

Proof: Let $(U, \tau_{N_{tr}})$ be a strongly $N_{tr}\Upsilon$ −normal space, $G$and H be $N_{tr}\Upsilon$ −closed sets such that $G\hat{q}H$. Then there exist N_{tr}open sets L and M in $(U, \tau_{N_{tr}})$ such that $G \subseteq L, H \subseteq M$ and $L\hat{q}M$. By theorem 2.7, L and M are $N_{tr}\Upsilon$ −open. Hence U is $N_{tr}\Upsilon$ −normal.

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Theorem 5.9: A neutrosophic topological space $\left(U, \tau_{N_{tr}}\right)$ is strongly $N_{tr}\Upsilon$ −normal if and only if for each pair of $N_{tr}\Upsilon$ −closed sets G and H in $(U, \tau_{N_{tr}})$ such that $G\hat{q}H$, there exist $N_{tr}\Upsilon$ −open sets L and M containing G and H respectively such that $N_{tr}\Upsilon cl(L)\hat{q}N_{tr}\Upsilon cl(M)$.

Proof: Let U be a strongly $N_{tr}\Upsilon$ −normal space, G and H be $N_{tr}\Upsilon$ −closed sets in U such that $G\hat{q}H$. Then, there exist N_{tr}open sets L and M such that $G \subseteq L, H \subseteq M$ and $L\hat{q}M \Longrightarrow L\hat{q}N_{tr}cl(M)$. Now, $N_{tr}cl(M)$ is N_{tr}closed and $G\hat{q}N_{tr}cl(M)$. Hence there exist N_{tr}open sets P and Q such that$G \subseteq P$ and $N_{tr}cl(M) \subseteq Q$ and $P\hat{q}Q$. Now, $M \subseteq N_{tr}cl(M) \subseteq Q \Longrightarrow Q^c \subseteq M^c$. Further, $P\hat{q}Q \Longrightarrow P \subseteq Q^c \subseteq M^c \Longrightarrow P\hat{q}M$. Since $N_{tr}cl(P)$is the smallest N_{tr}closed set containing $P, N_{tr}cl(P) \subseteq M^c$. Now, take $L = P$. Then $N_{tr}cl(L) = N_{tr}cl(P) \subseteq Q^c \subseteq (N_{tr}cl(M))^c$. Hence $N_{tr}cl(L)\hat{q}N_{tr}cl(M)$. Conversely, let G and H be a $N_{tr}\Upsilon$ −closed sets in U such that $G\hat{q}H$. Then, by assumption, there exist N_{tr}open sets L and M in U such that $G \subseteq L, H \subseteq M$ and $N_{tr}cl(L)\hat{q}N_{tr}cl(M)$. Now, $N_{tr}cl(L)\hat{q}N_{tr}cl(M) \Longrightarrow N_{tr}cl(L) \subseteq (N_{tr}cl(M))^c \Longrightarrow L \subseteq (N_{tr}cl(M))^c = (N_{tr}int(M))^c \subseteq M^c \Longrightarrow L\hat{q}M$. Therefore U is strongly $N_{tr}\Upsilon$ −normal.

Theorem 5.10: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a $N_{tr}\Upsilon$ −irresolute, N_{tr}open bijective function. If $(U, \tau_{N_{tr}})$ is strongly $N_{tr}\Upsilon$ −normal, then $(V, \rho_{N_{tr}})$ is also strongly $N_{tr}\Upsilon$ −normal.

Proof: Let $G$and H be a $N_{tr}\Upsilon$ −closed sets in V such that $G\hat{q}H$. Then $f_{N_{tr}}^{-1}(G), f_{N_{tr}}^{-1}(H)$ are $N_{tr}\Upsilon$ −closed in U and $f_{N_{tr}}^{-1}(G)\hat{q}f_{N_{tr}}^{-1}(H)$. Since U is strongly $N_{tr}\Upsilon$ −normal, there exist N_{tr}open sets L and M in U such that $f_{N_{tr}}^{-1}(G) \subseteq L$, $f_{N_{tr}}^{-1}(H) \subseteq M$ and $L\hat{q}M$. Then, $G \subseteq f_{N_{tr}}(L), H \subseteq f_{N_{tr}}(M)$ and $f_{N_{tr}}(L)\hat{q}f_{N_{tr}}(M)$. Again, since $f_{N_{tr}}$ is a N_{tr}open, $f_{N_{tr}}(L)$ and $f_{N_{tr}}(M)$ are N_{tr}open in V. Hence V is strongly $N_{tr}\Upsilon$ −normal.

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A Study on Kinds of Pairwise Micro (Near, Closer) relations in Micro Bitopological Space

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Abstract

Pairwise micro near and Pairwise micro closer are the names given to the relationships. We discuss different types of pairwise micro near and pairwise micro closer relations in micro bitopological space in this study. In addition to reviewing and providing examples, we also looked into some properties.

Keywords: Micro bitopological space, Pairwise micro near and Pairwise micro closer.

MSC: 54A10, 54E55.

1 Introduction

Bitopology origin is usually associated with the appearance of J. C. Kelly's [1] paper in 1963. In this fundamental paper, a bitopological space was clearly defined as a set with two arbitrary topological structure. R. Chitralekha [2]introduced the near and closer relations on bitopology. Micro topology was introduced by SakkraiveerananChandrasekar [3] (2019), and some of their properties are investigated. We discover a

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definition of micro bitopological space and different types of micro open sets, that paper is under on communication (Accepted).We defined a definition for pairwise micro β open set with example and their properties are discussed [5].

In this paper, we introduced pairwise micro near and pariwise micro closer relations in micro bitopological space with some examples in the current study. By using examples, we were able to teach micro $(1,2)-$ (semi,pre, α, β) set and micro $(2,1)-(\text{semi}, \text{pre}, \alpha, \beta)$set. On it, we discover the relationship.

2. Preliminary

Definition 2.1 [1].

Let X be a set. Let P and Q be topologies for X. Then the ordered triple(X, P, Q) is said to be a

bitopological space.

Definition 2.2 [2].

We say that A is $ij-$ near (resp. $ij-$ semi near, resp. $ij-$ pre near, resp. $ij-\alpha-$ near, resp. $ij-\beta-$ near) to B

if $Int_iA = Int_jB$ (resp. $SInt_iA = SInt_jB$, resp. $PInt_iA = PInt_jB$, resp. $\alpha Int_iA = \alpha Int_jB$, resp. $\beta Int_iA = \beta Int_jB$).

Definition 2.3 [2].

We say that A is $ij-$ closer (resp. $ij-$ semi closer, resp. $ij-$ pre closer, resp. $ij-\alpha-$ closer, resp. $ij-\beta-$ closer)

to B if $Cl_iA = Cl_jB$ (resp. $SCl_iA = SCl_jB$, resp. $PCl_iA = PCl_jB$, resp. $\alpha Cl_iA = \alpha Cl_jB$, resp. $\beta Cl_iA = \beta Cl_jB$).

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Definition 2.2 [3]

The micro topology $\mu_R(X)$ satisfies the following axioms

1. $U, \phi \in \mu_R(X)$.

2. The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

3. The intersection of the elements of any finite sub collection of $\mu_R(X)$is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$

is called micro topological space and the elements of $\mu_R(X)$ are called micro open sets

and the complement of a micro open set is called a micro closed set.

Definition 2.4 [4].

Let U be a non empty set and it contain two equivalence relationR_1, R_2. LetX_1, X_2 be any two subsets of U.Let $(U, \tau_{R_1}(X_1))$ is a nano topological space then $\mu_{R_1}(X_1) = \{S \cup (S' \cap \mu_1) / S, S' \in \tau_{R_1}(X_1)\}$ where $\mu_1 \notin \tau_{R_1}(X_1)$ is called a micro topology of $\tau_{R_1}(X_1)$ and let $(U, \tau_{R_2}(X_2))$ is an another nano topological space then $\mu_{R_2}(X_2) = \{T \cup (T' \cap \mu_2) / T, T' \in \tau_{R_2}(X_2)\}$ where $\mu_2 \notin \tau_{R_2}(X_2)$ is called a micro topology of $\tau_{R_2}(X_2)$. If $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2), \mu_{R_1}(X_1), \mu_{R_2}(X_2))$ briefly, $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is called a micro bitopological space then it is satisfy the following conditions:

1. $U, \phi \in \mu_{R_{1,2}}(X)$.

2. Arbitrary union of $\mu_{R_{1,2}}(X)$ is in $\mu_{R_{1,2}}(X)$.

3. Finite intersection of $\mu_{R_{1,2}}(X)$ is in $\mu_{R_{1,2}}(X)$.

Therefore $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is called a microbitopological space and the element of micro

bitopological space is called $M\mu_{R_{1,2}}(X)-$ open. The complement is called $M\mu_{R_{1,2}}(X)-$ closed set.

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Definition 2.5 [4].

Let H be a subset of a micro bitopological space. Then the micro closure of H is denoted

by $\mathrm{M}\mu_{\mathrm{R}_{1,2}}\mathrm{cl}(H) = \bigcap\{\mathrm{V}: H \subseteq \mathrm{V}$ and V is $\mathrm{M}\mu_{\mathrm{R}_{1,2}}(\mathrm{X}) - \text{closed}\}$. Then the micro interior of H is denoted by

$$\mathrm{M}\mu_{\mathrm{R}_{1,2}}\mathrm{int}(H) = \cup\{\mathrm{V}: \mathrm{V} \subseteq H \text{ and V is } \mathrm{M}\mu_{\mathrm{R}_{1,2}}(X) - \text{open}\}.$$

Definition 2.6 [4].

A subset A of $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is said to be pairwise micro semi open set in U

if A is micro $(1,2) -$ semi open if $\mathrm{A} \subseteq \mathrm{M}\mu_{\mathrm{R}_{1,2}}\mathrm{cl}\left(\mathrm{M}\mu_{\mathrm{R}_1}\mathrm{int}(\mathrm{A})\right)$ and micro $(2,1) -$ semi open if $A \subseteq \mathrm{M}\mu_{\mathrm{R}_{1,2}}\mathrm{cl}(\mathrm{M}\mu_{\mathrm{R}_2}\mathrm{int}(\mathrm{A}))$. A subset A of $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is said to be pairwise micro pre open set in U if A is micro $(1,2) -$pre open if $\mathrm{A} \subseteq \mathrm{M}\mu_{\mathrm{R}_1}\mathrm{int}(\mathrm{M}\mu_{\mathrm{R}_{1,2}}\mathrm{cl}(\mathrm{A}))$ and micro $(2,1) -$pre open if $A \subseteq \mathrm{M}\mu_{\mathrm{R}_2}\mathrm{int}(\mathrm{M}\mu_{\mathrm{R}_{1,2}}\mathrm{cl}(\mathrm{A}))$. A subset A of $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is said to be pairwise micro α open set in U if A is micro $(1,2) - \alpha$ open if $\mathrm{A} \subseteq \mathrm{M}\mu_{\mathrm{R}_1}\mathrm{int}(\mathrm{M}\mu_{\mathrm{R}_{1,2}}\mathrm{cl}(\mathrm{M}\mu_{\mathrm{R}_1}\mathrm{int}(\mathrm{A})))$ and micro $(2,1) - \alpha$ open if $A \subseteq \mathrm{M}\mu_{\mathrm{R}_2}\mathrm{int}\left(\mathrm{M}\mu_{\mathrm{R}_{1,2}}\mathrm{cl}\left(\mathrm{M}\mu_{\mathrm{R}_2}\mathrm{int}(\mathrm{A})\right)\right)$.

Definition 2.7 [5].

A subset H is said to be pairwise micro β open set then H is micro $(1,2)\,\beta -$open set if $H \subseteq \mathrm{M}\mu_{\mathrm{R}_{1,2}}\,\mathrm{cl}(\mathrm{M}\mu_{\mathrm{R}_1}\mathrm{int}(\mathrm{M}\mu_{\mathrm{R}_{1,2}}\,\mathrm{cl}(H)))$and micro $(2,1)\,\beta -$ open set if $H \subseteq \mathrm{M}\mu_{\mathrm{R}_{1,2}}\,\mathrm{cl}(\mathrm{M}\mu_{\mathrm{R}_2}\mathrm{int}(\mathrm{M}\mu_{\mathrm{R}_{1,2}}\,\mathrm{cl}(H)))$.

3 Kinds of Pairwise micro near

Pairwise micro near and its types are introduced and discussed in this session.

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Definition 3.1:

Let $\left(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X)\right)$ is a micro bitopological space, H and V are subsets of U. If H is pairwise micro near

to V then H is micro $(1,2)$ near to V and H is micro $(2,1)$ near to V that means, $M\mu_{R_1} Int\,(H) = M\mu_{R_2} Int\,(V)$ and $M\mu_{R_2} Int\,(H) = M\mu_{R_1} Int\,(V)$.

Definition 3.2:

Let $\left(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X)\right)$ is a micro bitopological space, H and V are subsets of U. Then

1. H is pairwise micro semi near to V then $M\mu_{R_{1,2}} SInt\,(H) = M\mu_{R_{1,2}} SInt\,(V)$.
2. H is pairwise micro pre near to V then $M\mu_{R_{1,2}} PInt\,(H) = M\mu_{R_{1,2}} PInt\,(V)$.
3. H is pairwise micro α near to V then $M\mu_{R_{1,2}} \alpha Int\,(H) = M\mu_{R_{1,2}} \alpha Int\,(V)$.
4. H is pairwise micro β near to V then $M\mu_{R_{1,2}} \beta Int\,(H) = M\mu_{R_{1,2}} \beta Int\,(V)$.

Example 3.3:

Let $\mathrm{U} = \{1,2,3,4\}$with $\mathrm{U/R}_1 = \{\{1,3\},\{2\},\{4\}\}$ and$\mathrm{X}_1 = \{1,2\} \Longrightarrow \tau_{R_1}(X_1) = \{\mathrm{U}, \phi, \{2\}, \{1,3\}, \{1,2,3\}\}$.

Then$\mu_1 = \{3\} \notin \tau_{R_1}(X_1)$. The $\mathrm{M}\mu_{R_1}(X_1) = \{\mathrm{U}, \phi, \{2\}, \{3\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$ and $\mathrm{U/R}_2 = \{\{1,3\},\{2\},\{4\}\}$ and $\mathrm{X}_2 = \{2,3\} \implies \tau_{R_2}(X_2) = \{\mathrm{U}, \phi, \{2\}, \{1,3\}, \{1,2,3\}\}$. Then$\mu_2 = \{3\} \notin \tau_{R_2}(X_2)$. Then the $\mathrm{M}\mu_{R_2}(X_2) = \{\mathrm{U}, \phi, \{2\}, \{3\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$. Then $M\mu_{R_{1,2}}(X) = \{\mathrm{U}, \phi, \{2\}, \{3\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$ is a micro bitopological space. Let take $H = \{3\}$ and $V = \{3,4\}$ be any two subsets of U. Then H is pairwise micro near to V.Its satisfy pairwise micro semi near, pairwise micro pre near, pairwise micro α near, pairwise micro β near.

Theorem 3.4: Intersection of any two sets of pairwise micro near is pairwise micro near.

Proof:

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Let H is pairwise micro near to V and R is pairwise micro near to S. Then $M\mu_{R_{1,2}}Int\,(H) = M\mu_{R_{1,2}}Int\,(V)$ and $M\mu_{R_{1,2}}Int\,(R) = M\mu_{R_{1,2}}Int\,(S)$. That implies $M\mu_{R_{1,2}}Int\,(H) \cap M\mu_{R_{1,2}}Int\,(R) = M\mu_{R_{1,2}}Int\,(V) \cap M\mu_{R_{1,2}}Int\,(S) \Rightarrow M\mu_{R_{1,2}}Int\,(H \cap R) = M\mu_{R_{1,2}}Int\,(V \cap S)$. The intersection of two pairwise micro near is pairwise micro near. But the converse is need not true. It's proved from the following example.

Example 3.5:

From the above example 3.3, $H = \{3\}$ and $V = \{3,4\}$ is pairwise micro near. Then the other pairwise micro near set, $R = \{2,4\}$ is pairwise micro near to $S = \{1,2\}$. Then take the union, $H \cup R = \{2,3,4\}$ and $V \cup S = \{1,2,3,4\} = U$. Therefore, $M\mu_{R_1}Int\,(H \cup R) = \{2,3\} \neq U = M\mu_{R_2}Int\,(V \cup S)$ and$M\mu_{R_2}Int\,(H \cup R) = \{2,3\} \neq U = M\mu_{R_1}Int\,(V \cup S)$. Hence proved.

Properties 3.6:

If H is micro $(1,2)$near to V then V is micro $(2,1)$ near to H. This is true for pairwise micro semi near,pairwise micro pre near, pairwise micro α near, pairwise micro βnear.

Theorem 3.7: Every pairwise micro near relation is pairwise micro semi near.

Proof:

Proof is obvious. But the converse is need not be true. Its explained by the following example.

Example 3.8:

Frome the example 3.3, Take $H = \{3\}$ and $V = \{1,3\}$. $M\mu_{R_{1,2}}SInt\,(H) = \{1,3,4\} = M\mu_{R_{1,2}}SInt\,(V)$. Therefore, H is pairwise micro semi near to V. Now we see pairwise micro near,

$M\mu_{R_1}\ Int\,(H) = \{3\} \neq \{1,3\} = M\mu_{R_2}Int\,(V)$and $M\mu_{R_2}\ Int\,(H) = \{3\} \neq \{1,3\} = M\mu_{R_1}Int\,(V)$. This is don't have apairwise micro near relation.

4 Kinds of Pairwise micro closer

Pairwise micro closer and its types are introduced and discussed in this session.

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Definition 4.1:

Let $\left(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X)\right)$ is a micro bitopological space, R and S are subsets of U. If R is pairwise micro closer to S then R is micro $(1,2)$ closer to S and R is micro $(2,1)$ closer to S that means, $M\mu_{R_1} cl\,(R) = M\mu_{R_2} cl\,(S)$ and $M\mu_{R_2} cl\,(R) = M\mu_{R_1} cl\,(S)$.

Definition 4.2:

Let $\left(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X)\right)$ is a micro bitopological space, R and S are subsets of U. Then

1. R is pairwise micro semi closer to S then $M\mu_{R_{1,2}} Scl\,(R) = M\mu_{R_{1,2}} Scl\,(S)$.
2. R is pairwise micro pre closer to S then $M\mu_{R_{1,2}} Pcl\,(R) = M\mu_{R_{1,2}} Pcl\,(S)$.
3. R is pairwise micro α closer to S then $M\mu_{R_{1,2}} \alpha cl(R) = M\mu_{R_{1,2}} \alpha cl(S)$.
4. R is pairwise micro β closer to S then $M\mu_{R_{1,2}} \beta cl(R) = M\mu_{R_{1,2}} \beta cl\,(S)$.

Example 4.3:

Let $\mathrm{U} = \{\mathrm{s,t,u,v,w}\}$with $\mathrm{U/R_1} = \{\{\mathrm{s}\},\{\mathrm{t,u}\},\{\mathrm{v,w}\}\}$ and$\mathrm{X_1} = \{\mathrm{s,v}\} \Longrightarrow \tau_{R_1}(X_1) = \{\mathrm{U}, \phi, \{\mathrm{s}\}, \{\mathrm{s,v,w}\}, \{\mathrm{v,w}\}\}\}$. Then$\mu_1 = \{\mathrm{w}\} \notin \tau_{R_1}(X_1)$. The $\mathrm{M}\mu_{R_1}(X_1) = \{\mathrm{U}, \phi, \{\mathrm{s}\}, \{\mathrm{w}\}, \{\mathrm{s,w}\}, \{\mathrm{v,w}\}, \{\mathrm{s,v,w}\}\}$ and $\mathrm{U/R_2} = \{\{\mathrm{s,t}\}, \{\mathrm{u}\}, \{\mathrm{v,w}\}\}$ and $\mathrm{X_2} = \{\mathrm{v}\} \Longrightarrow \tau_{R_2}(X_2) = \{\mathrm{U}, \phi, \{\mathrm{v,w}\}\}$. Then$\mu_2 = \{\mathrm{w}\} \notin \tau_{R_2}(X_2)$. Then the $\mathrm{M}\mu_{R_2}(X_2) = \{\mathrm{U}, \phi, \{\mathrm{w}\}, \{\mathrm{v,w}\}\}$. Then $M\mu_{R_{1,2}}(X) = \{\mathrm{U}, \phi, \{\mathrm{s}\}, \{\mathrm{w}\}, \{\mathrm{s,w}\}, \{\mathrm{v,w}\}, \{\mathrm{s,v,w}\}\}$ is a micro bitopological space. Let take $R = \{s,t\}$ and $S = \{s,u\}$ be any two subsets of U. Then R is pairwise micro closer to S. Let take $R = \{t,w\}$ and $S = \{u,w\}$ be any two subsets of U is pairwise micro semi closer, pairwise micro pre closer, pairwise micro α closer, pairwise micro β closer.

Properties 4.4:

If R is micro $(1,2)$closer to S then S is micro $(2,1)$ closer to S. This is true for pairwise micro semi closer, pairwise micro pre closer, pairwise micro α closer, pairwise micro β closer.

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Theorem 4.5: Union of any two sets of pairwise micro closer is pairwise micro closer.

Proof:

Let H is pairwise micro closer to V and R is pairwise micro closer to S. Then $M\mu_{R_{1,2}}cl\,(H) = M\mu_{R_{1,2}}cl\,(V)$ and $M\mu_{R_{1,2}}cl\,(R) = M\mu_{R_{1,2}}cl\,(S)$. That implies $M\mu_{R_{1,2}}cl\,(H) \cup M\mu_{R_{1,2}}cl\,(R) = M\mu_{R_{1,2}}cl\,(V) \cup M\mu_{R_{1,2}}cl\,(S) \Rightarrow M\mu_{R_{1,2}}cl\,(H \cup R) = M\mu_{R_{1,2}}cl\,(V \cup S)$. The union of two pairwise micro closer is pairwise micro closer. But the converse is need not true. It's proved from the following example.

Example 4.6:

For the example 3.3, $R = \{2\}$ and $S = \{2,4\}$ is pairwise micro closer. Then the other pairwise micro closer set, $H = \{3\}$ is pairwise micro closer to $V = \{3,4\}$. Then take the intersection, $H \cap R = \{\phi\}$ and $V \cap S = \{4\}$. Therefore, $M\mu_{R_1}Cl\,(H \cap R) = \{\phi\} \neq \{4\} = M\mu_{R_2}Cl\,(V \cap S)$ and$M\mu_{R_2}Cl\,(H \cap R) = \{\phi\} \neq \{4\} = M\mu_{R_1}Cl\,(V \cap S)$. Hence proved.

Theorem 4.7: Every pairwise micro semi closer relation is pairwise micro closer.

Proof:

Proof is obvious. But the converse is need not be true. Frome the example 4.3, Take $R = \{s,t\}$ and $S = \{s,u\}$. $M\mu_{R_{1,2}}Cl\,(R) = \{s,t,u\} = M\mu_{R_{1,2}}Cl\,(S)$. Therefore, R is pairwise micro closer to S. But it is not pairwise micro semi closer.

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FURTHER STUDY ON DERIVATIVE TOPOLOGICAL SPACES

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Abstract

A derivative topology on a differential ring R is defined by means of differential subsets of a differential ring. The derivative topology on Ris a collectionτ of differential subsets of R, which has the structure that $\emptyset$, R are in τ and is closed under arbitrary union and arbitrary intersection. The primary objective of this paper is to compare the derivative topology to various topologies such as general topology, supra topology, generalized topology, filter topology and interior topology. It is observed that the structure of derivative topology differs from that of other types of topologies. Finally, we prove that the intersection of any family of derivative topologies is again a derivative topology on a differential ring.

Key words: Derivative topology, differential subset, supra topology, filter topology, interior topology.

2020 Mathematics Subject Classifications: 54H13, 54A05, 54A10, 54B05.

INTRODUCTION

Throughout this paper, we focus only on differential rings. Let R denote the differential ring unless otherwise specified. The notion of the ring with derivation is quite old and plays a significant role in the integration of analysis, algebraic geometry and algebra. In 1950s a new part of algebra called differential algebra was initiated by the works of Ritt and Kolchin. In 1950 Ritt [14] and in 1973 Kolchin [12] wrote the well-known books on differential algebra in which differential rings, differential fields and differential algebras are rings, fields and algebras equipped with finitely many derivations. Kaplansky, too, wrote an interesting book on this subject in 1957 [11].

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Numerous researchers have introduced new topological structures via either the topological elements (open sets) or the topology itself (topology definition), such as Kelly [10] study of two topologies determine for the same set named Bitopology and introduced various separation properties into topological spaces, and obtained generalizations of some important classical results. For many years, various authors [1,3,7] constructed some topologies over algebraic structures and they investigated the relations between the algebraic properties of given algebraic structures (such as rings, modules, lattices and fuzzy structures) and topological properties of these topologies.

In this manner, we introduce a new topology on a differential ring, namely, derivative topology τ which has the structure that ∅, R are in τ and is closed under arbitrary union and arbitrary intersection.The structure of derivative topology differs from that of other types of topologies as general topology and supra topology etc. Also, we can study their topological properties to get a better understanding of the things around us.

2. Preliminaries

In this section, we present some basic definitions of differential algebra that are very useful in the subsequent discussions. Throughout the work, R denote the differential ring.

Definition 2.1 [11]: Let R be a commutative ring with identity. A derivation on R is a mapd:R→ R such that

(i) d(a+b)= d(a)+d(b)

(ii) d(a.b)= d(a).b+a.d(b)

A differential ring is a commutative ring with identity R together with the distinguished derivation d. If R is a differential ring and $x \in$R, then write x' for dx, x'' for d^2x and in general $x^{(n)}$for d^nx.

Let R be a commutative ring with identity. A derivation d on R is said to be the trivial derivation if $d(a) = 0$ for all $a \in R$.

The derivative of a nonempty set S is denoted by d(S) and defined as d(S) ={d(a)/a∈ S}.

Definition 2.2[11]: A subset S of a differential ring R is called a differential subset if it contains the derivative of each of its elements. Equivalently, d(S)⊆ S, where d(S) is the derivative of S.

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Definition 2.3 [15]:A derivative topology on a differential ring R with a derivation d is a collection τ of differential subsets of R having the following properties:

1. $\emptyset, R$are in τ.
2. The union of the elements of any subcollection of τ is in τ.
3. The intersection of the elements of any subcollection of τ is in τ.

If (R, d,τ) is a derivative topological space, we say that a differential subset U of R is a differential open set of R if U belongs to τ.

Definition 2.4 [13]: Let X be a nonempty set. Let P(X) be a power set of X. A subclass $\tau^* \subseteq$ P(X) is called supra topology on X if

1. $X \in \tau^*$.
2. τ^* is closed under an arbitrary union of elements of τ^*.

A supra topological space is set X together with supra topology τ^* on X.

Definition 2.5 [5]: Let X be a nonempty set.Let P(X) be a power set of X. A subclass $\mu \subseteq$ P(X) is called generalized topology on X if

1. $X \in \mu$.
2. μ is closed under an arbitrary union of elements of μ.

A generalized topological space is set X together with generalized topology μ on X.

Definition 2.6 [4]: Let X be a nonempty set. Let P(X) be a power set of X. A subclass $\mathcal{F} \subseteq$ P(X) is called a filter on X if the following is satisfied:

1. $\emptyset \notin \mathcal{F}$.
2. If A,B $\in \mathcal{F}$ then A$\bigcap$B $\in \mathcal{F}$.
3. If A $\in \mathcal{F}$ and A $\subseteq$ B$\subseteq$X then B $\in \mathcal{F}$.

Definition 2.7 [2]: Let X be a nonempty set. Let P(X) be a power set of X. A subclass$I_t \subseteq$ P(X) is called an Interior topology on X if the following is satisfied:

1. $\emptyset \notin I_t$.
2. It is closed under an arbitrary union of elements of I_t.

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3. It is closed under the arbitrary intersection of elements of I_t.

An Interior topological space is set X together with the Interior topology I_t on X.

3.INTERSECTION AND UNION OF DERIVATIVE TOPOLOGIES

In this section, we will check that whether the union and intersection of derivative topologies is a derivative topology or not.

Proposition 3.1: If $\{\tau_\alpha\}$ is a family of derivative topologies on a differential ring R, then $\cap \tau_\alpha$ is a derivativetopology on R.

Proof: Let$\{\tau_\alpha\}$ be a family of derivative topologies on a differential ring R. Let $\tau = \cap \tau_\alpha$. Since $\emptyset, R \in \tau_\alpha$ for each α, we have $\emptyset, R \in \tau$. Let $A_1, A_2, \ldots \ldots . . A_n, \ldots \ldots$ be in τ. Then $A_1, A_2, \ldots \ldots . . A_n, \ldots \ldots$ belongs to τ_α for each α. Since each τ_α is a derivative topology on R, we have $\cap A_i \in \tau_\alpha$ for each α. Therefore, $\cap A_i \in \tau$. Let $A_1, A_2, \ldots \ldots . . A_n, \ldots \ldots$ be in τ. Then $A_1, A_2, \ldots \ldots . . A_n, \ldots \ldots$ belongs to τ_α for each α. Since each τ_α is a derivative topology on R, we have $\cup A_i \in \tau_\alpha$ for each α. Therefore, $\cup A_i \in \tau$. Hence, $\cap \tau_\alpha$ is a derivativetopology on R.

Remark 3.2: If $\{\tau_\alpha\}$ is a family of derivative topologies on a differential ring R, then$\cup \tau_\alpha$ need not be a derivative topology on R. This is shown in the following example.

Example 3.3: Consider the differential ring Q[x], set of all polynomials with rational co efficients with the derivation d (usual differentiation).

1. Let $S_1 = \emptyset$.Then S_1 is a differential subset of Q[x].
2. Let S_2={0}, the set containing only the zero polynomials. Then S_2 is a differential subset of Q[x].
3. Let S_3= Q[x]. Then S_3 is a differential subset of Q[x].
4. Let $S_4 =$ Q, the set of all constant polynomials of Q[x]. Then S_4 is a differential subset of Q[x].
5. Let S_5 be the set of all polynomials of degree at most 1 ={$a_0 + a_1 x / a_0, a_1 \in Q$}. Then S_5 is a differential subset of Q[x].
6. Let $S_6 = Z[x]$, the set of all polynomials with coefficients in Z. Then S_6 is a differential subset of $Q[x]$.

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7. Let S_7 be the set of all polynomials in Q[x] with positive coefficients. Then S_7 is a differential subset of Q[x].

Let $\tau_1 = \{S_1, S_2, S_3, S_4, S_5\}$ and $\tau_2 = \{S_1, S_3, S_7\}$. Then τ_1 and τ_2 are derivative topology on Q[x]. Let $\tau = \tau_1 \cup \tau_2$. Then $\tau = \{S_1, S_2, S_3, S_4, S_5, S_7\}$ is not a derivative topology on Q[x] since $S_5 \cap S_7$ is the set of all linear polynomials with positive coefficients, which is not in τ.

Hence, in general the union of derivative topologies on a differential ring need not be derivative topology.

4.THE RELATION BETWEEN DERIVATIVE TOPOLOGY AND OTHER TOPOLOGIES

In this section, we will prove that there is no relation between derivative topology and topology, generalized topology, supra topology, interior topology and filter topology. This section aims to prove the independence and existence of derivative topology.

4.1. Topology and derivative topology

A derivative topology on a differential ring R is a topology on R but not conversely. Since by imposing trivial derivation on a ring, it will become a differential ring. In addition, the condition of arbitrary intersections mostly does not come true in topology.

4.2. Generalized topology and derivative topology

A derivative topology on a differential ring R is a generalized topology on R but not conversely. From the definition of generalized topology, it is clear that there is no need of μ is closed under arbitrary intersection. Also, the derivative topology has R but the generalized topology hasn't R.

4.3. Supra topology and derivative topology

A derivative topology on a differential ring R is a supra topology on R but not conversely. It follows from the definition of supra topology, it is not closed under arbitrary intersection. Also, the derivative topology has ∅ but the supra topology hasn't ∅.

4.4.Interior topology and derivative topology

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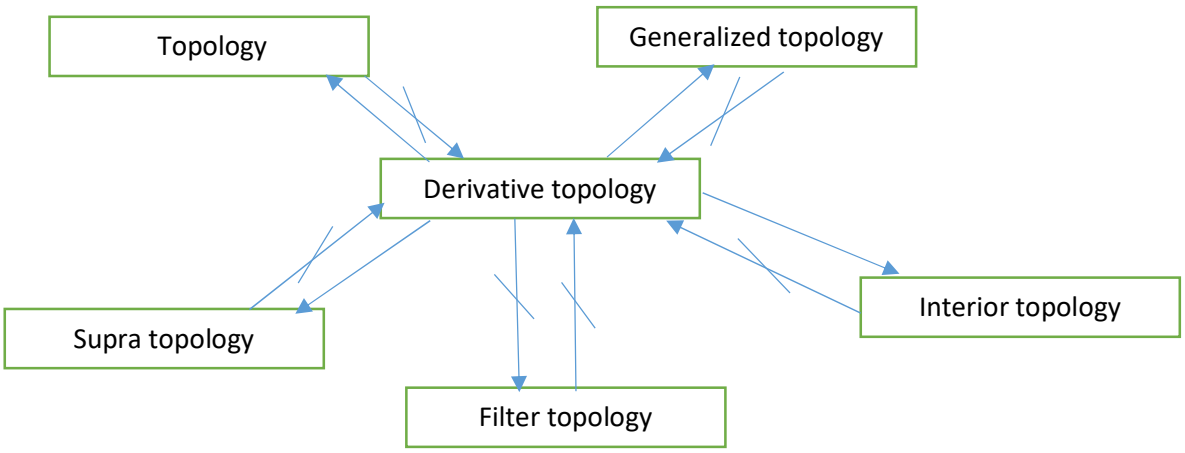
A derivative topology on a differential ring R is an interior topology on R but not conversely. Because the derivative topology has Ø but the interior topology hasn't Ø.

4.5. Filter topology and derivative topology

The concept of filter topology and derivative topology on R is independent. Since the arbitrary intersection and arbitrary union does not come true in filter topology. Also, in derivative topology a set which is a super set of a differential open set need not be differential open.Every derivative topology is impossible to become a filter topology and versa vice.

This is consolidated and shown in the following figure:



CONCLUSION

In this paper, we have shown the intersection of derivative topologies is again a derivative topology however the union of derivative topologies need not be a derivative topology. Also, we investigated its structure is independent from the previously indicated structure.

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Topological Polynomial Analysis of Imatinib Using Degree-Based Indices

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Abstract

The imatinib is an inhibitor used for the treatment of various types of cancer and bone marrow diseases. In this paper we used degree based topological indices to draw different polynomials like GA polynomial, ABC polynomials, Hyper Zagreb polynomials, Harmonic polynomials, Randic polynomials, Sum connectivity polynomials, first, second and third Zagreb polynomials, Augmented polynomials, Harmonic polynomials, Forgotten topological polynomials, inverse sum Indeg polynomials and Somber polynomials for $C_{29}H_{31}N_7O$.

Keywords: GA polynomial, ABC polynomials, Hyper Zagreb polynomials, harmonic polynomials, Randic polynomials, Sum connectivity polynomials.

2020 Mathematics Subject Classifications: 05C07, 14J80, 97K30

Introduction: A pair of sets that satisfy () ≤ |()| is called a graph, = (,). A molecular graph is one that is created by using edges to represent atom-to-atom bonds and vertices to represent each atom of a molecule. The degree is the number of edges incident to a vertex, and is represented by. A topological index is a mapping between the set of real numbers and the set of chemical compounds represented by molecular graphs. For additional information on topological indices and their uses, read [1-7, 9-11]. Among the topological indicators that are trending are [12, 13, 15][1].

Topologicalindices	**Formulae**
Geometric Arithmetic Index	GA ($C_{29}H_{31}N_7O$) = $\sum_{ab\in E} 2\left(\frac{\sqrt{AB}}{A+B}\right)$
ABC Index	ABC ($C_{29}H_{31}N_7O$) = $\sum_{ab\in E}\sqrt{\frac{A+B-2}{AB}}$
Randic Index	R ($C_{29}H_{31}N_7O$) = $\sum_{ab\,\epsilon\,E}(\frac{1}{\sqrt{AB}})$

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Sum Connectivity Index	$S(C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E}(\frac{1}{\sqrt{A+B}})$
First Zagreb Index	$Z_1\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} A + B$
Second Zagreb Index	$Z_2\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} AB$
Third Zagreb Index	$Z_3\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} \lvert A - B \rvert$
First multiple Zagreb index	$QZ_1(C_{29}\ H_{31}\ N_7\ O) = \prod_{ab\in E} A + B$
Second Multiple zagreb Index	$QZ_2\ (C_{29}\ H_{31}\ N_7\ O) = \prod_{ab\in E} AB$
Augmented Zagreb index	$AZ(C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} \left(\frac{AB}{A+B-2}\right)^3$
Hyper Zagreb Index	$HZ(C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E}(A + B)^2$
Harmonic Index	$H(C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E}(\frac{2}{A+B})$
Forgotten Index	$F(C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} A^2 + B^2$
Inverse sum indeg Index	$ISI(C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E}(\frac{AB}{A+B})$
Somber Index	$SO(C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E}\sqrt{A^2 + B^2}$

Basic definitions: Let G be a graph with a set of vertices V and a set of edges E. Let a€ V be the vertex in V and A be the degree of a. Let b€ E be the edges in E and B be the degree of b.

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Topological polynomials:

A,B where ab
E[1,3]= 2
E [2,2]= 11
E [2,3]= 26
E [3,3] = 1

1.The Geometric Arithmetic polynomial of $C_{29}\ H_{31}\ N_7\ O$ is given by,

$$GA\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{\frac{2\sqrt{AB}}{A+B}}$$

2. The ABC polynomial of $C_{29}\ H_{31}\ N_7\ O$ is given by ,

$$ABC\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\in E} x^{\sqrt{\frac{A+B-2}{AB}}}$$

3. The Randic polynomial of $C_{29}\ H_{31}\ N_7\ O$ is given by ,

$$R\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{\frac{1}{\sqrt{AB}}}$$

4. The Sum connectivity polynomial is given by,

$$S\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{\frac{1}{\sqrt{A+B}}}$$

5. The first Zagreb polynomial is given by,

$$Z_1\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{A+B}$$

6.The second Zagreb polynomial is given by,

$$Z_2\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{AB}$$

7. The third Zagreb polynomial is given by,

$$Z_3\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{\ |A-B\ |}$$

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8. The first multiple Zagreb polynomial is given by,

$QZ_1(C_{29}H_{31}N_7O) = \prod_{ab\in E} x^{A+B}$

9. The second multiple Zagreb polynomial is given by,

$QZ_2(C_{29}H_{31}N_7O) = \prod_{ab\in E} x^{AB}$

10. The Augmented Zagreb polynomial of $C_{29}H_{31}N_7O$ is given by

$AZ(C_{29}H_{31}N_7O) = \sum_{ab\,\epsilon\,E} x^{\left(\frac{AB}{A+B-2}\right)^3}$

11. The Hyper Zagreb polynomial of $C_{29}H_{31}N_7O$ is given by,

$HZ(C_{29}H_{31}N_7O) = \sum_{ab\,\epsilon\,E} x^{(A+B)^2}$

12. The Harmonic polynomial of $C_{29}H_{31}N_7O$ is given by,

$H(C_{29}H_{31}N_7O) = \sum_{ab\,\epsilon\,E} x^{\frac{2}{A+B}}$

13. The Forgotten polynomial of $C_{29}H_{31}N_7O$ is given by,

$F(C_{29}H_{31}N_7O) = \sum_{ab\,\epsilon\,E} x^{A^2+B^2}$

14. The Inverse sum indeg polynomials of $C_{29}H_{31}N_7O$ is given by,

$ISI(C_{29}H_{31}N_7O) = \sum_{ab\,\epsilon\,E} x^{\frac{AB}{A+B}}$

15. The Somber polynomial of $C_{29}H_{31}N_7O$ is given by,

$SO(C_{29}H_{31}N_7O) = \sum_{ab\,\epsilon\,E} x^{\sqrt{A^2+B^2}}$

Main result:

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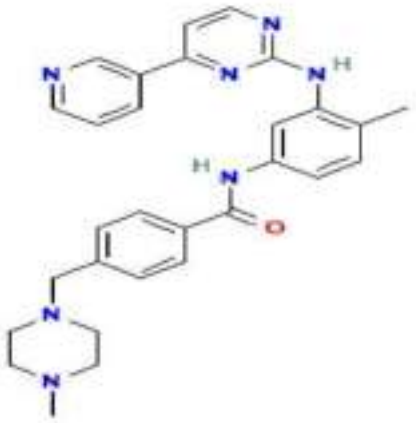

Theorem 1: The Geometric Arithmetic polynomial of C_{29} H_{31} N_7 O is $2x^{\frac{\sqrt{3}}{2}} + 26x^{\frac{2\sqrt{6}}{5}} + 12x$.

Proof: The Geometric Arithmetic polynomial of C_{29} H_{31} N_7 O is given by,

GA (C_{29} H_{31} N_7 O) = $\sum_{ab\,\epsilon\,E} x^{\frac{2\sqrt{AB}}{A+B}}$

$$= 2x^{\frac{2\sqrt{(1*3)}}{(1+3)}} + 11x^{\frac{2\sqrt{(2*2)}}{(2+2)}} + 26x^{\frac{2\sqrt{(2*3)}}{(2+3)}} + x^{\frac{2\sqrt{(3*3)}}{(3+3)}}$$

$$= 2x^{\frac{\sqrt{3}}{2}} + 26x^{\frac{2\sqrt{6}}{5}} + 12x$$

Theorem 2: The ABC polynomial of C_{29} H_{31} N_7 O is $2x^{\sqrt{\frac{2}{3}}} + 37x^{\frac{1}{\sqrt{2}}} + x^{\frac{2}{3}}$.

Proof: The ABC polynomial of C_{29} H_{31} N_7 O is given by ,

ABC (C_{29} H_{31} N_7 O) = $\sum_{ab\in E} x^{\sqrt{\frac{A+B-2}{AB}}}$

$$= 2x^{\sqrt{\frac{2}{3}}} + 11x^{\sqrt{\frac{2}{4}}} + 26x^{\sqrt{\frac{3}{6}}} + x^{\frac{2}{3}}$$

$$= 2x^{\sqrt{\frac{2}{3}}} + 37x^{\frac{1}{\sqrt{2}}} + x^{\frac{2}{3}}.$$

Theorem 3: The Randic polynomial of C_{29} H_{31} N_7 O is $2x^{\frac{1}{\sqrt{3}}} + 26x^{\frac{1}{\sqrt{6}}} + 11x^{\frac{1}{2}} + x^{\frac{1}{3}}$.

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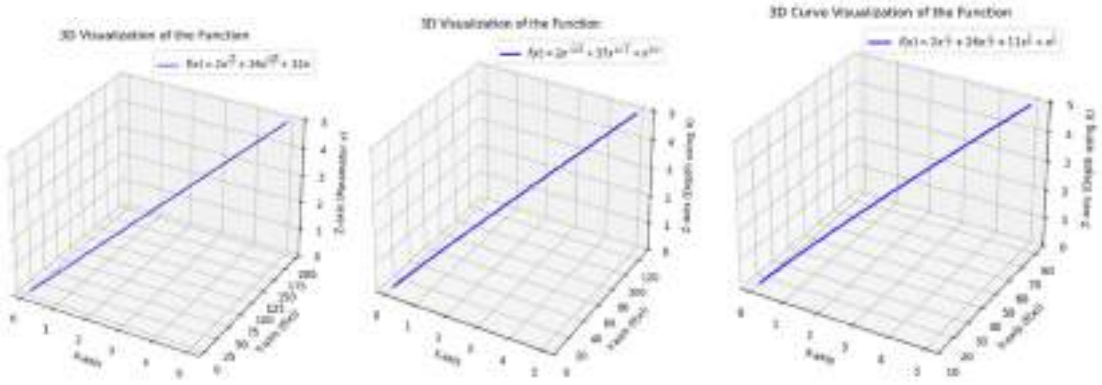
Proof : The Randic polynomial of $C_{29}\ H_{31}\ N_7\ O$ is given by ,

$R\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{\frac{1}{\sqrt{AB}}}$

$= 2\ x^{\frac{1}{\sqrt{3}}} + \ 11^{\frac{1}{\sqrt{4}}} + 26\ x^{\frac{1}{\sqrt{6}}} + \ x^{\frac{1}{\sqrt{9}}}$

$=2x^{\frac{1}{\sqrt{3}}} + 26x^{\frac{1}{\sqrt{6}}} + \ 11x^{\frac{1}{2}} + x^{\frac{1}{3}}.$



Theorem 4: The Sum Connectivity polynomial of $C_{29}\ H_{31}\ N_7\ O$ is $13(x^{\frac{1}{2}} + 2x^{\frac{1}{\sqrt{5}}}) + x^{\frac{1}{\sqrt{6}}}$.

Proof: The Sum connectivity polynomial is given by,

$S\ (C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{\frac{1}{\sqrt{A+B}}}$

$= 2x^{\frac{1}{2}} + \ 11x^{\frac{1}{2}} + \ 26x^{\frac{1}{\sqrt{5}}} + x^{\frac{1}{\sqrt{6}}}$

$= 13(x^{\frac{1}{2}} + 2x^{\frac{1}{\sqrt{5}}}) + x^{\frac{1}{\sqrt{6}}}.$

Theorem 5: The first, second and third Zagreb polynomial of $C_{29}\ H_{31}\ N_7\ O$ is

(a) $Z_1\ (C_{29}\ H_{31}\ N_7\ O) = x^4(x^2 + 26x + 13\ )$

(b) $Z_2\ (C_{29}\ H_{31}\ N_7\ O) = x^3(x^6 + \ 26x^3 + 11x + 2)$

(c) $Z_3\ (C_{29}\ H_{31}\ N_7\ O) = 2(x^2 + 13x + 6)$

Proof: The first Zagreb polynomial is given by,

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$Z_1(C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{A+B}$

$= 2x^4 + 11x^4 + 26x^5 + x^6$

$= x^4(x^2 + 26x + 13)$

The second Zagreb polynomial is given by,

$Z_2(C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{AB}$

$= 2x^3 + 11x^4 + 26x^6 + x^9$

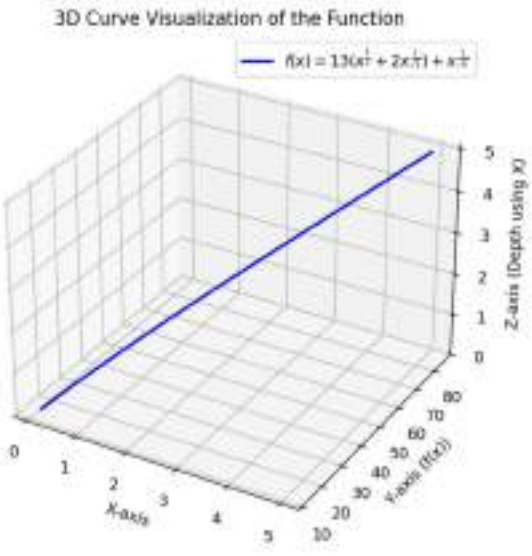
$= x^3(x^6 + 26x^3 + 11x + 2)$

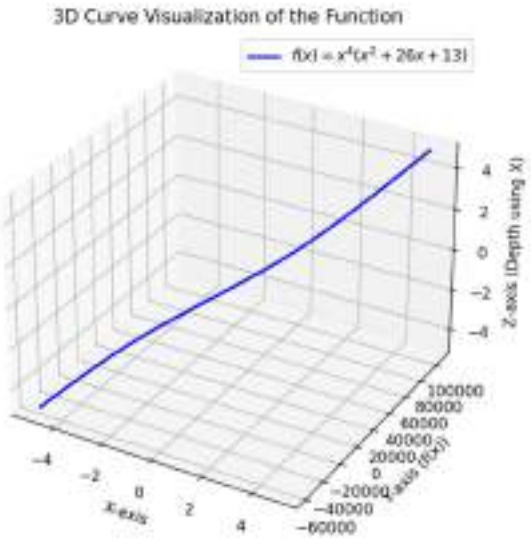
The third Zagreb polynomial is given by,

$Z_3(C_{29}\ H_{31}\ N_7\ O) = \sum_{ab\ \epsilon\ E} x^{|A-B|}$

$= 2x^{|1-3|} + 11x^{|2-2|} + 26x^{|2-3|} + x^{|3-3|}$

$= 2(x^2 + 13x + 6)$





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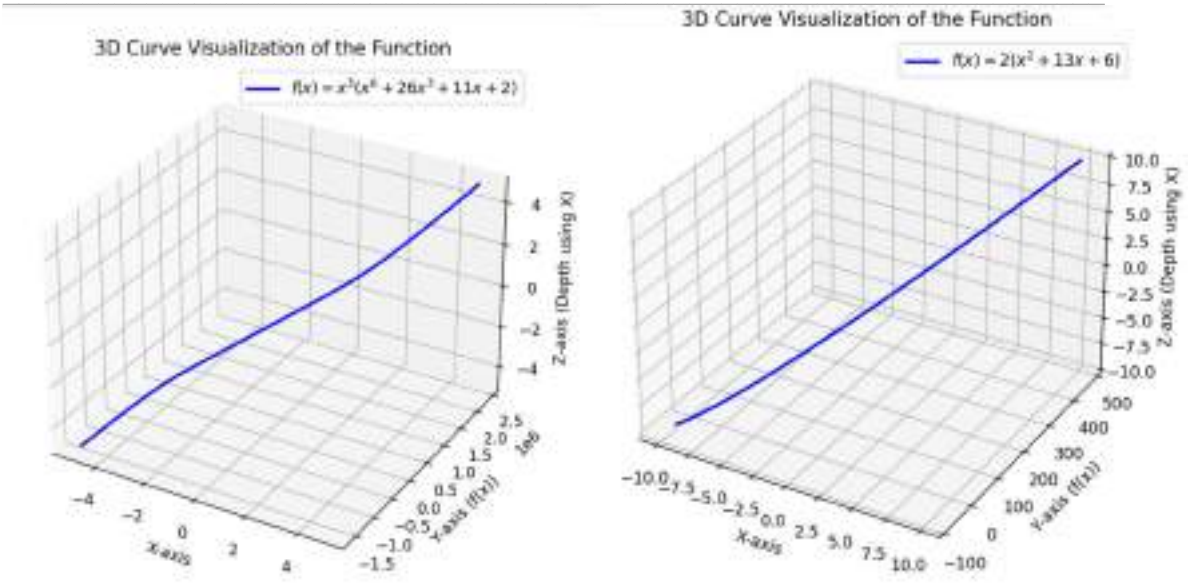

Theorem 6: The first and second multiple Zagreb polynomial of $C_{29}\ H_{31}\ N_7\ O$ is

(a) $QZ_1\ (C_{29}\ H_{31}\ N_7\ O) = 23040x^{106}$

(b) $QZ_2\ (C_{29}\ H_{31}\ N_7\ O) = 23040\ x^{286}$

Proof: The first multiple Zagreb polynomial is given by,

$QZ_1\ (C_{29}\ H_{31}\ N_7\ O) = \prod_{ab\in E} x^{A+B}$

$= x^{3+5}x^{3+6}(2)x^{4+4}(2)x^{4+5}(10)x^{5+5}(12)x^{5+6}(4)x^{5+7}(4)x^{6+6}(3)x^{6+7}x^{7+7}$

$=23040x^{106}$

The second multiple Zagreb polynomial is given by,

$QZ_2\ (C_{29}\ H_{31}\ N_7\ O) = \prod_{ab\in E} x^{AB}$

$=x^{3(5)}x^{3(6)}(2)x^{4(4)}(2)x^{4(5)}(10)x^{5(5)}(12)x^{5(6)}(4)x^{5(7)}(4)x^{6(6)}(3)x^{6(7)}x^{7(7)}$

$= 23040x^{286}$.

Theorem 7: The Augmented Zagreb polynomial of $C_{29}\ H_{31}\ N_7\ O$ is $2x^{\frac{27}{8}} + x^{\frac{729}{64}} + 37\ x^8$.

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Proof: The Augmented Zagreb polynomial of $C_{29}\ H_{31}\ N_7\ O$ is given by

AZ($C_{29}\ H_{31}\ N_7\ O$)= $\sum_{ab\ \epsilon\ E} x^{\left(\frac{AB}{A+B-2}\right)^3}$

$= 2x^{\left(\frac{3}{2}\right)^3} + 11x^{\left(\frac{4}{2}\right)^3} + 26x^{\left(\frac{6}{3}\right)^3} + x^{\left(\frac{9}{4}\right)^3}$

$=2x^{\frac{27}{8}} + x^{\frac{729}{64}} + 37\,x^8.$

Theorem 8: The Hyper Zagreb polynomial of $C_{29}\ H_{31}\ N_7\ O$ is $x^{16}(x^{20} + 26x^9 + 13)$.

Proof: The Hyper Zagreb polynomial of $C_{29}\ H_{31}\ N_7\ O$ is given by,

HZ($C_{29}\ H_{31}\ N_7\ O$)= $\sum_{ab\ \epsilon\ E} x^{(A+B)^2}$

$= 2x^{(4)^2} + 11x^{(4)^2} + 26x^{(5)^2} + x^{(6)^2}$

$=x^{16}(x^{20} + 26x^9 + 13)$.

Theorem 9: The Harmonic polynomial of $C_{29}\ H_{31}\ N_7\ O$ is $13x^{\frac{1}{2}} + 26x^{\frac{2}{5}} + x^{\frac{1}{3}}$.

Proof: The Harmonic polynomial of $C_{29}\ H_{31}\ N_7\ O$ is given by,

H($C_{29}\ H_{31}\ N_7\ O$)= $\sum_{ab\ \epsilon\ E} x^{\frac{2}{A+B}}$

$= 2x^{\frac{2}{4}} + 11x^{\frac{2}{4}} + 26x^{\frac{2}{5}} + x^{\frac{2}{6}}$

$= 13x^{\frac{1}{2}} + 26x^{\frac{2}{5}} + x^{\frac{1}{3}}$.

Theorem 10: The Forgotten polynomial of $C_{29}\ H_{31}\ N_7\ O$ is $x^8(26x^5 + 2x^2 + x^{10} + 11)$.

Proof: The Forgotten polynomial of $C_{29}\ H_{31}\ N_7\ O$ is given by,

F($C_{29}\ H_{31}\ N_7\ O$)= $\sum_{ab\ \epsilon\ E} x^{A^2+B^2}$

$= 2x^{1+9} + 11x^{4+4} + 26x^{4+9} + x^{9+9}$

$=x^8(26x^5 + 2x^2 + x^{10} + 11)$.

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Theorem 11: The Inverse sum indeg polynomials of C_{29} H_{31} N_7 O is $2x^{\frac{3}{4}} + 26x^{\frac{6}{5}} + x^{\frac{3}{2}} + 11x$.

Proof: The Inverse sum indeg polynomials of C_{29} H_{31} N_7 O is given by,

ISI(C_{29} H_{31} N_7 O)=$\sum_{ab \in E} x^{\frac{AB}{A+B}}$

= $2x^{\frac{3}{4}} + 11x^{\frac{4}{4}} + 26x^{\frac{6}{5}} + x^{\frac{9}{6}}$

=$2x^{\frac{3}{4}} + 26x^{\frac{6}{5}} + x^{\frac{3}{2}} + 11x$.

Theorem 12: The Somber polynomial of C_{29} H_{31} N_7 O is $2x^{\sqrt{10}} + 11x^{2\sqrt{2}} + 26x^{\sqrt{13}} + x^{3\sqrt{2}}$.

Proof: The Somber polynomial of C_{29} H_{31} N_7 O is given by,

SO(C_{29} H_{31} N_7 O)=$\sum_{ab \in E} x^{\sqrt{A^2+B^2}}$

= $2x^{\sqrt{1+9}} + 11x^{\sqrt{4+4}} + 26x^{\sqrt{4+9}} + x^{\sqrt{9+9}}$.

=$2x^{\sqrt{10}} + 11x^{2\sqrt{2}} + 26x^{\sqrt{13}} + x^{3\sqrt{2}}$.

Applications: Imatinib is a targeted cancer treatment that is mostly used to treat gastrointestinal stromal tumours (GIST) and chronic myeloid leukaemia (CML). It functions by blocking the BCR-ABL tyrosine kinase enzyme, which causes cancer cells to proliferate unchecked. Imatinib has transformed cancer treatment by selectively focussing on malignant cells, greatly increasing patient lifespan and quality of life.

Conclusion: Conclusion: Our research shows how to use degree-based topological indices to derive different Imatinib polynomials. By offering insightful information about the drug's molecular makeup and characteristics, these mathematical formulations help us comprehend its chemical behaviour on a deeper level. The investigation of imatinib's efficacy and possible alterations for improved therapeutic uses can be facilitated by such analyses.

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FUZZY ALTERNATIVE RANKING ORDER METHOD ACCOUNTING FOR TWO-STEP NORMALIZATION BASED MULTI-CRITERIA DECISION-MAKING APPROACH TO CHEMICAL SUPPLIER SELECTION

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Abstract

Multi-criteria decision making is an interdependent process comprising the core elements of alternatives, criteria and their inter associations. The intricacy of decisioning depends on the inter relational impacts between the elements of decision making. This research work addresses the problem of supplier selection with special reference to chemical industries. This study employs Fuzzy Alternative Ranking Order Method Accounting for Two-Step Normalization (FAROMAN) to rank eight suppliers using six criteria of cost, quality, delivery time, reliability, safety and flexibility. The ranking results using FAROMAN are compared with deterministic AROMAN to exhibit the efficacy of fuzzy integrated ranking approach. Sensitivity analysis is performed to determine the effects of the key parameters over ranking results. This decision approach shall be applied to other decisioning situations to design optimal solutions to real-world problems.

Keywords: Fuzzy AROMAN, Chemical Industries, Supplier Selection, Optimal Ranking

2020 Mathematics Subject Classifications: 94D05, 03B52, 03E72

Introduction

The chemical industry is an intrinsic sector that supports in transforming raw materials into consumable products with the utility of varied chemicals. This industry supplies materials to various industries such as pharmaceuticals, textiles, automotive, construction and others. Hence the supplier selection performs a critical role in enhancing the operational efficacy of the chemical industries. However, the selection process is intricate as it comprises several stages in obtaining the optimal supplier choice considering the criteria of cost, quality, delivery time, reliability, safety and flexibility. This decisioning problem shall be resolved with the application of multi-criteria decisioning approach (MCDM). A decisioning problem is primarily characterized by alternatives criteria and nature of the decision-making environment. In general, a deterministic kind of decision making do not exist always as the decisioning environment is influenced by uncertainty, impreciseness and vagueness. This has made the researchers to delve into fuzzy based MCDM approaches.

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Researchers have explored supplier selection problem with fuzzy based decisioning approach with reference to varied context of industrial demands. The decisioning methods are principally classified into two in which one is employed in criterion computation and other in alternative ranking. The fuzzy methods of Analytical Hierarchy process, Analytical Network Process, Entropy method and many other similar methods are applied in criterion weight computation. On other hand fuzzy based methods such as TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations), VIKOR (VlseKriterijumskaOptimizacija I KompromisnoResenje) are applied in ranking of the alternatives. In recent times, new MCDM approaches are also introduced and applied in decisioning process. One such method is Fuzzy Alternative Ranking Order Method Accounting for Two-Step Normalization (FAROMAN). FAROMAN is an extension of AROMAN method introduced by Boskovic et al [4]. The method of FAROMAN is applied in several decisioning environments but however to the best of our knowledge, this method is not applied in supplier selection. This has motivated the authors to apply FAROMAN method in supplier selection with special reference to chemical suppliers.

The remaining contents are organized into the following sections. The literature review of applications of fuzzy based decisioning methods in supplier selection is presented in section 2. The methodology of FAROMAN is discussed in section 3. The method of FAROMAN is applied to the decisioning problem of chemical supplier selection. The results obtained using deterministic and fuzzy based AROMAN are compared in section 4. The last section concludes the paper with future research directions and extensions.

2. Literature Review

This section presents the state of art of works related to the applications of fuzzy based decisioning methods applied in supplier selection. Researchers have applied different fuzzy MCDM in different industries to make optimal supplier selection. Some of the recent and noteworthy works of fuzzy based MCDM are presented as follows.

Jain et al delved integrated fuzzy based MCDM method in sustainable supplier selection [14]. Tsai et al developed new fuzzy based DEA approach in supplier selection [29]. Hoseini et al [12] applied interval type-2 fuzzy MCDM framework for resilient supplier selection. Göçer [11] modelled fuzzy based MCDM using Pythagorean fuzzy representations in sustainable supplier selection. Manupati et al [21] developed integrated fuzzy MCDM in supplier selection in Indian automotive industry. Dangkoub and Enzebati [8] applied integrated fuzzy MCDM in selecting suppliers with reference to electronic industry. Kao et al [15] applied in supply chain management. Nguyen et al [22] employed fuzzy data envelopment analysis in selection of sustainable supplier in steel industry. Wang et al [30] applied two-stage based fuzzy MCDM in green supplier in steel industry. Puska et al [26] applied Z-Numbers–Fuzzy LMAW–Fuzzy CRADIS model in green supplier selection with special reference to agriculture. Thanh and Lan [28] developed a hybrid MCDM in supplier selection in food-processing industry. Kokoç [17] applied interval-valued intuitionistic fuzzy environment in making supplier selection with special reference to textile industry.

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Chai et al [5] employed intuitionistic and interval-valued fuzzy MCDM approach based on cumulative prospect theory in sustainable supplier selection. Pinar [25] integrated sentiment analysis and q-rung orthopair fuzzy MCDM model for supplier selection in E-commerce. Ding and Hsu [9] applied fuzzy MCDM evaluation method in electrocardiogram monitor supplier selection. Kurniawan et al [19] developed distance-based fuzzy MCDM methods for a supplier selection. A general approach of hybrid fuzzy based MCDM is applied by Zahid et al [31] and Agarwal et al [1]. Kumar et al employed type-2 fuzzy based hybrid AHP-TODIM MCDM approach for green supplier selection [18]. Chakraborty et al employed intuitionistic fuzzy MCDM in healthcare supplier selection [6]. Lin et al [20] applied fuzzy based MCDM in green supplier selection. From the afore mentioned literature it is evident that Fuzzy AROMAN is not applied in supplier selection. For better understanding the state of art of fuzzy based MCDM in supplier selection is presented in Table 1

Table 1 State of Art of Fuzzy MCDM in Supplier Selection

Author & Year	**Fuzzy MCDM**	**Areas of Application**
Jain, N., Singh, A. R.(2020)	Fuzzy Inference System(FIS) - AHP	Supplier's sustainability performance
Göçer, F (2021)	Analytical Hierarchy Process (AHP) & TOPSIS	Sustainable supplier seletion
Tsai, C. M., Lee, H. S.(2021)	Novel fuzzy MCDM	Chain management and good supply chain management
Manupati, V. K., et al.(2021)	AHP-TOPSIS,VIKOR	Supplier selection in the automotive industry
Hoseini, S. A., et al.(2021)	Fuzzy Best Worst method-FTOPSIS	Potential supplier based on Hamming distance measure
Wang, C. N., Nguyen, (2021)	FAHP & FTOPSIS	Green supplier selection in Steel Industry
Kokoç, M. (2022)	Interval valued intuitionistic fuzzy -AHP & TOPSIS	Green Supplier Selection in the Textile Industry
Kao, J. C., et al.(2022)	FAHP – WASPAS(Weighted Aggregated Sum Product Assessment)	Supplier Selection in Chain Management
Nguyen, T. L., et al.(2022)	Spherical fuzzy AHP- Spherical fuzzy WASPAS(Weighted Aggregated Sum Product Assessment)	Supplier selection in Steel Industry
Puška, A., et al.(2022)	Step-wise Weight Assessment Ratio Analysis,Multi-Attributive Border Approximation Area Comparison, Multi-Attribute	Green Supplier selection in Agri Food Company

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	COmparison for Ranking and Ordering Solutions,Compromise Ranking of Alternatives with Distance-based Indicators of Similarity	
Thanh, N. V.,et al.(2022)	Fuzzy Analytical Hierarchy Process- Combined Compromise Solution algorithm	Food Processing Industry
Zahid, E., et al.(2023)	Fuzzy AHP- Decision-Making Trial and Evaluation Laboratory - VIKOR	Supplier Selection in Procurement Process
Chai, N., et al.(2023)	IVFS and Cumulative prospect Theory	Sustainable Supplier Selection in E-Bike Sharing Recycling
Pinar, A(2023)	q- Rung Orthopair Fuzzy set	Supplier Selection in E-Commerce (Amazon Product Reviews)
Kumar, R (2023)	Interval Type-2 Fuzzy AHP-Tomada de DecisãoInterativaMulticritério	Green Supplier Selection in Supply Chain Management
Kurniawan,V.R.B.,et al.(2023)	Fuzzy TOPSIS (FTOPSIS) & Fuzzy Evaluation Based on Distance from Average Solution (FEDAS)	Supplier Selection in a Small-Scale Retail Company
Ding, J. F., & Hsu, L. M.(2023)	Fuzzy AHP & Fuzzy TOPSIS	EKG Monitor Supplier Selection
Chakraborty, S., et al.(2024)	Weighted Aggregated Sum Product Assessment,Combinative Distance-based Assessment, Combined Compromise Solution	Healthcare Supplier Selection in the Indian Pharmaceutical Industry
Agarwal, R., et al.(2024)	Fuzzy Evaluation Based on Distance from Average Solution (F-EDAS)	Sustainable and Resilient Supplier Selection
Lin, T. Y., et al.(2025)	ZE-ELECTRE II (Extended Z-Numbers ELECTRE II)ELimination and Choice Expressing REality	**Green Supplier Selection**

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On other hand, FAROMAN based hybrid methods are applied in other decision making scenarios such as hybrid FAROMAN in professional driver selection [7], interval type-2 fuzzy AROMAN in enhancement of sustainability of the postal network [23], Interval-valued intuitionistic fuzzy AROMAN in selection of sustainable waste water treatment technology [2], integrated fuzzy Best Worst method and AROMAN in evaluation of entrepreneurship training [10], Intuitionistic fuzzy SWARA- AROMAN in sports events management [13], spherical fuzzy-based DIBR II-AROMAN in selection of wind energy plants [16], AI-Driven LOPCOW-AROMAN in healthcare [27], Fuzzy-MEREC-AROMAN in prioritizing of investments [24], Pythagorean fuzzy CRITIC-AROMAN [3] in intelligent transportation systems. To the best of our knowledge, we infer that the method of FAROMAN is not applied in supplier selection also no methods discussed above are applied in specific to determine an optimal supplier selection of chemical industries. This has motivated the authors to develop a decision -making model using fuzzy AROMAN with special reference to chemical industries.

3. Methodology of FAROMAN

This section presents the procedure of FAROMAN in ranking the alternatives considering equal criterion weightage and it is pictorially represented in Fig.1. [4]

Step 1: Construct the initial decision-making matrix G

The initial decision-making is constructed where m denotes alternatives and n denotes criteria as follows.

$$G = [g_{ij}]_{mxn} = \begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{m1} & \cdots & g_{mn} \end{bmatrix}$$

In this matrix G, each entry is a linguistic variable and it is quantified using triangular fuzzy numbers of the form (l,m,n). The matrix is quantified using triangular fuzzy numbers and then defuzzied using centroid method of the form $\frac{(l+m+n)}{3}$.

Step 2: Obtain the normalized decision matrix R.

Linear normalization technique normalizes the initial decision matrix elements, and normalized matrix R is formed as follows using linear max-min

Benefit criteria $r_{ij} = \frac{g_{ij} - min\ g_{ij}}{max\ g_{ij} - mi\ \ g_{ij}}$

Cost criteria $r_{ij} = \frac{max\ g_{ij} - g_{ij}}{max\ g_{ij} - min\ g_{ij}}$

Step 3 : Normalize the matrix again using Vector normalization methods

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Benefit criteria $n_{ij} = \frac{g_{ij}}{\sqrt{\sum_{i=1}^{m} g_{ij}^2}}$

Cost criteria $n_{ij} = 1 - \frac{g_{ij}}{\sqrt{\sum_{i=1}^{m} g_{ij}^2}}$

Step 4: Aggregated Normalization Value

The aggregated average normalization values are calculated by

$$t_{ij}^{norm} = \frac{\beta t_{ij} + (1-\beta) t_{ij}^{*}}{2} i = 1,2, \ldots . m; j = 1,2, \ldots . . n$$

β denotes the weight of the linear normalization methods, varies between 0 and 1.

Step 5: Weighted normalized matrix

The normalized matrix is converted to weighted normalized form considering equal weights

$$\hat{t}_{ij} = W_{ij} . t_{ij}^{norm} i = 1,2, \ldots . m; \;\; j = 1,2, \ldots n$$

The sum weighted normalized values of criteria are calculated by separating their types (max and min)

$$L_i = \sum_{j} \hat{t}_{ij}^{(\min)} \; i = 1,2, \ldots . m; \quad j \in cost\ criteria$$

$$A_i = \sum_{j} \hat{t}_{ij}^{(\max)} \; i = 1,2, \ldots . m; \quad j \in benefit\ criteria$$

Step 6: Ranking of the Alternatives

The final values of criteria function ($\mathrm{R_i}$) are computed.

$$R_i = L_i^{\lambda} + \; A_i^{(1-\lambda)} i = 1,2, \ldots \ldots . . m$$

Rank the $\mathrm{R_i}$ values by decreasing order. The highest value of $\mathrm{R_i}$ denotes the best alternatives.

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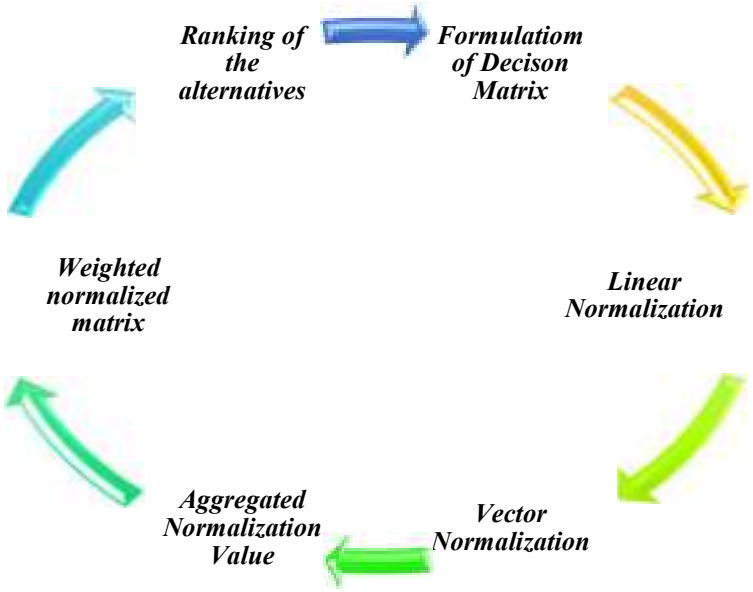

Fig. 1 FAROMAN Decisioning Process

4. Application of FAROMAN in Chemical Supplier Selection

Problem Description

Let us consider a chemical laboratory which requires chemicals for its routine works. The lab in-charge is hurdled with the task of choosing the optimal chemical suppliers from 8 alternatives namely S1, S2,S3,S4,S5,S6,S7 and S8 to meet out the requirements represented as criteria such as Cost, Quality, Delivery Time, Reliability, Safety and Flexibility. The decision maker considers all the criteria to assume equal weights. To facilitate the decisioning, the method of Fuzzy AROMAN is put forth considering the constraints of uncertainty. The data is collected from the persons in-charge of lab and the initial decision matrix is formulated based on their experience, perception and past data.

The initial decision matrix G with linguistic variable representations is presented in Table 2

Table 2 Linguistic Decision-Making Matrix

Criteria/Alternatives	S I	S II	S III	S IV	S V	S VI	SVII	SVIII
Cost (C1)	M	M	L	VH	H	L	L	M
Quality of Chemicals (C2)	VH	H	L	L	M	L	L	L

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Delivery Time (C3)	H	VH	H	H	M	H	M	H
Supplier Reliability (C4)	VH	H	VH	M	M	M	M	H
Safety Standards (C5)	H	VH	M	M	M	M	M	M
Flexibility in Supply (C6)	VH	VH	VH	M	H	H	M	H

In the above decision matrix, the criteria C1 and C3 are cost criteria and other criteria are benefit.

The equivalent triangular representation is presented in Table 3

Table 3 Quantification of Linguistic Variable

Linguistic Variable	**Triangular Fuzzy Representations**	**Defuzzified Forms**
Very Low (VL)	(0,0,20)	0.067
Low(L)	(10,25,40)	0.25
Moderate (M)	(30,50,70)	0.5
High (H)	(60,75,90)	0.75
Very High (VH)	(80,100,100)	0.934

The modified decision-making matrix obtained after defuzzification of the linguistic variable using triangular forms is presented in Table 4.

Table 4 Modified Decisioning Matrix

	S I	**S II**	**S III**	**S IV**	**S V**	**S VI**	**SVII**	**SVIII**
C1	0.5	0.5	0.25	0.934	0.75	0.25	0.25	0.5
C2	0.934	0.75	0.25	0.25	0.5	0.25	0.25	0.25
C3	0.75	0.934	0.75	0.75	0.5	0.75	0.5	0.75
C4	0.934	0.75	0.934	0.5	0.5	0.5	0.5	0.75
C5	0.75	0.934	0.5	0.5	0.5	0.5	0.5	0.5
C6	0.934	0.934	0.934	0.5	0.75	0.75	0.5	0.75

The Linear normalized matrix obtained using step 2 is presented in Table 5

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Table 5 Linear Normalized Matrix

	S I	S II	S III	S IV	S V	S VI	SVII	SVIII
C1	0.3655	0.3655	0	1	0.7310	0	0	0.3655
C2	1	0.7310	0	0	0.365	0	0	0
C3	0.5760	1	0.5760	0.5760	0	0.5760	0	0.5760
C4	1	0.5760	1	0	0	0	0	0.5760
C5	0.5760	1	0	0	0	0	0	0
C6	0	1	1	0	0.5760	0.5760	0	0.5760

The Vector form of normalization obtained using step 3 is presented in Table 6

Table 6 Vector Normalized Matrix

	S I	S II	S III	S IV	S V	S VI	SVII	SVIII
C1	0.3432	0.3432	0.1716	0.6411	0.5148	0.1716	0.1716	0.3432
C2	0.6607	0.5307	0.1769	0.1769	0.3538	0.1769	0.1769	0.1769
C3	0.3666	0.4566	0.3666	0.3666	0.2444	0.3666	0.2444	0.3666
C4	0.4748	0.3813	0.4748	0.2542	0.2542	0.2542	0.2542	0.3813
C5	0.4378	0.5452	0.2919	0.2919	0.2919	0.2919	0.2919	0.2919
C6	0.4710	0.4710	0.4710	0.2521	0.3782	0.3782	0.2521	0.3782

The aggregated normalized matrix obtained using step 4 is presented in Table 7.

Table 7 Aggregated normalized matrix

	S I	S II	S III	S IV	S V	S VI	SVII	SVIII
C1	0.1772	0.1772	0.0429	0.4103	0.3115	0.0429	0.0429	0.1771
C2	0.4153	0.3155	0.0442	0.0442	0.1799	0.0442	0.0442	0.0442

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C3	0.2357	0.3642	0.2357	0.2357	0.0611	0.2357	0.0611	0.2357
C4	0.3687	0.2394	0.3687	0.0636	0.0636	0.0634	0.0636	0.2392
C5	0.2535	0.3863	0.0730	0.0730	0.0730	0.0730	0.0730	0.0730
C6	0.3678	0.3678	0.3678	0.0631	0.2386	0.2386	0.063	0.2386

The aggregated weighted normalized matrix is obtained using step 5 and it is presented in Table 8.

Table 8 Aggregated weighted normalized matrix

	S I	**S II**	**S III**	**S IV**	**S V**	**S VI**	**SVII**	**SVIII**
C1	0.0295	0.0295	0.0072	0.0684	0.0519	0.0072	0.0072	0.0295
C2	0.0692	0.0526	0.0074	0.0074	0.0300	0.0074	0.0074	0.0074
C3	0.0393	0.0607	0.0393	0.0393	0.0102	0.0393	0.0102	0.0393
C4	0.0615	0.0400	0.0615	0.0106	0.0106	0.0106	0.0106	0.0400
C5	0.0423	0.0644	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122
C6	0.0613	0.0613	0.0613	0.0398	0.0398	0.0398	0.0105	0.0398

The ranking of the alternatives is made using step 6 and it is represented in Table 9

Table 9 Ranking of the Alternatives

Alternatives	Sum of the Cost Criteria	Sum of the Benefit Criteria	Score Values	Ranking
S1	0.0688	0.2343	0.7463	2
S2	0.0902	0.2183	0.7675	1
S3	0.0465	0.1424	0.5930	3
S4	0.1077	0.0407	0.5299	6
S5	0.0621	0.0926	0.5535	5
S6	0.0465	0.0700	0.4802	7
S7	0.0174	0.0407	0.3336	8

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S8	0.0688	0.0994	0.5776	4

From the above table it is inferred that the supplier S2 is optimal with high score values and the supplier S7 is non-preferential with the least score value. The preferential ordering of the suppliers is S2 > S1 > S3 > S8 > S5 >S4 > S6 > S7.

5. Sensitivity Analysis

The results obtained using Fuzzy AROMAN method is compared with deterministic AROMAN method to determine the efficacy of fuzzy representations over crisp forms. The deterministic decision matrix with score values ranging from 1-100 presented below in Table 10 considers the same alternatives and criteria and the score values are obtained from the same experts.

Table 10 Crisp Decision Matrix

Criteria/Alternatives	S I	S II	S III	S IV	S V	S VI	SVII	SVIII
Cost	55	50	20	90	75	15	10	50
Quality of Chemicals	95	75	20	20	55	15	15	15
Delivery Time	75	90	80	75	50	85	50	80
Supplier Reliability	90	85	95	50	55	50	50	85
Safety Standards	85	95	50	50	55	50	55	55
Flexibility in Supply	90	95	95	50	85	80	55	80

By applying the same procedure discussed in section 3, the ranking results and the final score values obtained are presented in Table 11.

Table 11 Ranking Results using Crisp AROMAN

Alternatives	Score Values	Ranking
S1	0.7628	2
S2	0.7824	1
S3	0.6236	3
S4	0.5272	4

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S5	0.5272	4
S6	0.5073	5
S7	0.3288	6
S8	0.6236	3

In the above ranking results, some of the rankings are repeated, for instance, the suppliers S3 and S8 occupy third ranking position, followed by the suppliers S4 and S5 occupy the fourth ranking position. However, in this ranking S2 occupies the first position and S7 occupies the last position. This shows that the ranking results obtained using crisp representations do not favor the decision makers in comparison with fuzzy based representations. Though the approach of AROMAN remains the same, the ranking results differ in the aspect of fuzzy and crisp representations and this is explicitly visualized in Table 12 and in Fig.2.

Table 12 Comparison of Fuzzy & Crisp AROMAN Ranking Results

Suppliers	**S I**	**S II**	**S III**	**S IV**	**S V**	**S VI**	**SVII**	**SVIII**
Fuzzy AROMAN	2	1	3	6	5	7	8	4
Crisp AROMAN	2	1	3	4	4	5	6	3

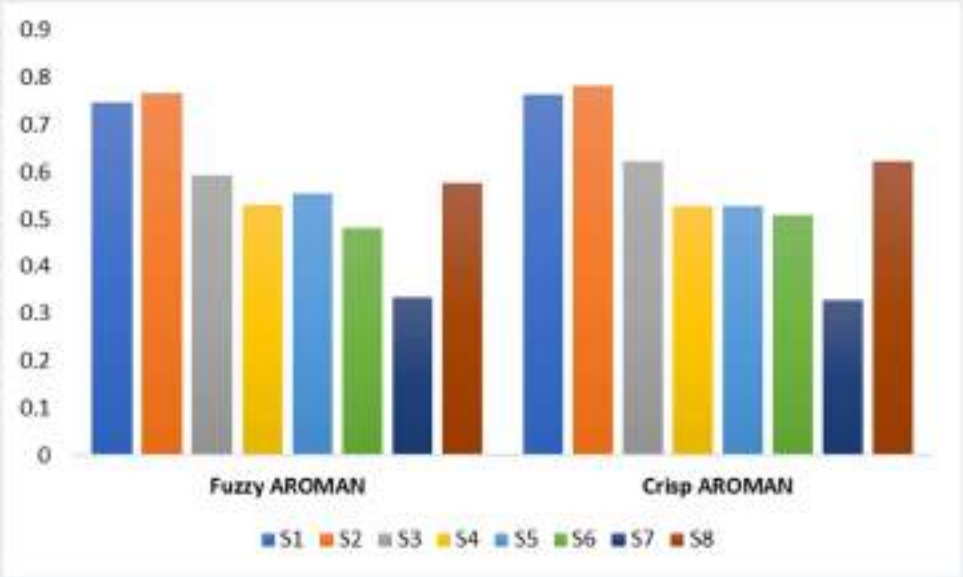

Fig.2 Comparison of Fuzzy & Crisp Score Values of Alternatives

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Conclusion

This research work presents a fuzzy AROMAN based decision making model for optimal selection of chemical supplier. The decisioning problem intends to rank eight alternatives based on six criteria. The ranking results are compared with the crisp representations of AROMAN. It is found that fuzzy based representations are more efficient in handling the uncertain representations. The method of fuzzy AROMAN is integrated with other MCDM methods. This work also shall be extended by integrating with other methods of criterion weight computation. Also, other kinds of fuzzy representations shall also be made to wider the decisioning approach. This model has crucial industrial applications as it serves as an excellent means of choosing the most suitable suppliers suiting to the demands of the decision makers.

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Analyzing Market Outcome using Game Theory: A Study of Nash Equilibrium in the Cournot Duopoly Model

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Abstract

The paper uses game theory to look at market outcomes using the Cournot duopoly model, with a focus on Nash equilibrium. The model, which is a basic idea in industrial organization, looks at how any firms decide how much to produce by considering how each firm will react to what its competitors do. This model looks at how companies consciously act to make the most money, taking into account how they are all linked. Asymmetric cost structures, similar cost structures, and product differentiation are some of the big changes that have been made to the model that help us understand in what way markets really work. The paper looks at the policy consequences and stresses how important the Cournot framework is for figuring out market efficiency, welfare effects, and regulatory measures. This research shows how important game theory is for understanding how competition works and how to act strategically in oligopolistic markets.

Keywords:*Game theory, Cournot duopoly model, Nash equilibrium, Oligopoly Markets*

2020 Mathematics Subject Classifications:91A11, 91A25, 91A86

Introduction

Oligopoly models have gained significant attention among scientists due to their widespread and importance in modern economies, such as food product markets, drugs markets, beer and light alcohol markets, and cell phone services markets [1]. However, analyzing these markets is considered the most difficult task [2]. The interaction of market participants is the main factor distinguishing oligopoly markets from perfect competition market structures, as actions of any market player directly affect market equilibrium and indirectly the results of all competitors. Increased seller concentration tends to raise industry-wide markups and profitability by

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facilitating collusion, thus earning the special attention of government antitrust services. Duopoly is a form of oligopoly market with only two participants: producers or sellers [3]. Every producer must consider not only the current strategy of the competitor but also their forthcoming responsive actions before making decisions on prices and quantities. The case of only two players in a market raises the chances for non-competitive agreements, which can lead to monopolistic pricing and unfavorable conditions for consumers [4-5]. The most famous oligopoly models were designed in the 19th century by French mathematician, philosopher, and economist A. Cournot, J. Bertrand, and H. Von Stackelberg. Later, mathematical methods, especially game theory, became mainstream in analyzing oligopoly markets. Hungarian mathematician J. Von Neumann and Austrian economist O. Morgenstern are considered the originators of game theory, releasing their main proceedings, "Theory of Games and Economic Behaviour," in 1944. The author of this article aims to fill the gap by providing a grounded opinion on the practical application of game theory in duopoly market research. The paper reviews the main game theory models commonly applied to analyze duopoly markets, aiming to provide a balanced and reasonable opinion on the extent to which theoretical results correspond to reality, that is, how appropriate game theory is for analyzing actions of real enterprises. The research focuses on the practical application of game theory models to analyze a real duopoly market. The research methods include systematic science literature analysis and comparative analysis of theoretical models. The paper is based on the works of J. Von Neumann and O. Morgenstern, K. Aiginger, J. Friedman, E. Rasmusen, D. Carlton and J. Perloff, M. Jackson, and S. Wilkie, who have made significant contributions to the development of game theory and its application.

Game theory originates from classical economics and philosophy, influenced by early concepts such as Adam Smith's "invisible hand" and John von Neumann's "On the Theory of Parlour Games." Von Neumann's minimax theorem, established in 1944, delineates optimum tactics for zero-sum games. His collaboration with Oskar Morgenstern extended game theory to cooperative games and created the idea of utility for quantifying player preferences [6]. The Nash equilibrium evolved as a fundamental idea for non-cooperative games without enforceable commitments [7]. During the late 20th century, game theory proliferated across several disciplines, exemplified by William Vickrey's auction theory and the general equilibrium theory developed by Kenneth Arrow and Gerard Debreu, highlighting its economic significance [8]. John Maynard Smith was a pioneer in the development of evolutionary game theory, which brought game-theoretic ideas to biological settings and shed light on how social behavior, competition, and cooperation among species have evolved. In the 21st century, game theory has advanced by incorporating insights from behavioural economics, psychology, and computer science [9]. This evolution has led to new models like algorithmic game theory,

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which addresses the computational challenges of strategic decision-making in complex systems. Game theory explores strategic interactions where each player's decisions influence others' outcomes. The field relies on key concepts like players, who can be individuals, organizations, or entities, all assumed to act rationally based on their preferences and available information [10]. Players choose strategies, which are possible actions they can take. These strategies can be pure, involving consistent choices, or mixed, where choices are randomized among multiple options [11]. Nash Equilibrium, Prisoner's Dilemma, Zero-Sum Games, and Mixed Strategies are some fundamental theories of game theory to make robust decisions. Here payoffs represent the benefits players receive from their chosen strategies, often quantified as utility [12]. Games are structured scenarios where players' strategies and outcomes are pre-defined, and they are classified as cooperative (with enforceable agreements) or non-cooperative, zero-sum (one player's gain equals another's loss) or non-zero-sum, and simultaneous (players decide simultaneously) or sequential (players decide in sequence) [13]. A fundamental concept is equilibrium, particularly Nash equilibrium, where no player benefits from altering their strategy. These concepts form the basis for analyzing various strategic interactions, offering mathematical insights into rational decision-making.

$$\mathrm{P} = \{1, 2, \dots, \mathrm{n}\} \tag{1}$$

The expression defines the set of participants involved when *P* players engage in a game. Strategy (*ST*) is crucial in games, with its feasibility and variety determined by players gathering information and resources during gameplay.

$$\mathrm{st} = \{\mathrm{st}_1, \mathrm{st}_2, \dots, \mathrm{st}_\mathrm{n}\} \tag{2}$$

$$\mathrm{ST} = \{\mathrm{ST}_1, \mathrm{ST}_2, \dots, \mathrm{ST}_\mathrm{n}\} \tag{3}$$

The formula depicts a strategy set comprising a player's policies combined with the strategy sets of all other players. Profit is the benefit player's gain from a game, often representing the maximum possible outcome, though it can also include losses. The following expression represents the collective advantage for all participants in the game.

$$\mathrm{x} = \{\mathrm{x}_1, \mathrm{x}_2, \dots, \mathrm{x}_\mathrm{n}\} \tag{4}$$

Game theory is extensively used across several disciplines, such as economics, political science, computer science, legal methods, psychology, environmental resources, and military tactics. It is classified into classical and evolutionary games, with classical games further subdivided into cooperative, non-cooperative, static, or

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dynamic categories. Nonetheless, it encounters criticism for its substantial dependence on the notion of rationality, which may not be relevant in practical situations owing to cognitive constraints and biases. Social values such as justice, charity, and reciprocity may diverge from the conventional emphasis on self-interest in game theory. Moreover, identifying equilibria in extensive, dynamic games may be computationally intensive, particularly in scenarios involving several participants or complex tactics. Algorithmic game theory was developed to tackle this problem, producing efficient methods for identifying equilibria in extensive games

Game theory and the nash equilibrium concept

Game theory is a theoretical framework that studies how individuals and entities make decisions in social scenarios. It is a key part of strategy and the science of optimal decision-making in strategic settings. Game theory is used in various fields to predict outcomes and set prices, make decisions about acquisitions, and handle lawsuits. It is essential for businesses to understand and navigate complex situations.

Nash equilibrium is a game theory concept where the game reaches an optimal outcome, with players knowing their opponent's strategy and remaining consistent with it. This state prevents players from deviating from their initial strategy, as they receive no incremental benefit from changing actions. A game can have multiple Nash equilibria or none at all.

The rule goes as follows: if the first payoff number, in the payoff pair of the cell, is the maximum of the column of the cell and if the second number is the maximum of the row of the cell - then the cell represents a Nash equilibrium shown in table 1.

Table 1: Example of game theory with nash equilibrium

Player 2	Option A	Option B	Option C
Player 1			
Option A	0, 0	25, 40	5, 10
Option B	40, 25	0, 0	5, 15
Option C	10, 5	15, 5	10, 10

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Game theory is the mathematical study of strategic decision making. It is used to find the optimal outcome from a set of choices by analyzing the costs and benefits to each independent party as they compete with each other. According to game theory, one always lose, and another player always wins.

Duopoly model concept:

A duopoly is an oligopoly consisting of two major corporations producing similar goods and services in a market or industry. The key components of a duopoly include their interactions and their impact on each other. The defining feature of a duopoly is that only two major players are considered major players, and their interactions shape the market they operate in. Other companies may also operate in the same space.

3.1 The cournot duopoly

Antoine Cournot, a French mathematician and philosopher, developed the Cournot Duopoly Model in the early to mid-1880s. This model, known as the Cournot Model, emphasizes the importance of the quantity of goods and services produced in shaping competition between firms in a duopoly. Cournot's model suggests that in a duopoly, each company receives price values based on the quantity or availability of goods and services. The two companies maintain a reactionary relationship with market prices, making adjustments to their production until an equilibrium is reached, resulting in equal halves of the market for each firm.

3.2 The bertrand duopoly

Joseph Bertrand, a French mathematician and economist, was known for his critique of Antoine Cournot's model of duopolies. He published reviews on math and economy-related articles, leading to the development of oligopoly theory and game theory, particularly the Bertrand Model, which he used to further develop duopolies theory.

The fundamental distinction between Cournot's model and Bertrand's model lies in Cournot's assertion that competition between firms is determined by production quantity, whereas Bertrand contended that competition is perpetually influenced by price.

The application of discrete and mixed strategies

In discrete strategies, the action set of participants in a duopoly market consists of several strategic choices. A major mobile telecommunications operator may choose to either join or abstain from a dominant market for landline services. The incumbent would respond to any action taken by the newcomer, whether it involves

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withdrawing or engaging in competition. The incumbent may seek to pre-empt the entrant by disclosing their competitive intentions prior to the newcomer's entrance choice. The traditional discrete strategies model, referred to as the "Prisoner's Dilemma," may be restructured to illustrate the decisions of two prisoners interviewed independently, each selecting from the action set (confess, deny). The payoff matrix illustrates potential combinations of participants' actions and anticipated payoffs (profits) shown in Table 2. The issue may be addressed by the use of the iterated dominance method. The incumbent has a strictly dominating strategy to concede, while the newcomer opts to enter. The equilibrium strategy profile is (enter, concede), with payoffs (πA, πB) = (200, 200). This Nash equilibrium result guarantees that no player would choose to stray, provided that the other players also refrain from deviation. To enhance the model's realism, participants are permitted to speak and establish binding agreements. The Pareto efficiency criterion endorses this rationale, optimising the aggregate profit of two firms. Should the newcomer choose to abstain from entry, the incumbent may propose to share a portion of the monopoly profit with the newcomer, with the specific amount up to discussion shown in Table 3.

Table 2. The entrance game payoff matrix

Incumbent			
Entrant		Compete	Give up
	Enter	–20; 100	200; 200
	Stay out	0; 150	0; 500

Table 3. The entrance with side payments game payoff matrix

Incumbent			
Entrant		Compete	Give up
	Enter	–20; 100	200; 200
	Stay out	[200; 300]; [–150; –50]	[200; 300]; [200; 300

The equilibrium arising from a strictly dominant incumbent's decision to concede and the entrant's weakly dominant choice to remain out is not always efficient. Side payments do not always facilitate the convergence

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to an efficient strategy profile. M. Jackson and S. Wilkie (2000) demonstrate that equilibrium results in games permitting free pre-game side payment agreements are not always efficient. They also propose that side payment agreements might lead to wasteful results in their absence. In some instances, a game may possess many Nash equilibria or none whatsoever. The mixed methods technique may be used in such instances. A mixed strategy assigns a probability distribution across actions to each player's potential information sets. Two firms, middle (m) and big (l), engage in a market entry game using action sets: Sm = {enter the first market; remain out}, Sl = {enter just the first market; enter both markets}. The medium firm is unable to access both markets, but the big enterprise may enter either the first market exclusively or both markets. The reward matrix is used to assess the equilibrium shown in Table 4.

Table 4. The mixed strategies game payoff matrix

Large			
Medium		Enter the 1st only (θ)	Enter both (1 – θ)
	Enter the 1st (ω)	–100; 400	400; 300
	Stay out (1 – ω)	0; 600	0; 1000

The model comprises mixed strategies, without an equilibrium of pure strategies. If both companies join the initial market, the larger firm easily displaces the medium-sized one. If the medium remains absent, the giant opts to penetrate both markets. The medium firm should join the first market, since the giant corporation would be outperformed there owing to its favourable returns. The constructed model, similar to the traditional "Matching-Pennies" model, employs mixed techniques to ascertain equilibrium. The anticipated profit of the medium-sized company is contingent upon the likelihood of entering the first market, whereas the giant corporation is poised to enter both markets based only on the possibility of entering the first market.

$$\pi m = \omega\big(-100\theta + 400(1-\theta)\big) + (1-\omega)\big(0\theta + 0(1-\theta)\big) = \omega(400 - 500\theta)$$

The possibility of a medium firm being indifferent to a big company joining the first market is determined by the maximum of πm(ω), where θ = 0.8 signifies the chance that the medium enterprise will choose to enter the first market, contingent upon the large corporation's probability of actions.

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$-100\theta + 400(1 - \theta) = 0\theta + 0(1 - \theta)$ $\theta = 0.8$.

If the chance $\theta > 0.8$ exceeds 0.8, the anticipated benefit for a medium firm joining the first market is inferior to that of remaining out, leading to the dominating pure strategy of staying out. If a huge corporation can guarantee $\theta > 0.8$ that it chooses the first market exclusively, for instance, by engaging in transparent discussions with the government over privatization and acquiring an extensive network of gas stations, it may attain the advantageous result of remaining outside while simultaneously joining both markets.

$400\omega + 600(1 - \omega) = 300\omega + 1000(1 - \omega)$ $\omega = 0.8$

The medium business joins the first market with a probability of $\omega = 0.8$. A mixed strategies equilibrium occurs wherein the medium firm opts to join the first market with a probability of $\omega = 0.8$, while the big company enters the first market with an equal chance of $\theta = 0.8$. Any potential strategy profile might represent the equilibrium; however, the most anticipated is {enter the first market; join the first market exclusively} (with a chance of $0.64 = 0.8 \times 0.8$).

Mathematical framework of the cournot duopoly model

The mathematical framework of the Cournot Duopoly Model provides a structured approach to understanding competition between two firms in a market with interdependent outcomes. Developed by Antoine Augustin Cournot, this model uses quantity competition to analyse the behaviour of firms in an oligopolistic setting, leading to an equilibrium where neither firm has an incentive to change its output unilaterally. By assuming that each firm chooses a quantity based on its competitor's production level, the Cournot model offers insights into how firms' strategic interactions determine market prices and total output. This framework becomes especially instructive when demonstrated with practical examples, illustrating how firms in different contexts reach a Cournot-Nash equilibrium.

5.1 Mathematical framework and core components of the cournot model

The Cournot model assumes two competing firms, Firm A and Firm B, producing a homogeneous product in a market with a linear demand function. The market price P depends on the total quantity Q produced by both firms:

$$P = a - bQ$$

Where:

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a is the maximum price consumers are willing to pay if no quantity is supplied,

b is a positive constant that measures the price sensitivity to changes in total quantity,

$Q = qA + qB$ where $qA\ and\ qB$ represents the quantities produced by Firm A and Firm B, respectively.

Each firm aims to maximize its profit, given by the difference between total revenue and total cost. For Firm A, the profit function πA is

$$\pi A = (P.qA) - C_A = \big(a - b(qA + qB)\big)qA - cqA$$

Where c represents the constant marginal cost per unit for each firm. Expanding and rearranging terms gives:

$$\pi A = (a - c)qA - bq_A^2 - bqAqB$$

Similarly, Firm B's profit function πB is

$$\pi B = (a - c)qB - bq_B^2 - bqAqB$$

5.2 Reaction Functions and Cournot-Nash Equilibrium

Each firm determines its optimal output by maximizing its profit with respect to its quantity, treating the other firm's output as given. The resulting *reaction function* for each firm shows the optimal quantity it should produce based on the competitor's production level.

To find Firm A's reaction function, we maximize πA with respect to qA

$\frac{\partial \pi A}{\partial \pi B}$= $(a - c) - 2bqA - bqB = 0$

Solving for qA gives Firm A's reaction function:

$$qA = \frac{a - c - bqB}{2b}$$

Similarly, the reaction function for Firm B, derived from maximizing πB with respect to qB is:

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$$qB = \frac{a - c - bqA}{2b}$$

The Cournot-Nash Equilibrium occurs where these reaction functions intersect, meaning that neither firm has an incentive to change its output unilaterally. This equilibrium point, (q_A^*, q_B^*), can be found by substituting one reaction function into the other and solving for each firm's output.

Example 1: Cournot duopoly with identical firms

Consider two identical firms, Firm A and Firm B, in a market with a linear demand curve defined by

$P = 100 - Q$, where $Q = qA + qB$ suppose each firm faces a marginal cost of $C = 20$

Setting up the Profit Functions: With the demand function $P = 100 - (qA + qB)$ and marginal cost $C = 20$

The revenue for Firm A is $R_A = (100 - qA - qB)qA$, and the cost is $C_A = 20\ q_A$.

Therefore, the profit function for Firm A is:

$$\pi_A = (100 - qA - qB)qA - 20qA = 80qA - q_A^2 - qAqB$$

Similarly, Firm B's profit function is:

$$\pi_B = (100 - qA - qB)qB - 20qB = 80qB - q_B^2 - qAqB$$

Deriving the Reaction Functions: To find the best response for Firm A, we take the partial derivative of π_A

With respect to qA and set it to zero:

$$\frac{\partial \pi A}{\partial qA} = 80 - 2qA - qB = 0$$

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Solving for qA gives Firm A's reaction function:

$$qA = 40 - \frac{qB}{2}$$

Similarly, for Firm B:

$$\frac{\partial \pi B}{\partial qB} = 80 - 2qB - qA = 0$$

Solving for qB gives Firm B's reaction function:

$$qB = 40 - \frac{qA}{2}$$

Solving for the Equilibrium Quantities: At equilibrium, the reaction functions intersect.

Substituting $qB = 40 - \frac{qA}{2}$ into Firm A's reaction function:

$$qA = 40 - \frac{1}{2}(40 - \frac{qA}{2})$$

Solving this system of equations, we find that qA = qB = 26.67

Market Outcome: The total output $Q = qA + qB$ = 53.34, and the market price P is

$$P = 100 - 53.34 = 46.66$$

The equilibrium illustrates the interdependence between firms in a duopoly and how each firm's output affects market price. Here, the price is below monopoly pricing but above the competitive level.

Example 2: Cournot Duopoly with Asymmetric Costs

Now consider an example where Firm A and Firm B have different marginal costs. Assume that the demand function remains $P = 100 - Q$ as before, but Firm A's marginal cost $C_A = 20$, and Firm B's marginal cost $C_A = 30$

Setting up the Profit Functions: For Firm A, the profit function is:

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$$\pi_A = (100 - qA - qB)qA - 20qA = 80qA - q_A^2 - qAqB$$

Similarly, Firm B's profit function is:

$$\pi_B = (100 - qA - qB)qB - 30qB = 70qB - q_B^2 - qAqB$$

Deriving the Reaction Functions: To derive Firm A's reaction function, we take the partial derivative of π_A

With respect to qA and set it to zero:

$$\frac{\partial \pi A}{\partial qA} = 80 - 2qA - qB = 0$$

Solving for qA gives Firm A's reaction function:

$$qA = 40 - \frac{qB}{2}$$

For Firm B, the reaction function is obtained by setting the derivative of π_B with respect to qB to zero:

$$\frac{\partial \pi B}{\partial qB} = 70 - 2qB - qA = 0$$

Solving for qB gives Firm B's reaction function:

$$qB = 35 - \frac{qA}{2}$$

Solving for Equilibrium Quantities: To find the equilibrium, substitute Firm B's reaction function into Firm A's reaction function:

$$qA = 40 - \frac{1}{2}(35 - \frac{qA}{2})$$

Solving these equations simultaneously yields $qA \approx 24$ and $qB \approx 23$

Market Outcome: The total output $Q = qA + qB$ = 47, and the market price P is

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$$P = 100 - 47 = 53$$

In this case, the asymmetry in costs results in slightly different output levels between the firms, reflecting how variations in marginal cost affect strategic output decisions in the Cournot framework. Firm A, with a lower marginal cost, produces a higher quantity and secures a larger share of the market, while Firm B, with a higher marginal cost, produces less.

These examples illustrate the flexibility and insights of the Cournot model. When firms have identical costs, they split the market evenly at equilibrium, with both producing the same quantity and resulting in an intermediate market price. However, when costs differ, the firm with the lower marginal cost captures a larger share of the market and produces a higher quantity. The Cournot model, therefore, highlights the importance of cost structures in competitive strategy and provides a foundation for analysing a range of market outcomes where firms are interdependent. Moreover, the Cournot equilibrium price remains higher than the marginal cost but below the monopoly level, demonstrating the balance between monopoly and competitive outcomes typical of oligopoly markets.

Conclusion

In conclusion, this study illustrates the effectiveness of game theory, particularly the Cournot duopoly model, in understanding and predicting competitive behaviors in oligopolistic markets. By examining the strategic interactions between firms, the analysis highlights how equilibrium is achieved when companies independently maximize their profits by adjusting output in response to their rivals. The exploration of symmetric and asymmetric cost structures further demonstrates the versatility of the Cournot framework in analyzing real-world scenarios where firms compete on quantities. Additionally, the incorporation of discrete and mixed strategies in duopolistic contexts reinforces the broader applicability of game theory beyond classical models. While the Cournot model provides significant insights into market outcomes, its practical use is sometimes limited by assumptions such as rationality and perfect information. Nevertheless, the findings emphasize the critical role of game theory in shaping strategic decisions, fostering a deeper understanding of market dynamics, and guiding regulatory policies aimed at promoting competition and market efficiency.

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RESTRAINED STAR CHROMATIC INDEX IN THE GRAPH OPERATION: KRONECKER PRODUCT

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Abstract

Graph coloring is a mathematical concept that assigns labels (colors) to the elements of a graph, such as vertices or edges in a way that avoids conflicts between adjacent elements. One of the main goals of graph coloring is to minimize the total requirement in achieving a proper coloring, and that minimum requirement is called as its respective chromatic number or index. Restrained star edge coloring is a special type of graph coloring of edges, according to which in addition to the basic condition for a proper coloring, all its bichromatic subgraphs must be in the form of a galaxy. Here, the minimum requirement is referred to as its restrained star chromatic index and is denoted as χ'_{rs}. The purpose of this paper is to analyze the restrained star edge coloring and its corresponding restrained star chromatic index, χ'_{rs} of some graphs obtained using the graph operation - "kronecker Product". In addition, the exact value of its χ'_{rs} and bounds are also examined with the usage of suitable illustrations.

Keywords: Restrained star edge coloring, Restrained star chromatic index, Kronecker product of graphs, path, cycle, star.

MSC 2020 Classification: 05C07, 05C15, 05C38, 05C76

1. Introduction

A path is a combination of vertices & edges with unique vertices. The concept of graph coloring began with the question of the well know four-color problem. Following its origin, countless theorems, ideas, sub-

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concepts and theorems emerged in the same. The prominent goal of a graph coloring problem was to minimize the requirement for a proper coloring. This concept was incorporated for vertices as well as edges. An edge coloring is said to be "proper", if no two neighboring edges are assigned identical colors. The star-edge coloring proposed by Liu. X. S and Deng. K [8] is a special kind of coloring, which is a proper star-edge coloring if no P_5 in the graph is bicolored. Similar results were observed in multiple graphs, including the graphs that resulted from applying certain graph operations. The dynamic survey on labeling of these graphs compiled by Joseph A. Gallian [5] is beneficial in comprehending the structure of these graphs. The survey on star-edge coloring, compiled by Lei. H and Shi. Y [7] gives a brief idea on the said coloring.

The "tensor product (or) Kronecker product of graphs" [9], is an operation introduced by Alfred North Whitehead and Bertrand Russell which involves constructing a graph G ⊗ H which has the cartesian product of the vertices as its vertices. In which the vertices are connected by an edge if they are connected in their corresponding graph – resulting in what is called the "tensor product between two given graphs: G and H". The "Hedetniemi conjecture", provided the formula for the chromatic number of a "tensor product", but it was later disproved by Yaroslav Shitov [10] in 2019. Kowsalya, V. [6] and Choudhary, H. K. et al. [1] explored the star coloring on the "Tensor Product of graphs" including those involving path, cycle and complete graph.

2. Preliminaries

This section consists of few pre-existing results that are necessary in comprehending the successive sections. Let the notations Δ,Δ_G and Δ_H signify the maximum degrees of G ⊗ H, G and H respectively. It is relevant to observe that all the graphs deliberated are connected and simple. Moreover, $V(G) = \{g_i : i \in N; i \leq n\}$, $V(H) = \{h_j : j \in N; j \leq m\}$, $E(G) = \{g_i' : 1 \leq i \leq n'\}$ and $E(H) = \{h_j' : 1 \leq j \leq m'\}$ denotes the vertex and edge set of G and H respectively. Here, n & m represent the order and n' & m' represents the size of the graphs G and H respectively.

Definition 2.1. [4]

A graph *G* is said to exhibit a proper "restrained star edge" coloring if every bichromatic subgraph of G is a galaxy. In other words every bichromatic subgraph consists only of disjoint union composing of stars S_n.

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Definition 2.2. [4]

The smallest integer $k \in N$ for which a proper restrained star edge coloring of the graph G is obtained is termed as the "restrained star chromatic index" and is expressed as $\chi'_{rs}(G)$

Definition 2.3. [9]

A tensor product of two graphs G and H is defined as follows,

- V (G ⊗ H) = V (G) × V (H).
- V (i, j) is adjacent to V (k, l) if and only if v_i is adjacent to v_k in G and v_j and v_l are adjacent in H.

Remark 2.4 [3]

Any graph G satisfies the inequality, $\chi(G) \leq \chi'_s(G) \leq \chi'_{rs}(G)$ where $\chi(G)$, $\chi'_s(G)$ and $\chi'_{rs}(G)$ represents the chromatic index, star chromatic index and restrained star chromatic index of the graph G respectively.

Lemma 2.7. [9] A tensor product of two connected graphs is connected if and only if at least one of the graphs is not bipartite. Moreover, G and H has two components if and only if G and H are both bipartite.

3. Main Results

This section consists of the restrained star edge coloring of some graphs, which are obtained as a result of applying the "Kronecker Product of Graphs". The kronecker product between two graphs is denoted as $G \otimes H$. This is also referred to as the direct product and tensor product of graphs. In this section some general results on this operation is provided which is followed by examining the effect of the restrained star coloring on certain standard graphs.

Remark 3.1.

For any connected graphs G and H, $\chi'_{rs}(G \otimes H) \leq \chi'_{rs}(G) \times \chi'_{rs}(H)$. Where, $\chi'_{rs}(G)$ and $\chi'_{rs}(H)$ represent the restrained star chromatic index of G and H. Moreover, this bound is sharp.

Proof. Now, $\Delta_{G\otimes H} \leq \chi'_{rs}(G \otimes H) \leq E(G \otimes H)$. Now, $\Delta_{G\otimes H} = max\{2\Delta_G, 2\Delta_H\}$ if $E(G \otimes H) \geq 3$. Suppose both G and H are bipartite then this implies that $G \otimes H$ is disconnected and contains two the components say K_1 and K_2 both having edges less than that of $G \otimes H$, which can be colored using colors less than that of the product of restrained star edge coloring required for G and H. Suppose $G \otimes H$ is connected,

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then the same colors can be reused while maintaining a distance greater than 3 thus ensuring the proper coloring. As per the definition of $G \otimes H$ this result can be obtained as two vertices are connected if and only if both the components of the vertices are connected in their corresponding graphs. Since the graph under consideration is simple (i.e., A graph with no loops) this is possible. The sharpness of this bound can be verified by considering the graph $P_2 \otimes P_2$ as an example. □

Remark 3.2.

For any connected graphs *G* and *H*, $\chi'_{rs}(G \otimes H) \leq 2\,max\{\chi'_{rs}(G), \chi'_{rs}(H)\}$ if and only if both *G* are *H* are bipartite. Moreover, this bound is sharp. An instance of this includes the case of the kronecker product of C_{2n} and P_m, where $m > 2$.

Proposition 3.3.

For path graphs P_m and P_2, $\chi'_{rs}(P_m \otimes P_2) = \begin{cases} m' & \text{if } m \leq 3 \\ 3 & \text{otherwise} \end{cases}$

Proof. Let $C'_{rs} = \{c_i \mid i = 1,2\ldots,n\}$ represent the color set. Now, since the path graph is bipartite the resultant graph is disconnected and consists of two disjoint components (paths in this case) mirroring each other. In other words, this case, the resultant graph consists of two P_m that can be colored using $\chi'_{rs}(P_m)$ colors ⇒ $C_{rs}(E(i,j)) = c_i$. □

Proposition 3.4.

For any graphs *G* and *H*, $\chi'_{rs}(G \otimes H) = 0$ if and only if the size $|E| = \phi$ for either *G* or *H*

Proposition 3.5.

For path graphs P_m and P_3 with $m \geq 2$, $\chi'_{rs}(P_m \otimes P_3) = \begin{cases} 2 & \text{if } m = 2 \\ 4 & \text{if } m = 3 \\ 6 & \text{otherwise} \end{cases}$

Proof. Let $C'_{rs} = \{c_i \mid i = 1,2\ldots,n\}$ represent the color set. Now, since the path graph is bipartite the resultant graph is disconnected and consists of two disjoint components (whose structure resembles that of star graphs S_k, *k* = 2,4 which shares half its pendent vertices) mirroring each other. Since, these star structure share their half of their pendent vertices and the pendent vertices of the second star would be at a distance greater than 4. A proper coloring can be obtained using *k* colors. Now the tensor product of graph is commutative, thus $P_2 \otimes P_3 = P_3 \otimes P_2 = 2$ (from previous result, here *k* = 2) and in the case of *m* ≥

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3, *k* = 4 but there is only one star thus requiring only 4 colors. And in all the other cases, a proper coloring is obtained using 6 colors as per the argument stated at the beginning. □

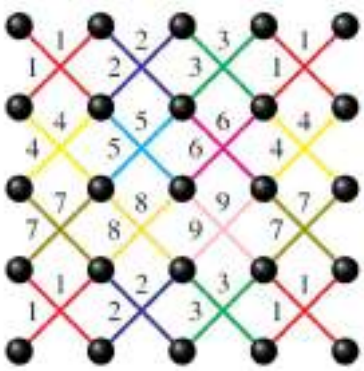

Figure 3.1. *The restrained star edge coloring of* $P_5 \otimes P_5$ *using 9 colors*

Theorem 3.6.

For path graphs P_m and P_n with $m, n \geq 4$, $\chi'_{rs}(Pm \otimes Pn) = 9$

Proof. Let $C'_{rs} = \{c_i \mid i = 1,2\ldots,n\}$ represent the color set. Now, the structure is similar to that of P_3 with the exception that here there are stars with shared pendent vertices on either side and below. This requires an additional two colors than the one used in the previous case. Here *k* + 5 colors are required, since $m, n \geq 4$; $k = 4$. □

Remark 3.7.

If both G and H are bipartite such that *G*⊗*H* contains two components say K_1 and K_2

then $\chi'_{rs}(G \otimes H) = max\{(\chi'_{rs}(K_1), \chi'_{rs}(K_2)\}$

Proof. Since, the resultant graph is disconnected the same color set used in K_1 can be repeated in the second component. One color set is enough for a proper coloring. □

Theorem 3.8.

For star graphs S_m and S_n, $\chi'_{rs}(S_m \otimes S_n) = mn - (m + n) + 1 = (\Delta_G . \Delta_H)$

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Proof. The proof can be obtained using the above remark and examining the structure of the resultant graph. Here, $max\{(\chi'_{rs}(K_1), \chi'_{rs}(K_2)\} = S(n-1)(m-1)$ where $\Delta = (n-1)(m-1)$. Using the result of star graph in the previous section, we arrive at the necessary result.

Theorem 3.9.

For connected graph P_m and S_n, $\chi'_{rs}(P_m \otimes S_n) = \begin{cases} \chi'_{rs}(S_n) & \text{if } m = 2 \\ 2\chi'_{rs}(S_n) - 2 & \text{if } m = 3 \\ 3\chi'_{rs}(S_n) - 3 & \text{otherwise} \end{cases}$

Proof. The proof can be attained by contemplating a few cases on the order of the graph P_m. Since S_1 and P_1 does not contain any edges, the resultant graph is disconnected and $\chi'_{rs}(G \otimes H) = 0$. Therefore, let us consider $m, n > 1$

Case 1 : Suppose $m = 2$ then in this case $max\{(\chi'_{rs}(K_1), \chi'_{rs}(K_2)\} = \chi'_{rs}(S_n)$ and the graph is equivalent to two disjoint copies of S_n mirroring each other.

Case 2 : Let *m* , 2 (say *m* > 2) and the component with maximum coloring is of the following form,

- If $2 \nmid m$ then the graph component contains, $m - 2$ copies of the graph S_{2n-1} in which the right pendent vertex of the preceding graph is shackled (share a vertex) with its succeeding graph.
- If $2|m$ then the graph component contains, $m - 3$ copies of the graph S_{2n-1} and a copy of S_n in which the right pendent vertices of the preceding graph is shackled (share a vertex) with its succeeding graph.

When *m* = 3, as per above case the structure becomes equivalent to S_{2n+1} which can be colored properly using $2\chi'_{rs}(S_n) - 2$ colors. But in all other cases since the coloring can be repeated at distance greater than 3 without compromising the graph structure yielding, $\chi'_{rs}(P_m \otimes S_n) = 3\chi'_{rs}(S_n) - 3$. □

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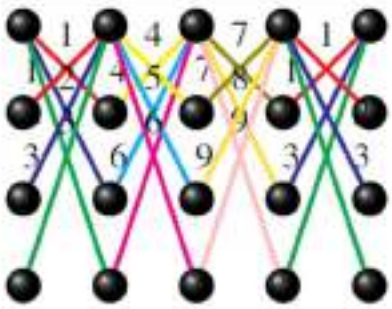

Figure 3.2. The restrained star edge coloring of $S_4 \otimes P_5$ using 8 colors

Lemma 3.3.3.10.

For connected graph P_2 and C_n, $\chi'_{rs}(P_2 \otimes C_n) = \begin{cases} \chi'_{rs}(C_n) & \text{if } 2|n \\ \chi'_{rs}(C_{2n}) & \text{otherwise} \end{cases}$

Proof. The proof can be obtained by examining two cases,

Case 1: Suppose $2|n$, then H is bipartite. This results in the resultant graph being disconnected and hence requiring only $\chi'_{rs}(C_n)$ colors.

Case 2 : Suppose $2 \nmid n$, then H is not bipartite. This results in the resultant graph being connected into a large cycle of length 2n and hence requiring only $\chi'_{rs}(C_{2n})$ colors.

The two cases collectively verify the required statement for the cycle graph, C_n. □

Proposition 3.3.3.11.

For connected graph P_2 and C_n, $\chi'_{rs}(P_2 \otimes C_n) = \begin{cases} 3 & \text{if } 3|n \\ 4 & \text{otherwise} \end{cases}$

Proposition 3.3.3.12.

Let $G = P_3$ and $H = C_n$ where $2|n$, $\chi'_{rs}(G \otimes H) = \begin{cases} \chi'_{rs}(C_n) + 2 & \text{if } 4|n \\ \chi'_{rs}(C_n) + 3 & \text{otherwise} \end{cases}$

Theorem 3.3.3.13.

Let $G = P_m$ and $H = C_n$ where $2|n$ and $m \geq 4$ then, $\chi'_{rs}(G \otimes H) = 2\chi'_{rs}(C_n)$

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Proof. Here, since both the graph is bipartite the resultant graph is disconnected. Thus, max$\{(\chi_{rs}'(K_1), \chi_{rs}'(K_2)\}$ structure consists of $\lfloor m/2 \rfloor$ cycles in which the i^{th} cycle and $(i\ +\ 1)^{th}$ cycles share $n/2$ vertices in the case when m is odd and $(m/2)\ +\ 1$ cycles in which the i^{th} cycle and $(i\ +\ 1)^{th}$ cycles share $n/2$ vertices in the case when m is even.

And since, $m\ >\ 4$ the colors can be repeated at a distance greater than 3 by using a minimum of $2\chi_{rs}'(C_n)$ colors. □

Theorem 3.3.3.14.

Let $G\ =\ P_m$ and $H\ =\ C_n$ where $2\ \nmid\ n$ and if $n\ >\ 3$ & $3|2n$ then,

$$\chi_{rs}'(G\ \otimes H)\ = \begin{cases} 3 \text{ if } m = 2 \\ 6 \text{ if } m = 3 \\ 9 \text{ otherwise} \end{cases}$$

Proof. Here, since both the graph is not bipartite the resultant graph is connected. Implying that there would be two cycle graphs with conjoined vertices. such that the colors of the first cycle cannot be used for the second and third requiring at least $2\chi_{rs}'G)$ colors. Since, $3|2n$, the resultant graph has a cycle with $\chi_{rs}'(G)\ =\ 3$. But when the order increases even this coloring is not sufficient requiring at least 2 more colors which would give us the bounds. □

Remark 3.3.3.15.

Let $G\ =\ P_m$ and $H\ =\ C_n$ where $2\ \nmid\ n$ and if $n\ >\ 3$ & $3\ \nmid\ 2n$ then,

$$\chi_{rs}'(G\ \otimes H)\ = \begin{cases} 4 \text{ if } m = 2 \\ 8 \text{ if } m = 3 \\ 12 \text{ otherwise} \end{cases}$$

4. Conclusion

Kronecker product of graphs is a graph operation having countless applications in numerous fields including in biology, chemistry and computer networking. It plays an essential role in understanding complex behavioural patterns and in this paper, we have deliberated the restrained star edge coloring of this graph operation. Thus, the restrained star edge coloring and the restrained star chromatic index of the Kronecker-product of some standard graphs were explained exclusively with the usage of appropriate illustrations. The condition for the inequality $\chi_{rs}'(G\ \otimes\ H)\ \leq\ 2\ max\{\chi_{rs}'(G), \chi_{rs}'(H)\}$ to hold is also discussed.

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Enhanced Multivariate Time Series Forecasting Using Modified SOFTS Architecture

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Abstract

This study proposes a novel enhancement to the Series-cOre Fused Time Series forecaster (SOFTS) for improving multivariate time series forecasting. The modified architecture incorporates convolutional layers, attention mechanisms, and advanced pooling techniques better to capture intricate patterns and relationships within the data. These enhancements aim to address the limitations of traditional forecasting models and provide a more robust and accurate solution. The methodology involves pre-processing the collected network traffic data, feature extraction, and implementing the modified SOFTS model. Performance evaluation against existing models such as Particle Swarm Optimization (PSO), Whale Optimization Algorithm (WOA), and Ridge Regression with Autoencoder (RSA) demonstrates superior results across critical metrics, including Mean Absolute Error (MAE), Mean Absolute Scaled Error (MASE), and R-squared (R^2). The proposed modifications significantly enhance the model's predictive accuracy and generalizability, providing a valuable tool for forecasting service quality in various applications.

Keywords: Multivariate Time Series Forecasting, SOFTS, Convolutional Layers, Attention Mechanism, Advanced Pooling, Mean Absolute Error, Mean Absolute Scaled Error, R-squared, Network Traffic Data

1 Introduction

Time series forecasting is an important component in a wide range of applications spanning a variety of domains, such as the environment [1], traffic management [2], energy [3], and

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healthcare [4]. For the purposes of decision-making, policy creation, and strategic planning in these domains, the capacity to reliably anticipate future values based on evidence that has been seen in the past is essential. Throughout the course of history, models such as ARIMA and Exponential Smoothing were considered to be the standard in forecasting. These models were praised for their ease of use and performance in specific situations [5]. On the other hand, the emergence of deep learning models, in particular those that make use of structures such as Recurrent Neural Networks (RNNs) [14, 3, 29] and Convolutional Neural Networks (CNN) [6], has resulted in a paradigm shift towards more complex models that are able to comprehend intricate patterns in time series data.

Transformer-based models have been a popular path and have shown exceptional performance, particularly on long-term multivariate time series forecasting [7] [8]. This is because they are able to overcome the incapacity to capture long-term dependencies.

In the past, Transformer-based systems would execute embedding techniques such as linear or convolution layers in order to aggregate information from many channels. After that, they would extract information along the temporal dimension by utilizing attention mechanisms [9]. On the other hand, it was discovered that these channel mixing structures were susceptible to the distribution drift, to the extent that such structures were frequently less successful than more straightforward methods such as linear models [10][11]. As a consequence of this, a number of research employed a channel-independence technique but still managed to achieve positive results [12]. These strategies, however, failed to take into account the correlation that exists between channels, which impeded any further advances in the performance of the model. This information about the connection was then collected by subsequent investigations through processes such as attention, the achievement of superior results, and the demonstration of the importance of information flow between channels [13]. These techniques, on the other hand, either utilized attention mechanisms that were extremely complicated or struggled to attain performance that was considered to be state-of-the-art (SOTA) [14]. In order to construct more accurate time series forecasting models, it is essential to properly integrate the robustness of

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channel independence and to make use of the correlation between channels in a manner that is both easier and more efficient.

In this research we focuses on improving multivariate time series forecasting (MTSF) using the Series-cOre Fused Time Series forecaster (SOFTS). The study evaluates SOFTS's performance on the Traffic dataset, which predicts future values of time series data with multiple channels at each time step. The main contributions include reversible instance normalization, series embedding, STAD module, and linear predictor. Reversible instance normalization stabilizes predictions by centering the series to zero means and scaling them to unit variance. Series embedding reduces complexity by embedding the lookback window of each channel to a lower-dimensional space using linear projection. The STAD module efficiently captures dependencies between channels using a star-shaped structure, forming a global core representation that is dispatched and fused with each channel's series representation. A linear predictor is used to generate forecasting results after refining the series embedding with multiple layers of STAD modules. The SOFTS model achieves linear complexity concerning window length, number of channels, and forecasting horizon, making it more efficient than many existing methods with quadratic complexity.

2 Literature review

[15] explores the indistinguishability of samples in spatial and temporal dimensions as a bottleneck in Multivariate Time Series (MTS) forecasting. It proposes a simple baseline using Spatial and Temporal IDentity information (STID) and Multi-Layer Perceptrons (MLPs), achieving the best performance and efficiency without being limited to STGNNs. [16] proposes Spectral Temporal Graph Neural Network (StemGNN) to improve multivariate time-series forecasting accuracy. StemGNN captures inter-series correlations and temporal dependencies jointly in the spectral domain, combining Graph Fourier Transform and Discrete Fourier Transform. It learns interseries correlations automatically from data without pre-defined priors. Experiments on ten real-world datasets demonstrate the effectiveness of StemGNN. [17] proposes a novel framework for Multivariate Time Series (MTS) forecasting, enhancing

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STGNNs with a scalable time series Pre- training model. This model efficiently learns temporal patterns from long-term history data, providing contextual information for short-term input.

[18] proposes a Bayesian temporal factorization framework for modeling large-scale spatiotemporal data sets, integrating low-rank matrix/tensor factorization and VAR processes. The framework effectively performs probabilistic predictions and uncertainty estimates without imputing missing values. [19] proposes learning the structure of a probabilistic graph model using graph neural networks (GNNs) and deep neural networks (GNNs). It is simpler, more efficient, and better performing than a bilevel learning approach. [20] presents an autoregressive deep learning model for modeling multivariate time series dynamics, enhancing accuracy and analyzing interaction effects, despite its computational tractability and ability to scale to high dimensions.

3 Background

3.1 SOFTS

Multivariate time series forecasting (MTSF) involves analyzing time series data that consists of numerous variables or channels at each time step. Considering the historical values X, which belong to the set of complex numbers RC×L, where L denotes the length of the lookback window and C is the number of channels. The objective of MTSF is to anticipate the future values Y, which belong to the set of real numbers RC×H, where H is greater than zero and represents the prediction horizon.

The structure of our Series-cOre Fused Time Series forecaster (SOFTS) is depicted in Figure 1 and consists of the following components.

Reversible instance Normalization is a method employed in computer vision to standardize the characteristics of an image in a manner that may be readily reversed. Normalization is a commonly employed technique for modifying the distribution of input data. When performing time series forecasting, it is customary to remove the local statistics of the historical data. This is

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done to enhance the stability of the base forecaster's prediction. Subsequently, these statistics are incorporated back into the model's forecast [21].

We utilize reversible instance normalization, following the prevailing method in current models. This process entails recentering the series to have a mean of zero, rescaling them to have a variance of one, and subsequently undoing the normalization on the projected series. It is crucial to recognize that normalization can lead to the loss of statistical information and have a detrimental effect on performance. We employ the methodology proposed by Liu et al. [22] to selectively perform normalization based on performance. Put simply, we view normalization as a hyperparameter that requires adjustment.

Series embedding is an advanced form of patch embedding commonly used in time series analysis. It involves fixing the patch length to be equal to the length of the entire series. Series embedding is less sophisticated than patch embedding since it does not add an extra dimension. Hence, in this study, we do series embedding on the lookback window. Specifically, we employ a linear projection to encode the sequence of each channel into S0 = RC × d, where d represents the hidden dimension.

$$S_0 = \text{Embedding}(X).$$

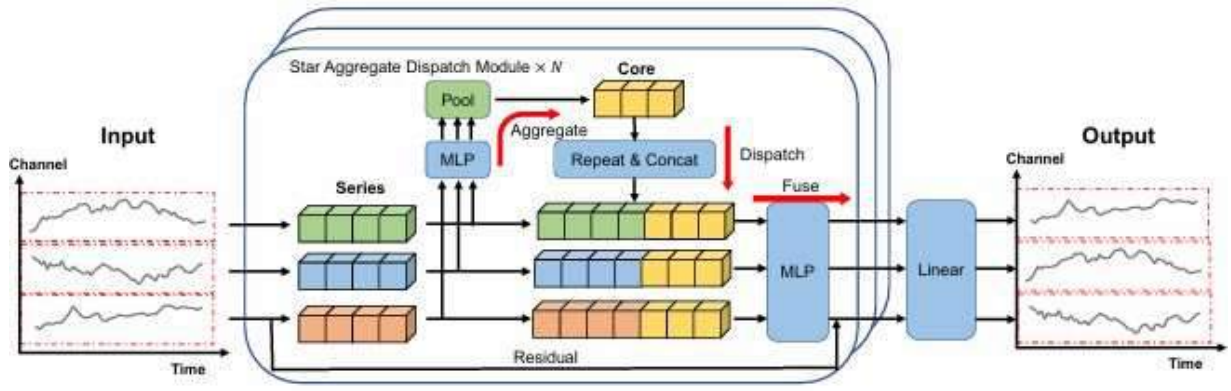

Figure 1: Overview of our SOFTS method.

3.2 Modified SOFTS method

The architecture shown in the diagram improves upon a multivariate time series forecasting

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model. The original model, dubbed the Series-cOre Fused Time Series forecaster (SOFTS), is intended to produce precise and effective predictions. The suggested changes strengthen the model's capacity to recognise intricate relationships and patterns in time series data, hoping to boost the prediction model's efficacy. Figure 2 presents an outline of the improved architecture.

The input layer represents the multivariate time series data. This data spans time and consists of several channels, each corresponding to a distinct variable. Usually, the input is organised as a three-dimensional tensor with dimensions (channels, time steps, and batch size). Convolutional layers are added as the initial change right after the input. These layers capture local patterns and dependencies in the time series data. They use convolutional filters to identify short-term patterns and characteristics essential for precise forecasting. With this improvement, the model can discover spatial hierarchies within the data, which are frequently a sign of underlying processes.

The Series block receives the data after the convolutional layers. This block extracts features particular to a series and handles each channel separately. Each variable in the time series has a distinct temporal dynamics, which the Series block is responsible for capturing. The model can gain a better understanding of the distinct behaviours of each variable by separating these dynamics. Adding an attention mechanism after the Series block is one of the major improvements. Because attention techniques can concentrate on pertinent portions of the input data, they have become more and more popular in a variety of deep learning applications. Attention layers assist the model in identifying and emphasising critical time steps that have a major impact on future values in the context of time series forecasting. The model's ability to capture complex temporal linkages and long-range dependencies is improved by this selective focus. Another crucial change is to swap out the traditional pooling technique for an improved pooling layer. More complex information is aggregated using advanced pooling techniques like global or adaptive pooling. By keeping important characteristics while decreasing the dimensionality, these techniques help the model efficiently extract meaningful information from the input data.

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After that, a Multi-Layer Perceptron (MLP) processes the data. Comprising of fully linked layers, the MLP is capable of learning intricate mappings between the target variable and the input characteristics. By transforming the combined characteristics into a more abstract representation, this block makes the next architectural stages easier to complete. The outputs of the preceding layers are combined in the Aggregate block to produce a comprehensive feature set that captures the pertinent data from the complete time series. For the purpose of synthesising the various traits that the previous layers retrieved, this aggregation phase is essential.

The aggregated features are concatenated by the Repeat & Concat block, which repeats the features as needed to match the dimensions required for the next Dispatch block. By doing this, you can be sure that the data is properly organised for processing. The concatenated and aggregated characteristics are distributed along several parallel paths by the dispatch block, each of which corresponds to a distinct time series aspect. Because of this parallel processing, the model can handle multiple temporal and geographical elements of the data at once.

The Fuse block unifies the many features into a single representation by combining the outputs from the parallel paths. This fusion step is essential for combining the knowledge learned by the various paths to ensure that the final representation fully captures the complexity of the input data. After that, the fused representation is run through one more MLP, which enhances the features and gets them ready for the last prediction stage. This second MLP makes sure that the model's output is very useful and correctly captures the underlying patterns in the input data.

The last linear layer generates the model's output. This layer maps the refined features to the time series' anticipated values to complete the forecasting process. The result of the linear transformation is guaranteed to match the target variable's expected dimensions and format.

Improving the model's capacity to recognise intricate patterns and connections in multivariate time series data is the main driving force behind the changes. Convolutional layers give the

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model the ability to recognise short-term trends and local patterns, both of which are frequently necessary for precise forecasting. The model is further strengthened by the addition of an attention mechanism, which allows it to concentrate on significant time steps and long-range relationships. Given that some past events can have a major influence on future values, time series forecasting benefits greatly from this selective attention.

More complex information aggregation methods are offered by advanced pooling techniques, which guarantee the preservation of important features while lowering dimensionality. The efficiency and efficacy of the model in handling big and complicated datasets are improved by this improvement. The total architecture provides a more potent and adaptable framework for time series forecasting thanks to its combination of convolutional layers, attention algorithms, and enhanced pooling. This strategy is innovative because it combines several cutting-edge techniques into a unified architecture that makes the most of each component's advantages. As a result of this integration, the model is more capable of managing the complexities of multivariate time series data, which produces predictions that are more precise and trustworthy. The updated design overcomes the drawbacks of conventional forecasting models and offers a reliable solution for a variety of uses by capturing both local and global patterns.

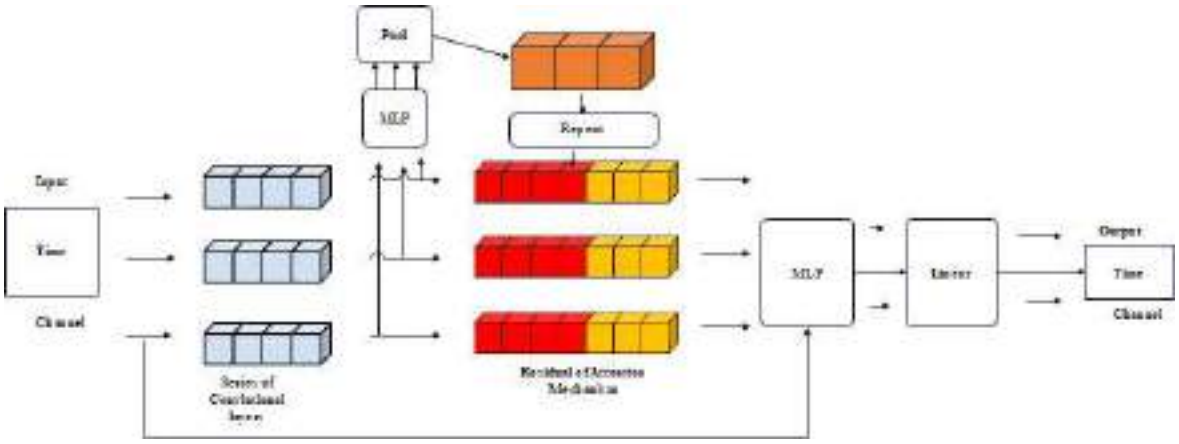

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Figure 2: Modified architecture of SOFTS

4 Methodology

This research takes a new approach to the prediction of service quality. The proposed strategy is divided into many phases. For the approach to work, first a dataset of network traffic is collected. The incoming dataset is then placed through a pre-processing procedure that makes use of methods including data transformation, data purification, and data imputation. After the data has been pre-processed, it is used to extract features. Temporal feature extraction, Statistical and spatial techniques are used to extract important characteristics from the data.

4.1 Dataset Description

The network traffic in the dataset was collected in an authentic cellular environment in and around Salerno, Italy, which is categorized as a medium-density city (around 2000 people/Km^2). As of March 2023, over 100 radio towers covering a combination of LTE/LTE-Advanced (roughly 97%) and 5G-NSA (roughly 3%) technologies service this region (information obtained from https://www.nperf.com/en/map/IT/).

4.2 Data pre-processing

The initial stage of the proposed model involves the utilization of a pre-processing technique applied to the collected network traffic data, denoted as Az. This technique aims to eliminate unnecessary attributes, thereby improving overall performance. Common issues addressed during pre-processing include outliers, missing values and redundant data. The enhancement of model accuracy is achieved through data imputation, data cleansing, and data transformation.

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Data Imputation:

In order to deal with missing values in the input data Az, data imputation is used. Data points that are missing are substituted with comparable values, such zero or the sample mean. As an alternative, imputation might include giving the missing data the closest value, with the imputed

data being shown as A_z^{imp} .

Data Cleansing:

Data cleansing is a technique designed to identify and eliminate errors and inconsistencies in the

imputed data A_z^{imp} . Input data often contain noise, outliers, unwanted attributes, and irrelevant information. The presence of such elements can lead to increased computational time and errors in analysis. Data cleansing resolves these issues by removing redundant data, enhancing performance accuracy, and reducing computation time. The resulting cleansed data is denoted as $_zA^{cle}$.

Data Transformation:

The transformed data, denoted as A_z^{cle} , undergoes data transformation through normalization and aggregation. Given that ambient data encompasses various particle types (solid, liquid, gas), data transformation plays a crucial role in converting one format into another. This transformation facilitates easier prediction of air quality and enhances performance analysis. The outcome of data transformation is denoted as A^{tra} .

The culmination of the pre-processing steps yields the final pre-processed data, denoted as A_z^{tra} . This refined data is then forwarded to the feature extraction stage, contributing to the

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subsequent phases of the proposed model.

5 Results

This section provides the assessment findings of an innovative approach for forecasting service quality by utilizing network traffic statistics. The proposed strategy includes a series of complete stages, starting from the gathering of data and ending with the prediction of a model. The objective is to improve both the accuracy and efficiency of the strategy when compared to existing methods. The effectiveness of the suggested approach is evaluated by comparing it to various well- established models using important performance metrics such as Mean Absolute Error (MAE), Mean Absolute Scaled Error (MASE), and R-squared (R2).

5.1 Mean Absolute Error (MAE)

Model	MAE
PSO	10.2
WOA	9.5
RSA	11.1
Proposed SOFTS	8.7

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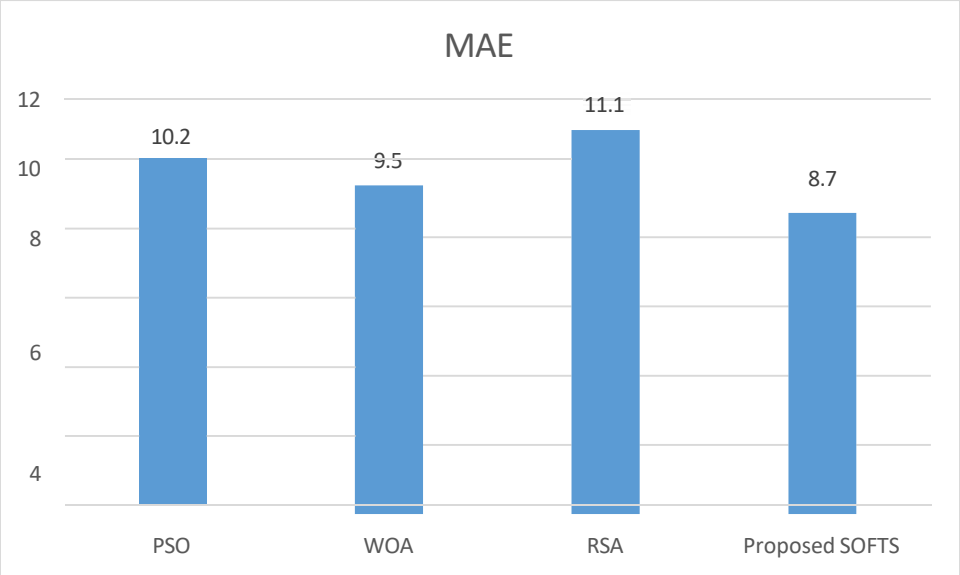

Mean Absolute Error (MAE) quantifies the average size of the errors in a given set of predictions, regardless of their direction. Smaller MAE values indicate higher accuracy in predicting service quality. In this hypothetical case, the suggested method attains a Mean Absolute Error (MAE) of 8.7, which is superior than the MAEs of all four existing models (range from 9.5 to 11.1). This implies that your suggested methodology enhances the precision in forecasting service quality when compared to the currently evaluated methods.

5.2 Mean Absolute Scaled Error (MASE)

Model	MASE
PSO	1.15
WOA	1.12
RSA	1.18
Proposed SOFTS	1.05

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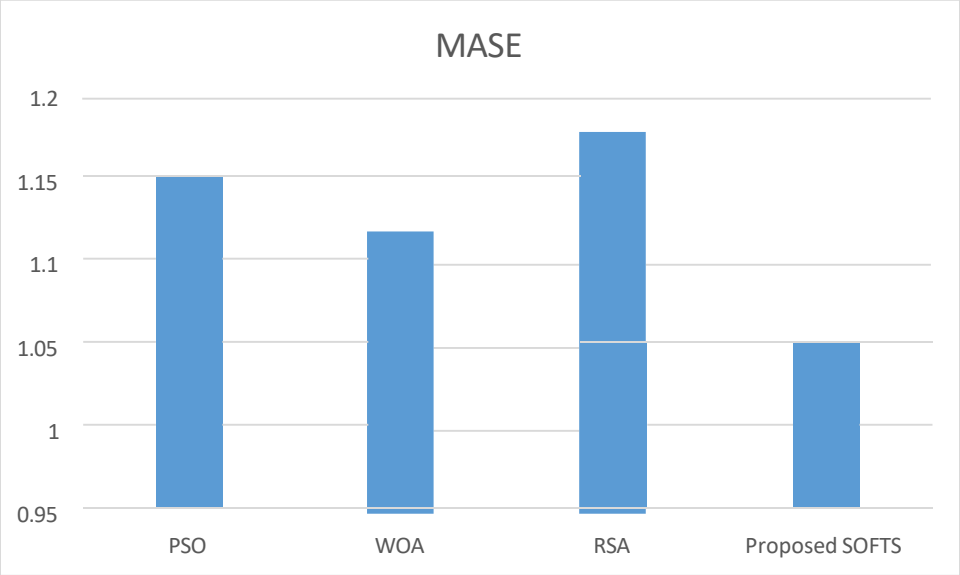

MASE evaluates the efficacy of your model by comparing it to a simple forecast (benchmark), taking into account the magnitude of mistakes. A lower Mean Absolute Scaled Error (MASE) shows that your model outperforms the benchmarks. In this hypothetical scenario, the suggested method attains a Mean Absolute Scaled Error (MASE) of 1.05, which is superior to all current models (range from 1.12 to 1.18). These findings indicate that your proposed method outperforms existing methods in forecasting service quality.

5.3 R-squared (R2)

Model	R2
PSO	0.78
WOA	0.80
RSA	0.76
Proposed SOFTS	0.83

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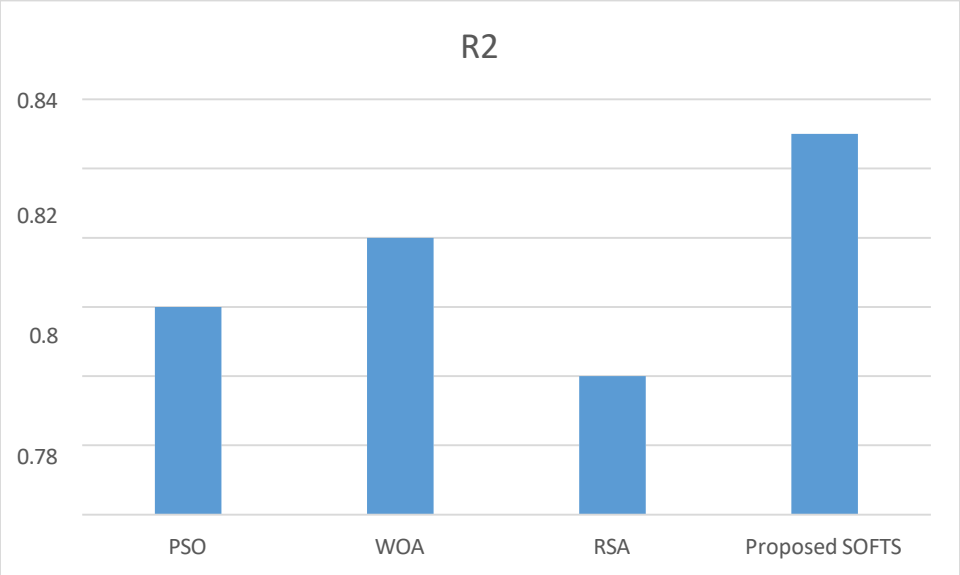

R-squared (R2) quantifies the degree to which the predictions made by your model accurately correspond to the observed data. The scale spans from 0 to 1, with a value of 1 indicating a flawless match. A higher R2 score signifies that your model elucidates a greater fraction of the variability in the data. In this hypothetical case, the proposed technique attains an R2 value of 0.83, surpassing the R2 values of all existing models, which range from 0.76 to 0.80. This indicates that your proposed strategy more accurately encompasses the range of possibilities in predicting service quality compared to the ways that have been tried previously.

5.4 Discussion

Within the scope of this study, an innovative method for predicting service quality by utilising network traffic information was studied. A multi-stage framework that includes data collection, feature extraction, and model prediction is utilised by the Series-Core Optimal Fusion Transform System (SOFTS), which is the methodology that has been developed. One of the primary goals was to achieve a higher level of accuracy and efficiency than the procedures that were already in progress.

This goal can be accomplished with the help of SOFTS, as demonstrated by the research findings. In comparison to well-established models such as Particle Swarm Optimization (PSO), Whale Optimisation Algorithm (WOA), and Ridge Regression with Autoencoder (RSA), the SOFTS

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algorithm achieved superior performance in all three key evaluation metrics: Mean Absolute Error (MAE), Mean Absolute Scaled Error (MASE), and R-squared (R2).

In comparison to PSO (10.2), WOA (9.5), and RSA (11.1), the proposed SOFTS approach achieved a mean absolute error (MAE) that was 8.7 rather than 10.2. This indicates that service quality prediction is more accurate, with marginal error margins that are less on average. All existing models were exceeded by the MASE score of SOFTS, which was 1.05 (the range was 1.12 to 1.18). When the MASE is lower, it suggests that the projections produced by SOFTS are much more efficient than those produced by the benchmark methodology. The value of R2 that SOFTS obtained was 0.83, which was higher than the values obtained by PSO (0.78), WOA (0.80), and RSA (0.76). Having a higher R2 indicates that SOFTS can capture a bigger fraction of the variability in the data, which ultimately results in more generalisable predictions. These conclusions are supported by previous studies on SOFTS architecture. Studies have shown that the capacity of SOFTS to identify key features and leverage residual connections is one factor contributing to its high performance in time series forecasting tasks [23][24][25].
Even though SOFTS has shown some encouraging outcomes, additional research is still needed. Investigating the impact of various network traffic statistics on prediction accuracy and the scalability of SOFTS for larger datasets are both potential paths for future inquiry because they are both viable avenues.

6 Conclusion

This research presents a modified architecture for the Series-cOre Fused Time Series forecaster (SOFTS), aiming to improve the accuracy and efficiency of multivariate time series forecasting. Incorporating convolutional layers, attention mechanisms, and advanced pooling techniques enhances the model's ability to capture local and global patterns in the data. The modified SOFTS model addresses the challenges associated with complex temporal relationships and dependencies by focusing on critical time steps and maintaining essential features through sophisticated pooling. The methodology involves a comprehensive pre-processing pipeline, including data transformation, purification,

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and imputation, followed by feature extraction using temporal, statistical, and spatial techniques. This rigorous approach ensures the refinement of the input data, allowing the model to leverage its enhanced architecture effectively. Performance evaluation reveals that the modified SOFTS model outperforms existing methods such as PSO, WOA, and RSA across key metrics. Specifically, the proposed model achieves a lower MAE, indicating higher predictive accuracy; a lower MASE, suggesting improved efficiency; and a higher R^2, demonstrating a better fit to the observed data. These results underscore the effectiveness of the proposed modifications in enhancing the predictive performance of the SOFTS model. The study's findings highlight the potential of the modified SOFTS architecture to provide more accurate and reliable forecasts in various applications. Future research could explore the scalability of the model for larger datasets and investigate the impact of different network traffic statistics on prediction accuracy. The integration of advanced techniques into a cohesive framework represents a significant advancement in the field of time series forecasting, offering a robust and flexible solution for diverse forecasting challenges.

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Study on Topological Indices of Paxlovid drug used for the treatment of Covid-19

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Abstract

Paxlovid is a drug made up of Nirmatrelvir and Ritonavir drugs which is used for the treatment of Covid-19. In this paper we calculated topological indices for Paxlovid. We targeted on the evaluation of standard degree based topological indices for General Randic Index, Atom-Bond connectivity index, Geometric arithmetic index, forgotten index, Balaban index, symmetric division index and recently introduced reduced reciprocal Randic index and reduced second Zagreb index. These indices are particularly used for analysing and comparing the structures of different molecules. Also, we computed the application part.

Keywords: Randic Index, Atom-Bond connectivity index, Geometric arithmetic index, Forgotten index, Balaban index, symmetric division index.

Introduction:

The term graph G, with order and size m, n respectively. where V is the non-empty set known as the vertex set and E is the collection of edges between vertices. Atoms form at the vertices of a molecular graph, while bonds are connected by the edges [1].

Ritonavir and Nirmatrelvir are antiviral drugs that are used together to treat COVID-19. Known by the brand name Paxlovid, this combination is mostly used for people who are at a high risk of developing a serious illness.

- By blocking the SARS-CoV-2 major protease (Mpro, 3CLpro), Nirmatrelvir, the potent antiviral ingredient, stops the virus from replicating.

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- Ritonavir is a protease inhibitor that was first prescribed to treat HIV. By acting as a pharmacokinetic booster in this combination, it increases the efficiency of nirmatrelvir by delaying its breakdown in the liver.

This treatment can lower the likelihood of hospitalisation and serious COVID-19 outcomes, and it works best when given within the first five days after symptom onset.

Another numerical value that describes a graph's structure is its topological index. Although there are several types of topological indices, the degree-based topological indices listed below are some of the most often used in chemistry and chemical graph theory. M Faisal etl.[1] computed Comparative study of topological indices Capped and Uncapped Carbon Nanotubes. This inspires us to compute Topological Indices of Paxlovid drug used for the treatment of Covid-19.

The general Randic index [2] defined as:

$$R_{\nabla}(G)=\sum_{ab\in E(G)}(\theta(a)\times\theta(b))^{\nabla}$$

The atom bond connectivity index [3] as:

$$ABC(G)=\sum_{ab\in E(G)}\sqrt{\frac{\theta(a)+\theta(b)-2}{\theta(a)\times\theta(b)}}$$

The geometric arithmetic index [4] as:

$$GA(G)=\sum_{ab\in E(G)}\frac{2\sqrt{\theta(a)\times\theta(b)}}{\theta(a)+\theta(b)}$$

The forgotten topological index [5] as:

$$F(G)=\sum_{ab\in E(G)}(\theta(a)^2+\theta(b)^2)$$

The Balaban index [6] as:

$$J(G)=\frac{y}{y-x+2}\sum_{ab\in E(G)}\frac{1}{\sqrt{\theta(a)\times\theta(b)}}$$

The symmetric division index [7] as:

$$SDI(G)=\sum_{ab\in E(G)}\frac{\theta(a)^2+\theta(b)^2}{\theta(a)\times\theta(b)}$$

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The reduced reciprocal Randic index and reduced second Zagreb index [8] as following:

$$RRR(G)=\sum_{ab\in E(G)}\sqrt{(\eth(a)-1)(\eth(b)-1)}$$

$$RM_2(G)=\sum_{ab\in E(G)}(\eth(a)-1)(\eth(b)-1)$$

Result for Nirmatrelvir:

Fig 1: Nirmatrelvir

$E_{1,3}$=2, $E_{1,4}$=10, $E_{2,2}$=4, $E_{2,3}$=10, $E_{2,4}$=2, $E_{3,3}$=2, $E_{3,4}$=2, $E_{4,4}$=3

Theorem 1.1

The general Randic index Nirmatrelvir are 228, 6.3264, 86.5441 & 14.9383

Proof: The general Randic index Nirmatrelvir is defined as

$$R_{\nabla}(\underline{C_{23}H_{32}F_3N_5O_4}) = \sum_{ab\in E(G)}(\eth(a)\times\eth(b))^{\nabla}$$

Case (1): When $\nabla = 1$,

$R_1(\underline{C_{23}H_{32}F_3N_5O_4})$ = 2(1*3) + 10(1*4) + 4(2*2) +10(2*3)+ 2(2*4)+ 2(3*3) +2(3*4) +3(4*4)

= 228.000

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Case (2): When ∇ = -1,

$R_{-1}(\underline{C_{23}H_{32}F_3N_5O_4}) = \frac{2}{(1*3)} + \frac{10}{(1*4)} + \frac{4}{(2*2)} + \frac{10}{(2*3)} + \frac{2}{(2*4)} + \frac{2}{(3*3)} + \frac{2}{(3*4)} + \frac{3}{(4*4)}$

$= 6.3264)$

Case (3): When $\nabla = {}^{1}/_{2}$

$R_{(1/2)}(\underline{C_{23}H_{32}F_3N_5O_4}) = 2\sqrt{(1*3)} + 10\sqrt{(1*4)} + 4\sqrt{(2*2)} + 10\sqrt{(2*3)} + 2\sqrt{(2*4)} + 2\sqrt{(3*3)} + 2\sqrt{(3*4)} + 3\sqrt{(4*4)}$

$= 86.5441$

Case (4): When $\nabla = -{}^{1}/_{2}$

$R_{(-1/2)}(\underline{C_{23}H_{32}F_3N_5O_4}) = \frac{2}{\sqrt{(1*3)}} + \frac{10}{\sqrt{(1*4)}} + \frac{4}{\sqrt{(2*2)}} + \frac{10}{\sqrt{(2*3)}} + \frac{2}{\sqrt{(2*4)}} + \frac{2}{\sqrt{(3*3)}} + \frac{2}{\sqrt{(3*4)}} + \frac{3}{\sqrt{(4*4)}}$

$= 14.9383$

Theorem 1.2

The Atom bond connectivity index of Nirmatrelvir is 26.0684

Proof: The Atom bond connectivity index Nirmatrelvir is defined as

$ABC(\underline{C_{23}H_{32}F_3N_5O_4}) = \sum_{ab \in E(G)} \sqrt{\frac{\theta(a)+\theta(b)-2}{\theta(a)\times\theta(b)}}$

$= 2\sqrt{\frac{(1+3)-2}{(1*3)}} + 10\sqrt{\frac{(1+4)-2}{(1*4)}} + 4\sqrt{\frac{(2+2)-2}{(2*2)}} + 10\sqrt{\frac{(2+3)-2}{(2*3)}} + 2\sqrt{\frac{(2+4)-2}{(2*4)}} + 2\sqrt{\frac{(3+3)-2}{(3*3)}} + 2\sqrt{\frac{(3+4)-2}{(3*4)}} + 3\sqrt{\frac{(4+4)-2}{(4*4)}}$

$= 26.0684$

Theorem 1.3

The Geometric arithmetic index of Nirmatrelvir is 32.3951

Proof: The Geometric arithmetic index Nirmatrelvir is defined as

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GA($\underline{C_{23}H_{32}F_3N_5O_4}$) = $\sum_{ab\in E(G)} \frac{2\sqrt{\theta(a)\times\theta(b)}}{\theta(a)+\theta(b)}$

$= (2*2)\frac{\sqrt{(1*3)}}{(1+3)} + (10*2)\frac{\sqrt{(1*4)}}{(1+4)} + (4*2)\frac{\sqrt{(2*2)}}{(2+2)} + (10*2)\frac{\sqrt{(2*3)}}{(2+3)}$

$+ (2*2)\frac{\sqrt{(2*4)}}{(2+4)} + (2*2)\frac{\sqrt{(3*3)}}{(3+3)} + (2*2)\frac{\sqrt{(3*4)}}{(3+4)} + (3*2)\frac{\sqrt{(4*4)}}{(4+4)}$

= 32.3951.

Theorem 1.4

The Forgotten topological index of Nirmatrelvir is 574

Proof: The Forgotten topological index Nirmatrelvir is defined as

F($\underline{C_{23}H_{32}F_3N_5O_4}$) = $\sum_{ab\in E(G)} (\theta(a)^2 + \theta(b)^2)$

$= 2(1^2+3^2) + 10(1^2+4^2) + 4(2^2+2^2) + 10(2^2+3^2) + 2(2^2+4^2) + 2(3^2+3^2) + 2(3^2+4^2) + 3(4^2+4^2)$

= 574

Theorem 1.5

The Balaban index of Nirmatrelvir is 104.5691

Proof: The Balaban index of Nirmatrelvir is defined as

J($\underline{C_{23}H_{32}F_3N_5O_4}$) = $\frac{y}{y-x+2}\sum_{ab\in E(G)}\frac{1}{\sqrt{\theta(a)\times\theta(b)}}$, Where y = 35 Where x = 32

$= \frac{35}{35-32}\left[\frac{2}{\sqrt{(1*3)}} + \frac{10}{\sqrt{(1*4)}} + \frac{4}{\sqrt{(2*2)}} + \frac{10}{\sqrt{(2*3)}} + \frac{2}{\sqrt{(2*4)}} + \frac{2}{\sqrt{(3*3)}} + \frac{2}{\sqrt{(3*4)}} + \frac{3}{\sqrt{(4*4)}}\right]$

= 104.5681

Theorem 1.6

The symmetric division index of Nirmatrelvir is 113.6667

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Proof: The symmetric division index of Nirmatrelvir is defined as

$SDI(\underline{C_{23}H_{32}F_3N_5O_4}) = \sum_{ab\in E(G)} \frac{\vartheta(a)^2+\vartheta(b)^2}{\vartheta(a)\times\vartheta(b)}$

$= 2\left[\frac{(1^2+3^2)}{(1*3)}\right] + 10\left[\frac{(1^2+4^2)}{(1*4)}\right] + 4\left[\frac{(2^2+2^2)}{(2*2)}\right] + 10\left[\frac{(2^2+3^2)}{(2*3)}\right] + 2\left[\frac{(2^2+4^2)}{(2*4)}\right]$

$+2\left[\frac{(3^2+3^2)}{(3*3)}\right] + 2\left[\frac{(3^2+4^2)}{(3*4)}\right] + 3\left[\frac{(4^2+4^2)}{(4*4)}\right]$

$= 113.6667.$

Theorem 1.7

The reduced reciprocal Randic index of Nirmatrelvir is 39.5052

Proof: The reduced reciprocal Randic index of Nirmatrelvir is defined as

$RRR(\underline{C_{23}H_{32}F_3N_5O_4}) = \sum_{ab\in E(G)} \sqrt{(\vartheta(a)-1)(\vartheta(b)-1)}$

$= 2\sqrt{(1-1)(3-1)} + 10\sqrt{(1-1)(4-1)} + 4\sqrt{(2-1)(2-1)} + 10\sqrt{(2-1)(3-1)} + 2\sqrt{(2-1)(4-1)} + 2\sqrt{(3-1)(3-1)} + 2\sqrt{(4-1)(2-1)} + 3\sqrt{(4-1)(4-1)}$

$= 39.5052$

Theorem 1.8

The reduced second Zagreb index of Nirmatrelvir is 77

Proof: The reduced second Zagreb index of Nirmatrelvir is defined as

$RM_2(\underline{C_{23}H_{32}F_3N_5O_4}) = \sum_{ab\in E(G)} (\vartheta(a)-1)(\vartheta(b)-1)$

$= 2[(1\text{-}1)\ (3\text{-}1)] + 10[(1\text{-}1)\ (4\text{-}1)] + 4[(2\text{-}1)\ (2\text{-}1)] + 10[(2\text{-}1)\ (3\text{-}1)] + 2[(2\text{-}1)\ (4\text{-}1)] + 2[(3\text{-}1)\ (3\text{-}1)] + 2[(3\text{-}1)\ (4\text{-}1)] + 3[(4\text{-}1)\ (4\text{-}1)]$

$= 77.$

Result for Ritronavir:

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Fig 2: Ritronavir

$E_{1,3}$=6, $E_{2,2}$=17, $E_{2,3}$=24, $E_{3,3}$=3

Theorem 2.1

The general Randic index Ritronavir are 257, 10.5833, 112.1800, 22.7620

Proof: The general Randic index Ritronavir is defined as

$R_{\nabla}(\underline{C_{37}H_{48}N_6O_5S_2}) = \sum_{ab \in E(G)} (\theta(a) \times \theta(b))^{\nabla}$

Case (1): When $\nabla = 1$,

$R_{(1)}(\underline{C_{37}H_{48}N_6O_5S_2}) = 6(1*3) + 17(2*2) + 24(2*3) + 3(3*3)$

$= 257.000$

Case (2): When $\nabla = -1$,

$R_{(-1)}(\underline{C_{37}H_{48}N_6O_5S_2}) = \frac{6}{(1*3)} + \frac{17}{(2*2)} + \frac{24}{(2*3)} + \frac{3}{(3*3)}$

$= 10.5833$

Case (3): When $\nabla = {}^{1}/_{2}$

$R_{(1/2)}(\underline{C_{37}H_{48}N_6O_5S_2}) = 6\sqrt{(1*3)} + 17\sqrt{(2*2)} + 24\sqrt{(2*3)} + 3\sqrt{(3*3)}$

$= 112.1800$

Case (4): When $\nabla = -{}^{1}/_{2}$, which is called the degree-based index

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$R_{(-1/2)}(\underline{C_{37}H_{48}N_6O_5S_2}) = \frac{6}{\sqrt{(1*3)}} + \frac{17}{\sqrt{(2*2)}} + \frac{24}{\sqrt{(2*3)}} + \frac{3}{\sqrt{(3*3)}}$

$= 22.7620$

Theorem 2.2

The Atom bond connectivity index of Ritronavir is 35.890

Proof: The Atom bond connectivity index Ritronavir is defined as

$ABC(\underline{C_{37}H_{48}N_6O_5S_2}) = \sum_{ab\in E(G)} \sqrt{\frac{\theta(a)+\theta(b)-2}{\theta(a)\times\theta(b)}}$

$= 6\sqrt{\frac{(1+3)-2}{(1*3)}} + 17\sqrt{\frac{(2+2)-2}{(2*2)}} + 24\sqrt{\frac{(2+3)-2}{(2*3)}} + 3\sqrt{\frac{(3+3)-2}{(3*3)}}$

$= 35.890$

Theorem 2.3

The Geometric arithmetic index of Ritronavir is 48.7112

Proof: The Geometric arithmetic index Ritronavir is defined as

$GA(\underline{C_{37}H_{48}N_6O_5S_2}) = \sum_{ab\in E(G)} \frac{2\sqrt{\theta(a)\times\theta(b)}}{\theta(a)+\theta(b)}$

$= (6*2)\frac{\sqrt{(1*3)}}{(1+3)} + (17*2)\frac{\sqrt{(2*2)}}{(2+2)} + (24*2)\frac{\sqrt{(2*3)}}{(2+3)} + (3*2)\frac{\sqrt{(3*3)}}{(3+3)}$

$= 48.7112$

Theorem 2.4

The Forgotten topological index of Ritronavir is 562

Proof: The Forgotten topological index Ritronavir is defined as

$F(\underline{C_{37}H_{48}N_6O_5S_2}) = \sum_{ab\in E(G)} (\theta(a)^2 + \theta(b)^2)$

$= 6(1^2+3^2) + 17(2^2+2^2) + 24(2^2+3^2) + 3(3^2+3^2)$

$= 562$

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Theorem 2.5

The Balaban index of Ritronavir is 189.683

Proof: The Balaban index of Ritronavir is defined as

$J(\underline{C_{37}H_{48}N_6O_5S_2}) = \frac{y}{y-x+2}\sum_{ab\in E(G)}\frac{1}{\sqrt{\theta(a)\times\theta(b)}}$, Where y = 50, Where x = 46

$$= \frac{50}{50-46+2}\left[\frac{6}{\sqrt{(1*3)}} + \frac{17}{\sqrt{(2*2)}} + \frac{24}{\sqrt{(2*3)}} + \frac{3}{\sqrt{(3*3)}}\right]$$

$$= 78.3333[22.7620]$$

$$= 189.683$$

Theorem 2.6

The symmetric division index of Ritronavir is 700.333

Proof: The symmetric division index of Ritronavir is defined as

$SDI(\underline{C_{37}H_{48}N_6O_5S_2}) = \sum_{ab\in E(G)}\frac{\theta(a)^2+\theta(b)^2}{\theta(a)\times\theta(b)}$

$$= 6\left[\frac{(1^2+3^2)}{(1*3)}\right] + 17\left[\frac{(2^2+2^2)}{(2*2)}\right] + 24\left[\frac{(2^2+3^2)}{(2*3)}\right] + 3\left[\frac{(3^2+3^2)}{(3*3)}\right]$$

$$= 700.333.$$

Theorem 2.7

The reduced reciprocal Randic index of Ritronavir is 56.9411

Proof: The reduced reciprocal Randic index of Ritronavir is defined as

$RRR(\underline{C_{37}H_{48}N_6O_5S_2}) = \sum_{ab\in E(G)}\sqrt{(\theta(a)-1)(\theta(b)-1)}$

$$= 6\sqrt{(1-1)(3-1)} + 17\sqrt{(2-1)(2-1)} + 124\sqrt{(2-1)(3-1)}$$

$$+ 3\sqrt{(3-1)(3-1)}$$

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= 56.9411.

Theorem 2.8

The reduced second Zagreb index of Ritronavir is 77

Proof: The reduced second Zagreb index of Ritronavir is defined as

$RM_2(\underline{C_{37}}\underline{H_{48}}\underline{N_6}\underline{O_5}\underline{S_2}) = \sum_{ab\in E(G)}(\theta(a)-1)(\theta(b)-1)$

= 6[(1-1) (3-1)] + 17[(2-1) (2-1)] + 24[(2-1) (3-1)] + 3[(3-1) (3-1)

= 77.

Applications:

Drug Structure Analysis: By comprehending Paxlovid's structural characteristics, the calculated topological indices can help with drug creation, optimisation, and modification for increased efficacy. Comparative Molecular Studies: By comparing Paxlovid to other antiviral medications, the measured indices enable researchers to examine molecular attributes for similarities, differences, and possible improvements. Pharmacokinetics and Drug creation: By forecasting the biological activity, stability, and molecular interactions of Paxlovid, the study's results can aid in the creation of new pharmaceuticals.

Conclusion:

In this study, we computed different degree-based topological indices for the medication Paxlovid, which is used to treat COVID-19 and is made up of Ritonavir and Nirmatrelvir. Paxlovid's molecular structure can be better understood by using the calculated indices, which include the General Randic Index, Atom-Bond Connectivity Index, Geometric Arithmetic Index, Forgotten Index, Balaban Index, Symmetric Division Index, Reduced Reciprocal Randic Index, and Reduced Second Zagreb Index. Understanding molecular characteristics, forecasting biological activity, and contrasting the structural features of various medications all depend on these indices. The findings of this work support the analysis and possible development of antiviral drugs, contributing to the expanding fields of computational chemistry and drug design.

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Applications of some topological indices for Raloxifene drug

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Abstract:

The drug Raloxifene has been widely used in the prevention and treatment of osteoporosis and breast cancer in postmenopausal disease for women. In this paper we focused on the calculation of degree based topological indices of first and second Zagreb Indices, Atom-Bond connectivity Index, Hyper Zagreb Index, updated second Zagreb Index and Max-Min Rodeg Index. These indices are effective tools for analysing and understanding the structure of molecules. Also, we computed some polynomials for some index drawn 3-dimensional graph using Python software.

Keywords:

Zagreb Indices, Atom-Bond connectivity Index, Hyper Zagreb Index, updated second Zagreb Index and Max-Min Rodeg Index.

Introduction:

A branch of discrete mathematics and graph theory was initially presented by Euler in 1736. Numerous disciplines, including physics, biology, computer science, and chemistry, have looked into it [2]. Graph theory and mathematical modelling are combined in chemical graph theory. It focuses on topological indices that have a close relationship with molecular or chemical compound characteristics. Topological indices are frequently used in quantitative structure–property/structure–activity relationship (QSPR/QSAR) modelling to forecast the physicochemical and bioactivity characteristics of a molecule [3]. The topological structure of a molecule or chemical compound can be accurately described by its topological index [4]. In 1947, the Wiener index—the first topological index known to exist—was employed to ascertain the physical properties of petroleum [5]. The vertices of the molecule are in for its

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atoms. The edges indicate the chemical bonds between the atoms. Role of topological indices in predictive modelling and raking of drugs treating eye disorder were recently calculated by Nazeran etl. [1], this motived us to calculate topological indices for Raloxifene drug.

Preliminaries:

Topological Index	**Formula**
First Zagreb Index	$M_1(N) = \sum_{a,b \in E} (h_a + h_b)$
Second Zagreb Index	$M_2(N) = \sum_{a,b \in E} (h_a h_b)$
Atom-Bond connectivity Index	$ABC(N) = \sum_{a,b \in E} \sqrt{\frac{h_a + h_b - 2}{h_a h_b}}$
Hyper Zagreb Index	$HM(N) = \sum_{a,b \in E} (h_a + h_b)^2$
Updated second Zagreb Index	$ReZG_2(N) = \sum_{a,b \in E} \frac{h_a h_b}{h_a + h_b}$
Max-Min Rodeg Index	$mMsde(N) = \sum_{a,b \in E} \sqrt{\frac{max\{h_a, h_b\}}{min\{h_a, h_b\}}}$

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Main result:

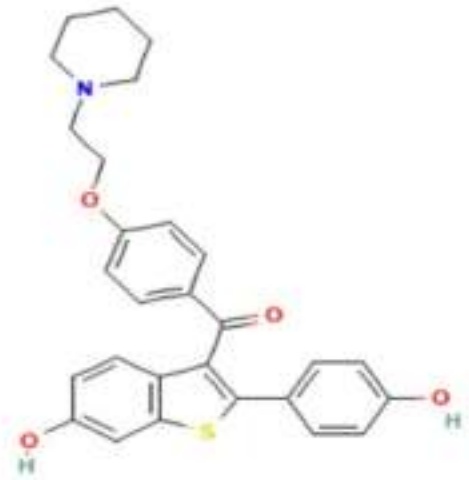

Fig 1: Raloxifene

Theorem 1.1: The first Zagreb Index of Raloxifene is 176.

Proof: The first Zagreb Index for Raloxifene is defined [6] as,

$M_1(C_{28}H_{27}NO_4S) = \sum_{a,b\in E} (h_a + h_b)$

$= 2(1+3) + 11(2+2) + 20(2+3) + 4(3+3)$

$= 176$

Theorem 1.2: The first Zagreb polynomial of Raloxifene is $13x^4 + 20x^5 + 4x^6$.

Proof: $M_1(C_{28}H_{27}NO_4S) = \sum_{a,b\in E} x^{(h_a + h_b)}$

$= 2x^{(1+3)} + 11x^{(2+2)} + 20x^{(2+3)} + 4x^{(3+3)}$

$= 13x^4 + 20x^5 + 4x^6$

Theorem 2.1: The second Zagreb Index of Raloxifene is 206.

Proof: The second Zagreb Index for Raloxifene is defined as,

$M_2(C_{28}H_{27}NO_4S) = \sum_{a,b\in E} (h_a h_b)$

$= 2(3) + 11(4) + 20(6) + 4(9)$

$= 206$

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Theorem 2.2: The second Zagreb polynomial of Raloxifene is $2x^3 + 11x^4 + 20x^6 + 4x^9$.

Proof: $M_2(C_{28}H_{27}NO_4S) = \sum_{a,b\in E} x^{(h_a h_b)}$

$= 2x^{(1*3)} + 11x^{(2*2)} + 20x^{(2*3)} + 4x^{(3*3)}$

$= 2x^3 + 11x^4 + 20x^6 + 4x^9$

Theorem 3.1: The Atom-Bond connectivity Index of Raloxifene is 26.2200.

Proof: The Atom-Bond connectivity Index for Raloxifene [7] is defined as,

$ABC(C_{28}H_{27}NO_4S) = \sum_{a,b\in E} \sqrt{\frac{h_a + h_b - 2}{h_a h_b}}$

$= 2\sqrt{\frac{1+3-2}{3}} + 11\sqrt{\frac{2+2-2}{4}} + 20\sqrt{\frac{2+3-2}{6}} + 4\sqrt{\frac{3+3-2}{9}}$

$= 26.2200$

Theorem 3.2: The Atom-Bond connectivity polynomial of Raloxifene is $4x^{\left(\sqrt{\frac{4}{9}}\right)} + 31x^{\left(\sqrt{\frac{1}{2}}\right)} + 2x^{\left(\sqrt{\frac{2}{3}}\right)}$.

Proof: $ABC(C_{28}H_{27}NO_4S) = \sum_{a,b\in E} x^{\left(\sqrt{\frac{h_a + h_b - 2}{h_a h_b}}\right)}$

$= 2x^{\left(\sqrt{\frac{1+3-2}{3}}\right)} + 11x^{\left(\sqrt{\frac{2+2-2}{4}}\right)} + 20x^{\left(\sqrt{\frac{2+3-2}{6}}\right)} + 11x^{\left(\sqrt{\frac{3+3-2}{9}}\right)}$

$= 4x^{\left(\sqrt{\frac{4}{9}}\right)} + 31x^{\left(\sqrt{\frac{1}{2}}\right)} + 2x^{\left(\sqrt{\frac{2}{3}}\right)}$

Theorem 4.1: The Hyper Zagreb Index of Raloxifene is 852.

Proof: The Hyper Zagreb Index for Raloxifene [8] is defined as,

$HM(C_{28}H_{27}NO_4S) = \sum_{a,b\in E} (h_a + h_b)^2$

$= 2(1+3)^2 + 11(2+2)^2 + 20(2+3)^2 + 4(3+3)^2$

$= 852$

Theorem 4.2: The Hyper Zagreb polynomial of Raloxifene is $13x^{16} + 20x^{25} + 4x^{36}$.

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Proof: HM ($C_{28}H_{27}NO_4S$) = $\sum_{a,b\in E} x^{(h_a+h_b)^2}$

$= 2x^{(1+3)^2} + 11x^{(2+2)^2} + 20x^{(2+3)^2} + 4x^{(3+3)^2}$

$= 13x^{16} + 20x^{25} + 4x^{36}$

Theorem 5.1: The updated second Zagreb Index of Raloxifene is 42.5.

Proof: The updated second Zagreb Index for Raloxifene [9] is defined as,

$ReZG_2$ ($C_{28}H_{27}NO_4S$) = $\sum_{a,b\in E} \frac{h_a h_b}{h_a + h_b}$

$= 2\left(\frac{3}{4}\right) + 11\left(\frac{4}{4}\right) + 20\left(\frac{6}{5}\right) + 4\left(\frac{9}{6}\right)$

$= 42.5.$

Theorem 5.2: The updated second Zagreb polynomial of Raloxifene is $2x^{\left(\frac{3}{4}\right)}$ + 11x + $20x^{\left(\frac{6}{5}\right)}$ + $4x^{\left(\frac{3}{2}\right)}$.

Proof: $ReZG_2$ ($C_{28}H_{27}NO_4S$) = $\sum_{a,b\in E} x^{\left(\frac{h_a h_b}{h_a + h_b}\right)}$

$= 2x^{\left(\frac{3}{4}\right)} + 11x^{\left(\frac{4}{4}\right)} + 20x^{\left(\frac{6}{5}\right)} + 4x^{\left(\frac{9}{6}\right)}$

$= 2x^{\left(\frac{3}{4}\right)} + 11x + 20x^{\left(\frac{6}{5}\right)} + 4x^{\left(\frac{3}{2}\right)}$

Theorem 6.1: The Max-Min Rodeg Index of Raloxifene is 42.9590.

Proof: The Max-Min Rodeg Index for Raloxifene [10] is defined as,

mMsde ($C_{28}H_{27}NO_4S$) = $\sum_{a,b\in E} \sqrt{\frac{max\{h_a,h_b\}}{min\{h_a,h_b\}}}$

$= 2\sqrt{\frac{max\{1,3\}}{min\{1,3\}}} + 11\sqrt{\frac{max\{2,2\}}{min\{2,2\}}} + 20\sqrt{\frac{max\{2,3\}}{min\{2,3\}}} + 4\sqrt{\frac{max\{3,3\}}{min\{3,3\}}}$

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$$= 2\sqrt{\frac{3}{1}} + 11\sqrt{\frac{2}{2}} + 20\sqrt{\frac{3}{2}} + 4\sqrt{\frac{3}{3}}$$

$$= 42.9590.$$

Theorem 6.2: The Max-Min Rodeg polynomial of Raloxifene is $15x + 20x^{\left(\sqrt{\frac{3}{2}}\right)} + 2x^{(\sqrt{3})}$.

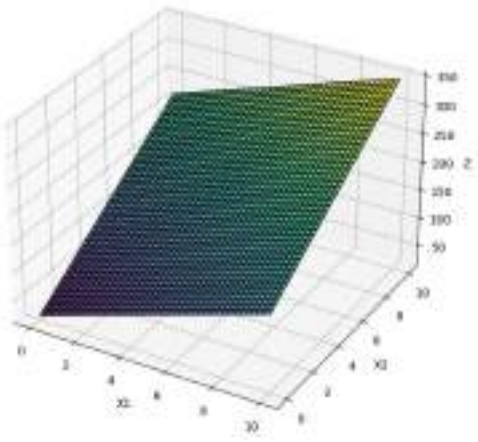
Proof: mMsde ($C_{28}H_{27}NO_4S$) = $\sum_{a,b\in E} x^{\left(\sqrt{\frac{max\{h_a,h_b\}}{min\{h_a,h_b\}}}\right)}$

$$= 2x^{\left(\sqrt{\frac{max\{1,3\}}{min\{1,3\}}}\right)} + 11x^{\left(\sqrt{\frac{max\{2,2\}}{min\{2,2\}}}\right)} + 20x^{\left(\sqrt{\frac{max\{2,3\}}{min\{2,3\}}}\right)} + 11x^{\left(\sqrt{\frac{max\{3,3\}}{min\{3,3\}}}\right)}$$

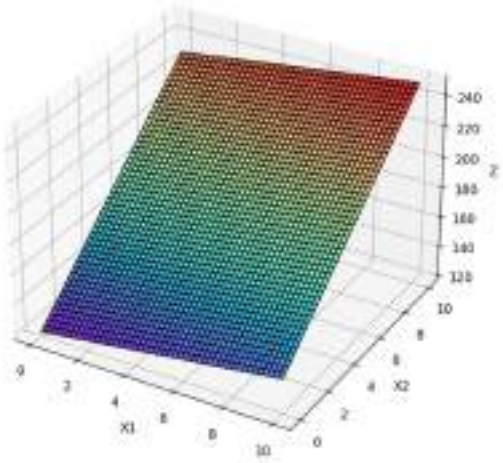
$$= 2x^{\left(\sqrt{\frac{3}{1}}\right)} + 11x^{\left(\sqrt{\frac{2}{2}}\right)} + 20x^{\left(\sqrt{\frac{3}{2}}\right)} + 11x^{\left(\sqrt{\frac{3}{3}}\right)}$$

$$= 15x + 20x^{\left(\sqrt{\frac{3}{2}}\right)} + 2x^{(\sqrt{3})}.$$

Three-Dimensional mesh graphs for polynomial equations:



$13x^4 + 20x^5 + 4x^6$



$2x^3 + 11x^4 + 20x^6 + 4x^9$

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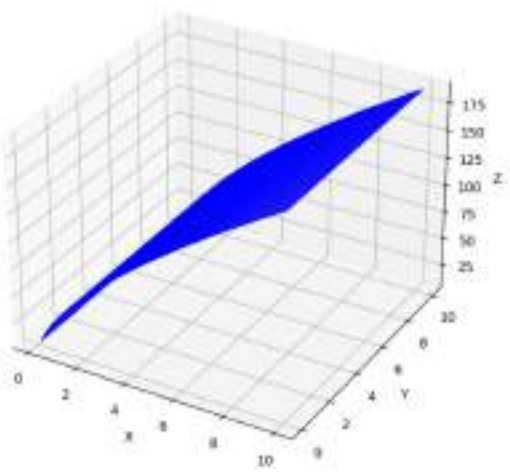
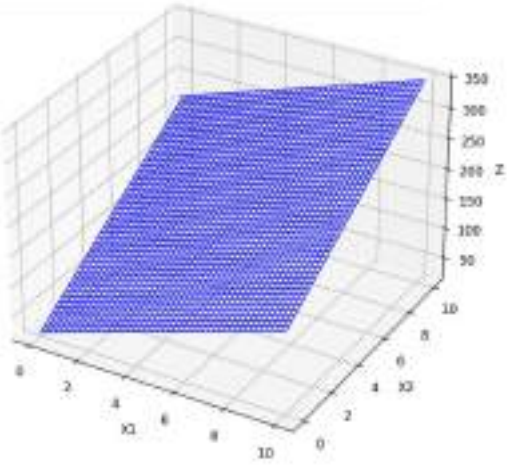
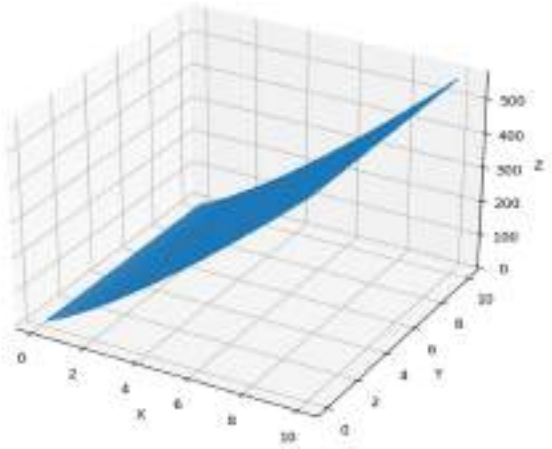
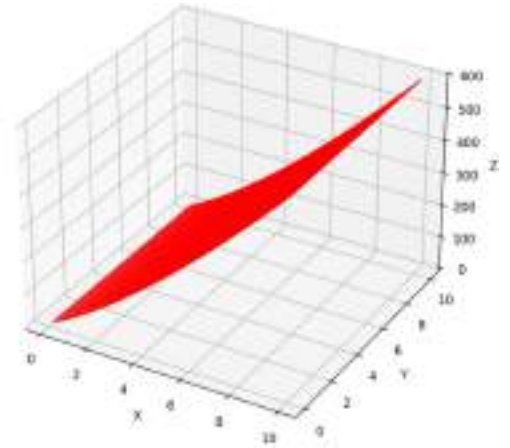


$$4x^{\left(\sqrt{\frac{4}{9}}\right)} + 31x^{\left(\sqrt{\frac{1}{2}}\right)} + 2x^{\left(\sqrt{\frac{2}{3}}\right)}$$



$$13x^{16} + 20x^{25} + 4x^{36}$$





$$2x^{\left(\frac{3}{4}\right)} + 11x + 20x^{\left(\frac{6}{5}\right)} + 4x^{\left(\frac{3}{2}\right)}$$

$$15x + 20x^{\left(\sqrt{\frac{3}{2}}\right)} + 2x^{(\sqrt{3})}$$

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Applications:

Raloxifene's chemical and physical characteristics are better understood. Thanks to the calculated topological indices and polynomial properties, which aid in pharmacokinetic research, drug design optimisation, and QSAR/QSPR modelling. These discoveries advance our knowledge of molecular interactions and help scientists create novel medication analogues that are safer and more effective. Additionally, the efficiency of structural analysis in cheminformatics and bioinformatics is improved by the use of computational tools. Chemists can examine molecule stability and bonding properties with the use of a three-dimensional surface or contour map of the Zagreb polynomial values. By comparing topological indicators, it aids in the prediction of biological activity and interaction potential. It helps in drug interaction predictions, ensuring better formulation and receptor binding efficiency. Mesh graphs, also known as wireframe plots, are frequently used in many different domains to visualise mathematical functions, surfaces, and 3D data.

Conclusion:

Using degree-based topological indices, such as the Atom-Bond Connectivity (ABC) Index, Hyper Zagreb Index, updated second Zagreb Index, Max-Min Rodeg Index, and the first and second Zagreb indices, we examined the molecular structure of Raloxifene in this study. These indices are essential resources for comprehending the drug's physical and chemical characteristics. Furthermore, we used Python software to visualise a three-dimensional mesh molecular graph and compute pertinent polynomials, improving structural analysis through computational techniques.

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STRONG CO-SECURE DOMINATION NUMBER OF SILICATES FAMILY CHEMICAL GRAPH STRUCTURES

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Abstract

A graph $G = (V, E)$ is finite, connected and the set $D \subseteq V$ is *dominating set,* if all vertices V in D guards remaining vertices $V - D$ of G with minimum cardinality. A set $D \subseteq V(G)$ is said to be *strong dominating set* if *e* is an edge with end vertices *u* and *v* and degree of *u* is greater than or equal to degree of *v*, then *u strongly dominates v.* If every vertex of V - D is strongly dominated by some vertex of D, then D is strong dominating set. A graph G is *strong co - secure dominating set* $SCSD$, if for each $u \in D$ there exists a vertex $v \in V - D$ such that $v \in N(u)$, $uv \in E(G) and\ (D \setminus \{u\} \cup \{v\})$ is a dominating set of G with minimum cardinality. The chemical graphs here used are organized from silicate family where nodes are ions and bonds are edges. Various uses are invented when silicates combine with many other materials or chemicals.

2D view of silicates considered in general and the strong co - secure domination number of chain silicate and cyclic silicate is studied in this paper.

Keywords: Domination, Strong Domination, Strong co - secure Domination, Chain and Cyclic silicates.

AMS Subject Code: 05C72, 05C69

1. Introduction

All the graphs considered in this paper are finite, non trivial, connected, undirected and constructed in the model of chemical structure of silicates types, where silicon ions and oxygen ions are nodes $V(G)$ and bonds between these ions are edges $E(G)$. The silicates are most complicated and randomly used materials in regular use. The style of its chemical structure is resemble of some phase

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arrangements, basic connections on databases, and the extension of its network has wide range of applications in planting PMUs. Ions act as defenders, processors and the link between ions act as connectors, bondages. Graphical model of chemical structure of Chain silicates $CS(N)$ and cyclic silicate $CC(N)$ has Power domination number $\left\lceil \frac{N}{2} \right\rceil$ [7]. Graphs and digraphs by Chartrand G, Lesniak L in 2004 determined the terminology of graphs in theoretic way. Dominating set of G, $S \subseteq V$ of plenty of graphs has been extensively studied in enormous amount of papers and basics and bounds and rules are detailed in [1][2]. The minimum cardinality $\gamma(G)$ is the representation of domination number.

A set S of vertices is *independent* if no two vertices in S are adjacent. Roman domination in article entitled "Defend the Roman Empire!"[10] explained the work of Guards in a set. Strategy for protection of a graph G by placing one or more guards at every vertex of a subset S of V is detailed in [2]. A guard *v* can protect any vertex in its closed neighborhood. Severe topics emerge from this concept and raises to Roman domination, secure domination, co-secure domination and many more. A guard needs to be substituted by another when the guard get attacked leads to process the definition of secure domination and co - secure domination [3].

In paper [8], the strong co-secure domination number of basic graphs are investigated. In paper [7] Power domination of silicate family is discussed. The chemical structure of silicate designed in graphs is peculiar than other graphs that stimulate to protect several neighbors with very few guards.

Various silicate chemical structures when combined with many chemical structures obtained the solid materials such as cement, ceramic, glass, detergents, paper, water treatment etc. In this paper the chemical structure of silicates are considered as graphs where oxygen ions and silicones as vertices and their bonds as edges. Using chemical structure as graph models, we can connect or protect many neighborhoods from attackers by few defenders. The strong co-secure dominating sets of Chain silicates and Cyclic silicates are investigated here.

The basic structural unit of all silicate minerals is the silicon tetrahedron in which one silicon atom is surrounded by and bonded to four oxygen atoms, each at the corner of a regular tetrahedron. All silicates contain SiO_4 tetrahedron, but sharing the silicon ions and oxygen ions are different. The chemical formula contains multiple of SiO_4 such as Zircon, Topaz, Olivine and many more. Various types of silicates such as Nesosilicates, Sorosilicates, Cyclic or ring silicates, Chain silicates, Double chain silicates, Sheet silicates, Tectosilicates.

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2. Basic silicate family strong co-secure dominating set

Proposition 2.1: The minimum defenders required for protecting Orthosilicates is one and for Pyrosilicates is two.

Proposition 2.2: The strong domination number and the strong co-secure domination number of Ring silicate denoted by RS with three tetrahedrons is equal.

Proof: The corner vertices are oxygen ions sharing 2 oxygen ions for each and center vertices are silicon ions. Some oxygen ions have degree 6 and all remaining oxygen ions and silicon ions of degree 3.

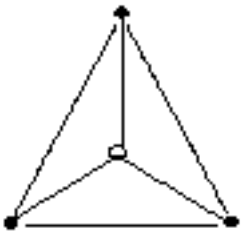

Fig 1 – Orthosilicates (SiO_4^{4-}) , Fig 2 – Pyrosilicates ($Si_2O_7^{6-}$)

The possible strong dominating sets of this ring silicate are $D = \{2,3\},\{2,5\},\{3,5\}$. Thus the domination number is 2.

2.1 Strong co - secure domination of Chain Silicate structure

The single and double chain silicates are collectively known as Inosilicates denoted as $CS(N)$. The number of vertices of $CS(N)$ is $3N + 1$ and the number of edges is $6N, N = 1,2,3,\ldots$ The structure of chain silicates resembles the double triangular graph structure with special case of oxygen ions in center and extension of chain silicates is addition of tetrahedrons in only alternate manner and not both side.

The domination number of chain silicates is $\gamma(CS(N)) = \left\lceil \frac{N}{2} \right\rceil$ [7]. The domination number is the strong domination number of chain silicates ie)$\gamma = \gamma_s$. Through strong dominating set we can

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generate co-secure dominating set of chain silicates. Note that we need not consider the center silicates for replacing oxygen ions as these are of least degree but these should be protected from attackers. So choosing co - secure dominating set is also based on silicates.

Theorem 2.1.1:

For odd N of $CS(N), N = 3N$ and $3N - 1$ with $3N + 1$ nodes leaving $3N + 1$ chain silicates, the strong co - secure domination number is $2N$ where $N = 1,2,3..$

Proof: The strong domination number is $\left\lceil \frac{N}{2} \right\rceil$ for all N.

Start from the last pyrosilicates which is single in the path of chemical structure joined in chain. Obviously the strong node $(N - 2)$ is presented in strong dominating set. Proceeding from this, remaining sets of pyrosilicates with dominating nodes is either even or odd. Consider the paired tetrahedron as odd N has $N - 1$ pyrosilicates with $\frac{N-1}{2}$ domination number.

Claim 1: Dominating set along with last tetrahedron failed to satisfy the strong co - secure domination number with minimum cardinality.

Dominating set is distinctly the nodes of the center of each pyrosilicates.

Each of these nodes has exactly 6 neighbors. Replacement process is by rearranging attacked node by any one of these neighbors.

Unfortunately this process failed to satisfy the domination property to secure remaining neighbors. Hence strong dominating set is not strong co - secure dominating set.

Claim 2: Strong dependent dominating nodes only selected.

If not, let D' be the set larger than D and some of nodes in D' is independent. As D' definitely contains the last tetrahedron strong node and let it be v_{n-2} selected and independent. So if this node is destroyed by attackers, suddenly any one of the neighbors of v_{n-2} is used to replace.

Let it be v_n. This node not securing the node v_{n-4} as it is secured by v_{n-2}. Let consider another neighbor node v_{n-4} which is independent. v_n now not secured by any other nodes. Similar neighbors of v_{n-2} results the same.

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Hence independent dominating vertex in D' is impossible to secure the whole graph. Hence all nodes in D' is dependent.

The first node in D' is v_{n-2}. Form above claim, strong dependent node of v_{n-2} is placed next. ie) v_{n-4}.

The next pair is $v_{n-8}\ and\ v_{n-10}$ leaving v_{n-6} from D' to minimize the securities. Proceeding this way, $D' = \{v_{n-2}, v_{n-4}, v_{n-8}, v_{n-10,} \ldots\ldots v_5,\ v_3\}$.

Note that the origin node v_1 is not in D' but being protected by v_3 vertex of the chain. From this consequence, D' is dependent and even nodes are enough to secure the remaining vertices including oxygen ions. Thus the D' examined with minimum cardinality when replaced by its neighbors uninterrupted the domination property to protect $V - D$'.

Hence, odd chain silicates of $N = 3N$ and $3N - 1$, $N = 1,2,3 \ldots$ with $3N + 1$ vertices has strong co - secure domination number $2N$ with minimum cardinality.

Theorem 2.1.2: The strong co - secure domination number is $2N$ for even N of $3N$ and $3N - 1$ with $3N + 1$ vertices where $N = 1, 2, \ldots$

Proof:

The chemical structure of these chain silicates is all pyrosilicates is strong bonded with no single tetrahedron in chain path. Number of pairs is either odd or even. Consequently, the domination number is equal to number of pairs of tetrahedron. So the domination number is $\frac{N}{2}$. Strong co - secure dominating set D' is without N^{th} node as it is of least degree. Proceeding the next step similar to previous theorem.

ie) $D' = \{ v_{n-2}, v_{n-4}, v_{n-8}, v_{n-10,} \ldots\ldots v_3,\ v_2\}$. Dependent co - secure domination arrangements are followed. Any of the neighbors when restored in place of attacked node satisfies the domination process also.

Hence the strong co-secure domination number is $2N$ for even N of $3N$ and $3N - 1$.

Theorem 2.1.3: For Chain silicates N of $3N + 1$, dependent dominating vertices $2N + 1$ co -secured the whole vertices with minimum cardinality where $N = 0,1,2 \ldots$

Proof: The chain silicates of $N = 3N + 1$, has $3(2N + 1)$ vertices where $N = 0,1,2 \ldots$ excluding the silicon ions in center but to be protected.

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It is enough to prove the strong co - secure domination number of Chain silicates is one by three of the total vertices. ie) $\gamma_{scsd}\big(CS(N)\big) = 2N + 1. for\ N = 0,1,2 \ldots$

We are determining the securities of these N by two cases based on tetrahedrons in it.

Case 1: let N be even. ie) $3N + 1$ is odd.

The pyrosilicates arranged in this $CS(N)$ is resemble of even chemical structure of $CS(N)$ of previous theorem where no single tetrahedron is single in the path of the chain. The strong dominating set is not varied as each pair has one strong node.

Now to enlarge this D as D' with some more nodes to satisfy minimum cardinality of strong co - secure dominating set.

Let starts with $N - 2^{nd}$ node. Clearly $N - 4^{th}$node is selected to D' based on dependent factor.

For the contrary, if any one of the D' node is independent, let the node be $N - 2$. Then if it get attacked, then n^{th}node or $N - 1^{th}$ node failed to satisfy the strong domination property when replaced. Hence D' nodes are dependent.

Now continue the search on nodes of D'. Neglect $N - 6^{th}$ node to minimize the securities. Then next two nodes selected on the path are $N - 8$ and $N - 10$.

Continuing in this procedure,

$D' = \{v_{N-2}, v_{N-4}, v_{N-8}, v_{N-10}, \ldots\ldots v_5, v_3\}$ is picked out. Now this D' carried atmost one pyrosilicates from protection.

Enumerate atmost one strong node to any one of these paired dominating nodes results in cover of all pyrosilicates with least number of defenders.

ie) $D' = \{v_{n-2}, v_{n-4}, v_{n-6}, v_{n-8}, v_{n-10,} \ldots\ldots v_5, v_3\}$ or

$D' = \{v_{n-2}, v_{n-4}, v_{n-8}, v_{n-10,} v_{n-12} \ldots\ldots v_5, v_3\}$ or

$D' = \{v_{n-2}, v_{n-4}, v_{n-8}, v_{n-10,} \ldots\ldots v_{11,} v_{9,} v_7, v_5, v_3\}$.

These are some possible ways to obtain minimum strong co-secure dominating set for even N of $3N + 1$.

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Hence the minimum cardinality of strong co - secure dominating number of $CS(N)$ of $3N + 1$ is $2N + 1$ where N is odd.

Case 2: let N be odd. ie) N is even

The pyrosilicates arranged in this $CS(N)$ is resemble of odd chain silicates of previous chemical structure with one tetrahedron singled in the path of $CS(N)$. The domination number of this case is sum of atmost one node in addition to half of the n total vertices. Relying on nodes in D as $D' = \{v_{n-2}, v_{n-4}, v_{n-8}\ v_{n-10}, \ldots\ v_{11}, v_9, v_7, v_5, v_3\}$.

Clearly from all these procedure, atmost one pair of nodes in D' has one more node to satisfy the domination property of the co - secure set with minimum cardinality. Hence $\gamma_{scsd}(CS(N)) = 2N + 1$ for odd N where N is even.

2.2 Strong co-secure domination of Cyclic Silicate structure

A cyclic silicate denoted by $CC(N)$ is obtained by connecting *N* tetrahedra into a cyclic structure. The number of vertices and edges of $CC(N)$, $N \geq 3$ is $3N$ where n is number of tetrahedrons in it. Each tetrahedron has 4 vertices including silicon ions in center. The center vertex is not considered in defender list but to be protected by defenders compulsorily. Chain silicate chemical structure and cyclic silicate chemical structure is same in addition of tetrahedron during extension and indifferent in end node. The interesting fact is the results of finding defenders for both silicates is also same but vary in their claims and selection of vertices.

Theorem 2.2.1: Cyclic formation of chain silicate without alternate opposite tetrahedron of $N = 3N + 1, N = 1,2,3 \ldots$ is protected by strong dominating vertices with minimum cardinality $2N + 1$.

Proof: Cyclic silicate $CC\ (N)$ starts the cycle with 4 vertices of tetrahedron and 3 times N tetrahedrons with 2 more vertices in last tetrahedron. Let D' denote co-secure dominating set.

To claim,

(i) D' is dependent,

(ii) (ii) $D\ (N) < D'\ (N)$,

(iii) Minimum cardinality is $2n + 1$.

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The strong dominating set D of both even and odd N is independent and its minimum cardinality is $\frac{N}{2}\ and\ \left\lceil\frac{N}{2}\right\rceil$.

On the contrary, let D' be independent.

Consider the strong node $v_1\ and\ v_3$ in D'.

Neighbors of these nodes are $N\ (v_1) = \{\ v_2,\ v_n\ \}$ ∪ remaining 4 vertices in first and last tetrahedron with least degree and $N\ (\ v_3) = \{v_2\ , v_4\ \}$ ∪ remaining 4 vertices in 2nd and 4th tetrahedron with least degree. $\{\ v_2,\ v_4\ ,\ v_n\ \}$ are not in D'.

Now $D'\ /\ \{\ v_1\ \}$ ∪ $\{\ v_2\ \}$ and $D'\ /\ \{\ v_1\ \}$ ∪ $\{\ v_n\ \}$ is not a dominating set. Also all least degree nodes in place of D' nodes is not a dominating set.

Thus D' is not a strong co - secure dominating set. Therefore all D' is dependent.

Now to claim (ii) $D(N)\ <\ D'(N)$. Since all the dominating sets possible for this $CC(N)$ is independent and from claim 1, it is clear D' itself is not enough to secure the whole vertices in $CC(N)$. Hence $D(N)\ <\ D'(N)$.

Now to claim (iii) minimum cardinality is $2n\ +\ 1$.

On the contrary let $2N$ be the minimum cardinality of the strong co - secure dominating set. For odd N of $3N\ +\ 1$, $2N$ dependent strong dominating vertices failed to protect atmost one tetrahedron. For even N of $3N\ +\ 1$, 2N dependent strong dominating vertices failed to protect atmost 2 tetrahedrons. Our assumption failed.

Hence the minimum cardinality of the strong co-secure dominating number of N with $3N\ +\ 1$ tetrahedron is $2N\ +\ 1$.

Theorem 2.2.2: For Cyclic silicate of $N\ =\ 3N$ and $3N\ -1$, the strong co-secure domination number is $2N$, where $N\ =\ 1, 2, \ldots$

The proof followed in the order of the above theorem.

Another way of finding co - secure dominating set is the next theorem and this is for all N of cyclic silicates. The dominating sets vary large when N increases. This theorem estimates the average number of strong dominating sets needed when N is large.

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Theorem 2.2.3: $\left\lfloor \frac{3N}{4} \right\rfloor - 1 \leq \gamma_{scsd}(CC(N)) \leq \left\lfloor \frac{3N}{4} \right\rfloor$ for all N except for $CC(5)$ where $\gamma_{scsd}(CC(5)) = \left\lfloor \frac{3N}{4} \right\rfloor + 1$.

Proof: By choosing less than $\left\lfloor \frac{3N}{4} \right\rfloor - 1$ nodes, atleast one of the tetrahedron is not protected.

Observation 2.2.4: For any cyclic silicate $CC(N)$ of $N > 3$,

$$\gamma\left(CC(N)\right) < \gamma_S(CC(N)) < \gamma_{sccd}(CC(N))$$

Theorem 2.2.5: Cyclic silicate of some N ≥ 12 and for all N ≥ 18 have strong co - secure dominating set less than one than strong co-secure dominating set of N ≥ 11. ie) Increase in tetrahedrons reduce the securities by atleast one.

Proof: Claim for N ≥ 18, where

$$N = \begin{Bmatrix} 3N+1, n = 6,7,.. \\ 3N, n = 6,7,\ldots \\ 3N-1, n = 7,8,\ldots \end{Bmatrix}$$

have minimum cardinality of strong co-secure domination number is $\left\lfloor \frac{3N}{4} \right\rfloor - 1$.

Case (i) Let $N = 3N,\ 3N-1$.

For even N, we have $D' = \{v_1, v_2, v_4, v_5, v_7, v_8, \ldots\ldots\ v_{n-2}, v_{n-1}\}$ where D' is dependent implies even multiple of n vertices in D' results in $\left\lfloor \frac{3N}{4} \right\rfloor - 1$ vertices as minimum cardinality.

For odd N, we have $D' = \{v_1, v_2, v_4, v_5, v_7, v_8, \ldots\ldots\ v_{n-2}, v_{n-1}\}$ and also dependent and even number of vertices in D' results in $\left\lfloor \frac{3N}{4} \right\rfloor - 1$ vertices as minimum cardinality.

Case (ii) Let $N = 3N + 1$.

For both even and odd N, we have $D' = \{v_1, v_2, v_4, v_5, v_7, v_8, \ldots\ldots\ v_{n-2}, v_{n-1}\}$ in addition with any one node which should be adjacent to any one pair of above vertices in D' results in $\left\lfloor \frac{3N}{4} \right\rfloor - 1$ vertices as minimum cardinality. Hence claim is proved.

For some N ≥ 12, we prove by example.

Example 2.2.6:

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For $N = 12, D' = \{v_1, v_2, v_4, v_5, v_7, v_8, v_{10}, v_{11}\}$

$$ie)\ \gamma_{sccd}(CC(12)) = 8 = \left\lfloor \frac{3N}{4} \right\rfloor - 1,$$

For $N = 13, \gamma_{sccd}(CC(13)) = 9 = \left\lfloor \frac{3N}{4} \right\rfloor$

Conclusion

Nowadays, small faults on strong minimum securities results severe damage to expensive materials, products, databases etc. Wide range of applications being more sensitive to attackers and hence to rectify without any change in the process motivated us to do work on this type of domination number.

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